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A NEW APPROACH TO DYNAMICAL EVOLUTION OF INTERPLANETARY DUST

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ABSTRACT

We introduce the continuity equation written in the coordinate space of the orbital elements (e.g., semimajor axis a versus eccentricity e, etc.). This equation can serve as an effective tool to analyze the transport of interplanetary dust particles as well as their dynamical evolution, and offers a very useful complement to the approach using purely numerical integration of orbits. Using the continuity equation and suitable analytical and numerical approximations, statistically useful results can be achieved very quickly, and new integrals of the motion can be sought to simplify the description of large-scale phenomena. This paper describes the method, illustrates it with a simple example of multiple gravitational scatterings of particles on planets in circular orbits in two dimensions, and outlines the program for further development with more accurate approximations. We describe the particle dynamical evolution due to gravitational scattering by means of the "scattering matrix" $W(a, e, a', e')$ in the continuity equation. This matrix determines both the probability of transition and the value of the particle's shift from the point a, e to the point a', e'. For purposes of illustration, two cases of the zodiacal particle diffusion due to gravitational scattering are computed, which are characterized by the initial conditions (1) (asteroid case), $a_0 = 2.5$ AU, $e_0 = 0.4$ (resonance 1:3 with Jupiter), and (2) (comet case), $a_0 = 2.22$ AU, $e_0 = 0.846$ (comet Encke). We discuss the approximations of the example and how they might be improved in future work. These include treatments of the Poynting-Robertson drag, interparticle collisions, secular perturbations, three-dimensional orbits, and resonance capture by the planets. For instance, analytical expressions are available for the rates of gradual change of orbital elements due to the Poynting-Robertson and solar wind drags, which can be incorporated easily in the "div" terms of the continuity equation.

Subject headings: infrared: solar system — interplanetary medium

1. INTRODUCTION

Interplanetary dust, the existence of which was revealed long ago by the zodiacal light, has become a subject of extensive studies since the discovery of the infrared zodiacal emission. Interpretation of the zodiacal light as a scattering process suffers from poor knowledge of the basic scattering parameters. The zodiacal emission is reradiation, which offers broad opportunities for understanding both the spatial distribution and the physical parameters of the grains. Progress in these fields could open new prospects for analyzing the dynamics and origins of the interplanetary particles. During the last decade or so, this was a strong motivation for an extensive theoretical analysis of the dynamical evolution of the interplanetary dust.

The problem of dynamical evolution of interplanetary particles aims at finding the number density of the particles, $n(r, t)$, as a function of appropriate coordinates and time. As is known, four basic effects determine that evolution: (1) the Poynting-Robertson (PR) drag; (2) resonance effects associated chiefly with Jupiter, Mars, Earth, and Venus; (3) gravitational encounters with these planets, which occur in the form of elastic gravitational scattering of the particles by the planets; and (4) mutual collisions of the particles (for reviews see, e.g., Dermott et al. 1993; Marzari & Vanzani 1994; Liou, Zook, & Jackson 1995). While extensive numerical simulations (mostly employing the Monte Carlo technique) have so far remained the basic tool for studying these complex phenomena, it would be of great interest to combine them with analytical approaches. This paper offers and explores such an approach.

A difficult problem in tackling these phenomena analytically is that effects 1 and 2 result in a *continuous* motion of particles in the phase space, which can be described by a velocity at each point in the phase space, whereas effects 3 and 4 produce practically instantaneous "*jumps*."

A convenient tool to describe both the continuous motions and the jumps is the continuity equation. In this paper, we outline a new approach to the dynamical evolution of interplanetary particles based on both analytical and computational analysis of the continuity equation. As a particular illustration of our approach, we analyze the gravitational scattering problem. The latter is of interest for a number of applications, including the evolution of a cometary clouds (discussed in this paper), moon systems, and planetesimal swarms in protoplanetary systems.

The paper is organized as follows: In § 2 we discuss the basic equation of the problem. In § 3 we consider in detail gravitational scattering of dust particles by the planets.

Section 4 deals with the computational analysis of two particular cases of particle dynamics when the particles are of either asteroidal or cometary origin. In § 5 we summarize our results and discuss the underlying assumptions as well as further work to relax the limitations. Some of our results were presented previously at IAU Colloquium 150 (Gor'kavyi et al. 1996).

2. BASIC EQUATION

For a given size of dust particles, the distribution function is a function of six additional variables: semimajor axis, eccentricity, inclination to the ecliptic, time of perihelion passage, direction of the perihelion, and direction of the ascending node. In our approach, instead of dealing with the motion of single particles, we explore their dynamics hydrodynamically by using the transport equation written in the form of the continuity equation:

$$\frac{\partial n(x_i, t)}{\partial t} + \frac{\partial (n v_i)}{\partial x_i} = N^+(x_i, t) - N^-(x_i, t) , \qquad (1)$$

where $i = 1, \ldots, 6$, and v_i is velocity along the ith axis in the phase space.

The continuity equation governs the density evolution of the entire cloud of dust particles in the coordinate space of the orbital elements. In the above equation, the "div" term $\partial(nv_i)/\partial x_i$ describes *slow* processes of change of particle orbital elements, such as the particle transport due to the Poynting-Robertson drag, resonance effects, and secular perturbations of the angular variables by the planets. Available analytical expressions for the rates v_i (i.e., $\partial a/\partial t$, $\partial e/\partial t, \ldots$) due to the PR-drag (as well as due to the solar wind drag in the presence of the radiation pressure) have been recently discussed by Liou et al. (1995). In the present paper, we emphasize the terms $N^+(x_i, t)$ and $N^-(x_i, t)$, which are responsible for *fast* processes such as the gravitational scattering of particles by the planets and the contact particle collisions (both mutual and with the planets). The term $N^+(x_i, t)$ accounts for the "birth" of particles at the given point x_i due to the jumps from the other points, and $N^-(x_i, t)$ accounts for the "death" of particles due to their escape from x_i.

The Poynting-Robertson effect can be accounted for analytically; mutual collisions can also be described analytically in the framework of a two-body problem. Meanwhile, accounting for resonance effects as well as for gravitational scattering requires computations in the framework of (at least) a three-body problem. Therefore, an analysis of equation (1) with all four effects included would be very complicated. We have chosen the gravitational scattering to be a subject of this analysis. Indeed, while the resonance effects have been discussed in numerous papers (see Dermott et al. 1994; Marzari & Vanzani 1994; Liou et al. 1995; and references therein), there is a lack of detailed analysis of gravitational scattering effects. The next section describes our approach to this problem.

3. GRAVITATIONAL SCATTERING

The problem of gravitational scattering of particles on planets is quite general. It is of importance for a number of astronomical applications, including the evolution of a cometary cloud, the asteroidal belts, and planetesimals in the protoplanetary disk. Below we consider the process of gravitational scattering of the dust particles in the simplest two-dimensional case, i.e., considering particles with zero inclination and neglecting the resonances.

3.1. *Equations*

For particles with zero inclination and a specified size, the distribution function is a function of four additional variables: semimajor axis, eccentricity, time of perihelion passage, and direction of the perihelion. One can simplify the equations further by integrating the density over two of the independent variables, time of perihelion passage and direction of the perihelion. This integration leads to a probabilistic treatment, since it ignores long-term coherent phenomena, but the simplified problem still includes many interesting effects. Then the dynamical evolution of the dust particles due to gravitational scattering by the planets is described by the equation

$$\frac{\partial n(a, e)}{\partial t} = N^+(a, e) - N^-(a, e) . \qquad (2)$$

For direct scattering with the planets due to close encounters,

$$N^+(a, e) = \int W(a', e', a, e) n(a', e') da' \, de' , \qquad (3)$$

where $W(a', e', a, e)$ is the probability of particle transfer from the point (a', e') to the point (a, e) by a jump. The value of $N^-(a, e)$ is given by

$$N^-(a, e) = P(a, e) n(a, e) , \qquad (4)$$

where

$$P(a, e) = \int W(a, e, a', e') da' \, de' \qquad (5)$$

is the integral probability of the particle escaping, due to the gravitational scattering, from the point (a, e) to the point (a', e').

Gravitational scattering requires a sufficiently close encounter between the particle and the planet. The necessary condition for such an encounter is given by

$$a \le \frac{a_P}{1 - e} \quad \text{if } a > a_P , \qquad a \ge \frac{a_P}{1 + e} \quad \text{if } a < a_P , \qquad (6)$$

where a_P is the semimajor axis of a planet. The regions described by equation (6) are shown, for all the inner planets, in Figure 1. It can be seen that the particles able to be scattered by a planet belong to the triangle-like region with the vertex on the planet.

3.2. *The Tisserand Criterion*

Gravitational scattering of a dust particle by a planet, being an elastic process, keeps the total energy of the particle constant in a coordinate system moving with the planet. As a result, the dynamical trajectory of the particle in the (a, e)-space is described by the Tisserand criterion (e.g., Roy 1978):

$$\frac{1}{2}\left(\frac{a}{a_P}\right)^{-1} + \sqrt{\frac{a}{a_P}(1 - e^2)} \cos i = C , \qquad (7)$$

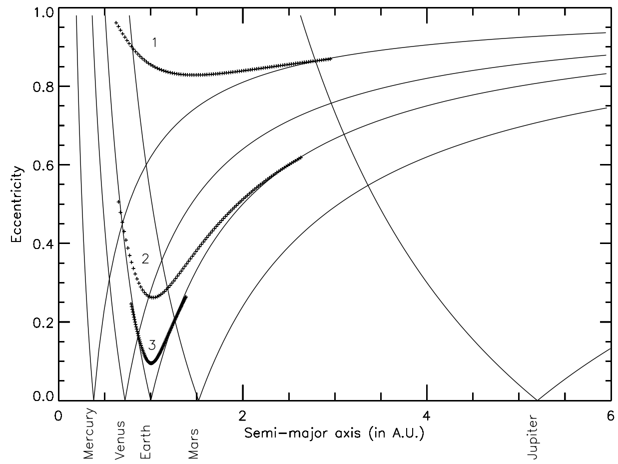

FIG. 1.—Regions of gravitational scattering of particles around Jupiter and the inner planets. Lines denote the boundaries of these regions. Labels at the Tisserand criterion curves for Earth correspond to the following initial conditions: (1) $a_0 = 2.22$, $e_0 = 0.846$ (comet Encke); (2) $a_0 = 2.5$, $e_0 = 0.6$ (a 1:3 resonance with Jupiter); (3) $a_0 = 1.26$, $e_0 = 0.205$.

where cos $i = 1$ in our two-dimensional consideration, and C is a constant related to the Tisserand parameter T by $2C = T$. In a fast process such as gravitational scattering, C can be taken to be a constant. The use of the Tisserand criterion is very helpful, since it allows reducing the number of computer-calculated orbital elements of particles by 1.

In practice, it is much more convenient to analyze gravitational scattering of particles in the (C, a)-space rather than in the (a, e)-space. In this case, the scattered particles are stretched along the straight line C = constant, which is much more convenient from a computational point of view than dealing with the curved lines in the (a, e)-space. It is even more convenient to introduce (J, a)-space, where

$$J = \sqrt{3 - 2C} \tag{8}$$

changes within the range $0 < J < 3^{1/2}$ when e runs in the range $0 < e < 1$. In the (J, a)-space the gravitationally scattered particles are distributed rather uniformly, and an inverse transition to the (a, e)-space is more straightforward.

The Tisserand criterion yields the trajectory of a particle in the (a, e)-space or in the (J, a)-space but not the probability of gravitational scattering from a given point to another point. Those values can be determined numerically using equations (2)–(5). When we deal with scattering of particles in the (a, e)-space or in the (J, a)-space, the variables in equations (2)–(5) are J and a, so that one needs only to take the integrals in a and not in e. The price we pay for that is that one needs to account for both "birth" and "death" of all the particles in the entire (J, a)-space and to deal with several matrices W, the number of which is the number of significant planets as sources of gravitational scattering.

3.3. *Computations*

We introduce a coordinate system centered on the Sun and rotating with the Earth (Fig. 2). The initial position of the particle is characterized by the line of apsides that has an angle θ_0 with the Sun-Earth line. The initial angle θ_0 is chosen in such a way that it corresponds to the maximal scattering by the Earth.

As a result of the first scattering by Earth, the particle jumps from the point (a_0, e_0) to the point (a_1, e_1). For $\theta \neq \theta_0$, we get different sets of points, which form a path between the points (a_0, e_0) and (a_1, e_1) on the curve shown in Figure 1. The evolutionary track of the dust particle turns out to lie very close to the analytical curve given by the Tisserand criterion (eq. [7]).

We now consider the evolution of dust particles in the (J, a)-space. Diffusion in this space of dust particles from comet Encke due to gravitational perturbations from all the inner planets is shown in Figure 3. Particles with initial $a_0 = 2.22$ and $e_0 = 0.846$ are scattered on Mars, Earth, Venus, and Mercury along the lines A, B, C, and D, respectively, which correspond to $J = 1.0132$, 0.9802, 0.8978, and 0.5215. As can be seen from equations (7) and (8), whenever a, e, and i are fixed, J only depends on the semimajor axis of the planet.

In order to determine the probability of the jump from the point (a_0, e_0) to the region between the points (a_1, e_1)

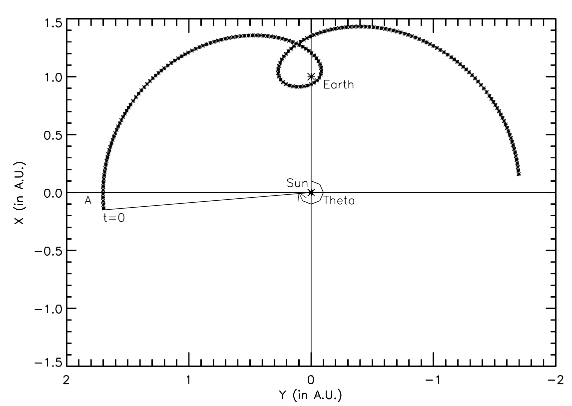

FIG. 2*a*

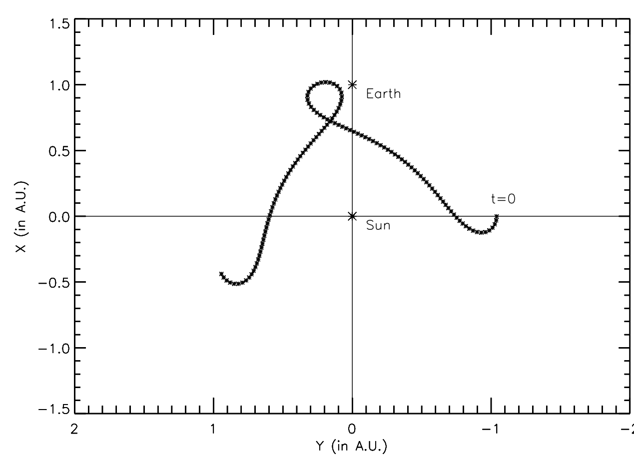

FIG. 2*b*

FIG. 2.—The trajectory of a particle orbiting around the Sun in the frame centered on the Sun and corotating with the Earth's mean motion. Changes in the particle's position in the (a, e)-space are due to the gravitational perturbations from Earth. (*a*) $a_0 = 1.31$ (near a 2:3 resonance), $e_0 = 0.3$. Shown for one revolution of the particle. The angle between the x-axis and the line (0, A) serves as an independent variable in the analytical function (eq. [9]) that approximates the scattering. Generally the maximal scattering occurs at two possible angles (at one angle for tangential orbits). (*b*) $a_0 = 0.80$, $e_0 = 0.3$. Shown for two revolutions of the particle.

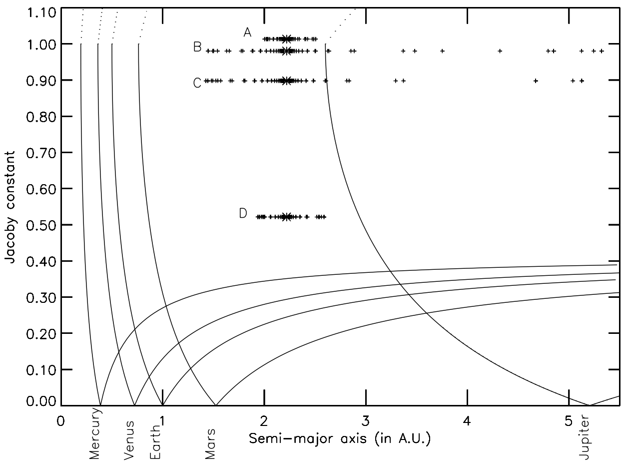

FIG. 3.—Regions in the (J, a)-space for gravitational scattering of comet Encke particles due to Jupiter and the inner planets. Particles with initial $a_0 = 2.22$, $e_0 = 0.846$ from comet Encke are scattered by Mars, Earth, Venus, and Mercury (lines *A*, *B*, *C*, and *D*, respectively). Dotted lines show the boundaries of $e = 1$ for every planet: $e > 1$ to the left side of the boundary (therefore, particles would leave the solar system), and $e < 1$ to the right side of the boundary (therefore, particles remain here).

and $(a_1 + \Delta a, e_1 + \Delta e)$, one finds the corresponding interval of angles from θ_0 to $\theta_0 + \Delta\theta$. The probability of this jump is $W = \Delta\theta/360°$.

In this example the problem has been simplified by the assumption that the phase-space density is independent of the angular variables. In the case of the zodiacal dust particles this might be expected to be correct, since the PR drag changes the phase angles quickly. Also, the real particles have a wide range of sizes, and the PR drag affects them all differently. Exceptions clearly occur for resonance phenomena, but we neglect them in this example. The details of the scattering events can be approached analytically using an impact parameter formulation, or they can be done numerically as outlined below.

For convenience of computations, we replace W as a continuous function by a matrix with a number of rows N_r and columns N_c. The values of N_r and N_c are functions of the required accuracy, $(\delta a, \delta e)$, for the determination of the particle's orbital elements: $N_r \sim 1/\delta e$, $N_c \sim \Delta R/\delta a$, where $\Delta R \sim$ a few AU is the region containing the bulk of the zodiacal particles.

Formally, the total number of elements in the matrix W is $N_r^2 N_c^2$, i.e., very large. However, in practice there is no need to compute all the matrix elements in advance: it is necessary to compute those (and only those) matrix elements that contain the positions of the particles after the scattering. Therefore, the result of computations of the particle evolutionary tracks determines the elements of the matrix W. In other words, this matrix turns out to be self-filling.

The matrix W is written in the form of a computer file that contains the complete information about the fate of a particle. A typical structure of such a file is shown in Table 1.

TABLE 1

A SAMPLE OF THE DATA FILE WRITTEN FOR THE (a, e)-SPACE: a_0, e_0 $(t = 0) = 2.5, 0.60$

a_j, e_j ($t = 1$ revolution)	$W(a_0, e_0, a_j, e_j)$ (%)	Comments
2.5, 0.60	97	No scattering
2.4, 0.58	1.8	
2.3, 0.57	0.9	
2.2, 0.55	0.2	
2.1, 0.53	0.1	
2.0, 0.51	2×10^{-2}	
1.9, 0.49	5×10^{-3}	
1.8, 0.47	10^{-4}	
1.7, 0.44	10^{-5}	
1.6, 0.41	10^{-6}	
1.5, 0.38	2×10^{-7}	Maximal scattering
1.0, 0.00	4×10^{-7}	Capture to Earth

4. NUMERICAL RESULTS

4.1. *Approximations and Simplifications*

To illustrate how our method works, we consider two examples of the evolution of particles whose initial positions in the (a, e)-space are given as follows:

Case A: $a_0 = 2.5$, $e_0 = 0.4$, which corresponds to inflow of particles into the Mars zone from the 1:3 resonance with Jupiter.

Case B: $a_0 = 2.22$, $e_0 = 0.846$, which corresponds to inflow of particles from comet Encke.

Since our purposes are just illustrative, it is reasonable to simplify the problem as much as possible: The orbits of all the planets are taken to be circular; the problem is two-dimensional ($i = 0$ for both the particles and the planets); the effect of solar radiation pressure and the resonance effects are neglected; an accuracy of determination of the particle's coordinates in the (a, J)-space is $\delta a, \delta J = 0.01$.

As a rule, the change in the particle's semimajor axis due to gravitational scattering is a smooth function of an initial angle θ_0 (for the meaning of θ see Fig. 2). This enables us, by using a series of several tens of calculated orbits, to approximate the value of a after the scattering via an analytical function $\mathscr{F}(\theta_0)$:

$$a = \mathscr{F}(\theta_0) = a_0 + \frac{A}{\theta - \theta_0} + \frac{B}{(\theta - \theta_0)^2}, \qquad (9)$$

where a_0 is an initial (before scattering) semimajor axis of the particle, θ_0 is an initial angle, and the coefficients A, B, and θ are to be determined from the computations. All the information about the gravitational scattering of the particle is contained in a compact form in the data file, the structure of which can be seen from Table 2.

As can be seen from Table 2 and equation (9), a strong scattering at which $\Delta a \sim 1$ occurs, for scattering by the Earth, in the range $\Delta\theta \sim A_1/\Delta a \sim A_1$, which implies the probability of such scattering $W \sim \Delta\theta/360° \sim 2 \times 10^{-5}$. This requires about $5 \times 10^4 T_p$ yr, where T_p (in years) is the period of the particle orbit. In turn, a change in a by $\Delta a = 0.01$ AU requires about $500 T_p$ yr. For scattering by Mars, the corresponding evolutionary changes occur a factor of 10 more slowly.

4.2. *Case A* (*Asteroid Case*)

Figure 4 shows the scattering of asteroidal particles with initial $a_0 = 2.5$, $e_0 = 0.4$ from the main asteroid belt that lies in the region of the 1:3 resonance with Jupiter. Particles are transported into this region due to a resonance with Jupiter. During the early stages of the solar system's formation, this diffusion could produce one of the Kirkwood gaps.

TABLE 2

A SAMPLE OF THE DATA FILE WRITTEN FOR THE (J, a)-SPACE

1. Label of planet	3 (Earth)	4 (Mars)
2. Running number of crossing of planetary orbit by particle	1	1
3. Initial a (AU)	1.20	2.00
4. Constant J	0.1500	0.2215
5. Initial angle at which particle will meet center of planet[a] (deg)	242.0799	279.2716
6. Initial angle for flyby at which trajectory is tangential to planet (deg)	242.0722	279.2704
7. a after the above scattering (AU)	1.3632	2.6645
8. Initial angle for maximal scattering (deg)	242.0719	279.2704
9. a after the above scattering (AU)	1.3759	2.6645
10. Coefficients of $\mathscr{F}$ for this scattering (AU):		
A_1	0.66617E−2	0.87653E−3
B_1	−0.40225E−4	−0.21266E−7
11. Same as item 6 above for the opposite side of the planet	242.0875	279.2731
12. a after the above scattering (AU)	0.7941	1.4912
13. Same as item 8 for the opposite side of the planet	242.0875	279.2731
14. Minimal a after the above scattering (AU)	0.7941	1.4912
15. Coefficients of $\mathscr{F}$ for this scattering (AU):		
A_2	0.70854E−2	0.89803E−3
B_2	0.31701E−4	0.20263E−6

[a] This angle is used as θ in eq. (9).

From their initial position (label 0), the particles undergo the first, second, and third scatterings by Mars (labels 1, 2, and 3 denote the positions of the corresponding maximal scattering). From every point (including those labeled by 0, 1, and 2) the particles can be scattered both to and away from the Sun. Figure 4 shows that the semimajor axes of particles are bounded by a barrier at the point of the intersection of the Tisserand line and the right boundary of the region of particles' gravitational scattering (see eq. [6]). This point is located at 2.8 AU. After the third scattering by Mars, a small fraction of the particles lies in the region of Earth's gravitational influence. The arrows show the directions of the evolutionary tracks due to the first gravitational scattering by Earth: the particles are quickly spread out within this region. Observations of the so-called AAA-group of asteroids indicating the presence of numerous asteroidal particles near the Earth are plotted in Figure 5. An appreciable part of the zodiacal cloud includes particles from the main asteroid belt. It is tempting to speculate that those particles could be transferred into the cloud in the way just discussed.

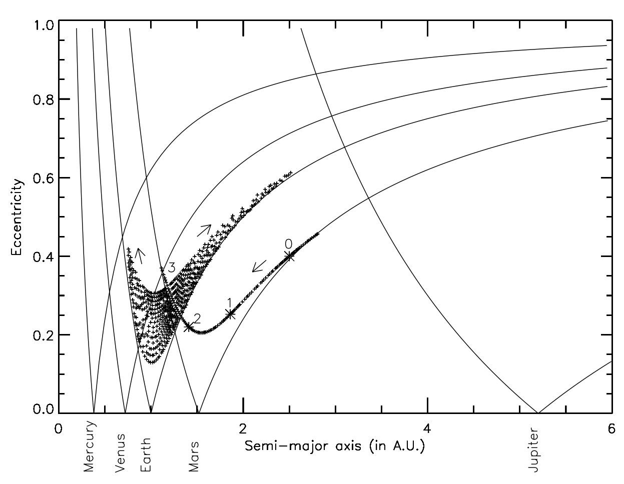

FIG. 4.—Stages of scattering of particles from a resonance region (a 1:3 resonance by Jupiter) with initial $a_0 = 2.5$, $e_0 = 0.4$. Zero labels an initial position; 1, 2, and 3 are the points of the first, second, and third maximal scattering by Mars. After the third scattering, the particles reach the Earth's gravitational influence region. Arrows show the evolutionary tracks of these particles due to the first scattering by Earth.

4.3. *Case B* (*Comet Case*)

Figure 6 shows several stages of scattering of particles from comet Encke ($a = 2.22$, $e_0 = 0.846$) in the (a, e)-space.

Stage A.—Comet Encke particles are scattered by all the inner planets. After scattering by Venus and Earth, the particles reach Jupiter's region of gravitational influence.

Stage B.—After getting into Jupiter's region, the particles are scattered by the powerful gravitational field of Jupiter.

Stages C and D.—Stage C stands for the next scattering path of the comet Encke particle due to perturbations from Venus, and stage D shows that due to perturbations from Earth.

To sum up, Figure 6 illustrates one of the most effective ways of the dynamical evolution of cometary particles due to gravitational scattering by Jupiter and the inner planets. It is of interest to compare these results with observations indicating the presence of numerous meteor streams (Cook 1973), fireballs (Terentjeva 1990), and single meteors (Terentjeva 1966); available data on meteors are plotted in

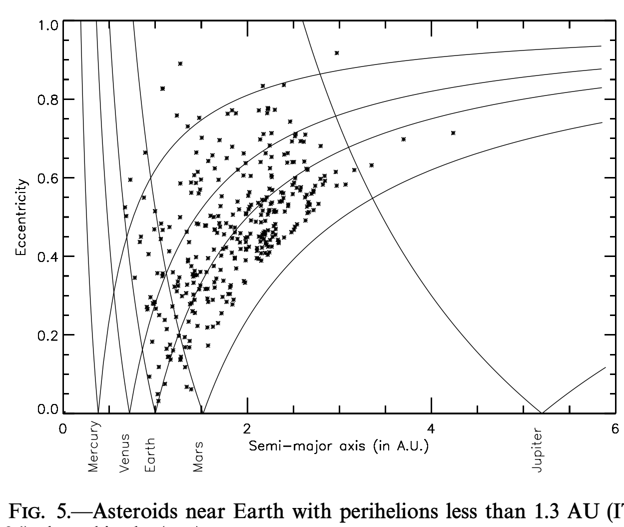

FIG. 5.—Asteroids near Earth with perihelions less than 1.3 AU (ITA 1994) plotted in the (a, e)-space.

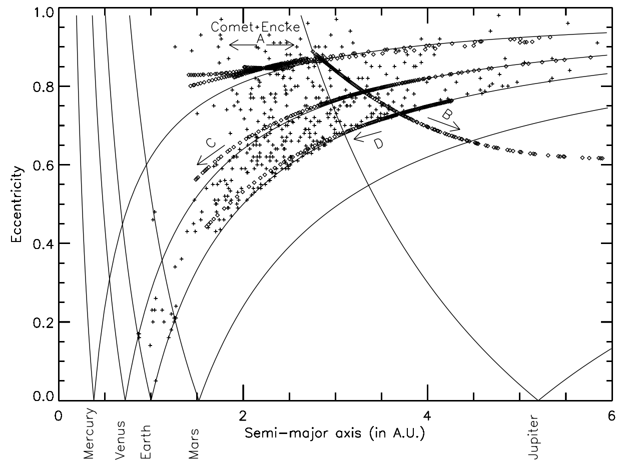

FIG. 6.—Three stages of the scattering of comet Encke particles with initial $a_0 = 2.22$, $e_0 = 0.846$ in the (a, e)-space. Computer-calculated particle orbits are shown by open diamonds. First stage (labeled A): scattering due to perturbations from all four inner planets. Second stage (labeled B): scattering by Jupiter. Third stage (labeled C): the next scattering path due to perturbations from Venus. D labels the next scattering path of the particles due to gravitational scattering by Earth. Crosses show meteor orbits from photographic, radar, and visual observations (Terentjeva 1966).

Figure 6. The neglect of radiation pressure effects on dynamical evolution in the above computations is well justified, since the β-parameter of solar pressure for meteor bodies with radii about 0.01–0.1 cm is very small: $\beta = 0.0026$–0.00026 (Liou et al. 1995).

5. CONCLUSIONS AND DISCUSSION

In this paper, our purpose has been to introduce an innovative use of the transport equation, to show that it can achieve statistically useful results very quickly, and to explore its approximations.

Our approach to the dynamical evolution of interplanetary particles makes wide use of the transport equation, for which we employ *the continuity equation* written in the (semimajor axis, eccentricity)-space or related spaces. This approach, a basically hydrodynamical one, permits powerful simplifications and approximations that preserve many interesting phenomena, and allows physically intuitive descriptions. The main results are the following.

1. The continuity equation makes it possible to describe both the *continuous* motion of particles in the phase space (due to the Poynting-Robertson drag and the resonance effects) and *jumps* (due to particle collisions and gravitational scattering of particles by the planets).

2. The introduction of the "*scattering matrix*" W enables one to solve in a convenient way a number of problems related to the dynamical evolution of the zodiacal particles due to gravitational scattering.

3. The evolutionary tracks of scattered particles in the (a, e)-space *are located very close to the Tisserand criterion curves.* Our numerical computations yield *the rate of the evolution* along the Tisserand curves.

4. Our codes compute the elements of the scattering matrix W that describes the transport of interplanetary particles in the (a, e)- or (J, a)-space.

5. *Trial computations* of the dynamical evolution of the cometary and asteroid particles *demonstrate a high efficiency of this approach.* Scattering of an element of (J, a)-space into all other points of this space due to gravitational perturbations by one planet takes a few tens of seconds of CPU time of a DX4/100 MHz processor.

So far, a Monte Carlo method has actually been the only effective method used for studying the dynamical evolution of the dust cloud with all its complexities. The semianalytic approach we propose has many advantages compared with the Monte Carlo approach. The computations can be much more rapid and less expensive, and the results more general, although the accuracy of the analytical approximations must be carefully confirmed. For example, a recent numerical approach (Liou et al. 1995) deals with the evolution of only 800 particles on a timescale of only 20,000 yr. On such a short timescale, the effects of the rare but very effective events of gravitational scattering on the planets are not revealed at all. By contrast, our approach includes these effects and shows that they are quite important on large timescales, as appropriate for relatively large particles.

The numerical calculations we performed serve as an illustration to our analytical approach and can be appreciably improved in further applications. Therefore, instead of discussing whether the numerical results outlined in § 4 could be considered as more than just an illustration and might have possible astronomical applications, we discuss the approximations and how they could be relaxed in further work.

Neglecting the PR drag.—This effect tends to slowly decrease the semimajor axis and eccentricity of the particle's orbit. As particles spiral in to the Sun, they can make close approaches to the planets and make large jumps in their orbit parameters. However, the more rapid the PR motion, the shorter the time available and the smaller the probabilities of jumps. For jumps to be important, the time of the orbit evolution due to the PR drag should be much larger than the time of the orbit periods, which implies that the particle's size should not be too small.

Since the PR drag is a slow process, it can easily be incorporated in further analysis. Indeed, success in calculating both the matrix W and the terms $N^+(x_i, t)$ and $N^-(x_i, t)$ in the continuity equation makes it possible to solve the continuity equation for the density of the zodiacal particles in more sophisticated conditions. This is done by incorporating the "div" term $\partial(nv_i)/\partial x_i$ that describes the particle transport due to the PR drag, solar wind, etc. Convenient analytical expressions for the rates of gradual change of orbital elements due to the PR and solar wind drags have recently been discussed by Liou et al. (1995).

Neglecting resonances.—We integrated the density function over two variables, yielding a statistical description of the flow. This approach neglects coherent long-term phenomena like resonances, which must then be included as separate "div" terms. In further work, we will include them.

Numerical approximations.—The major approximation was to replace $W(a_k, e_k, a_l, e_l)$, which is a continuous function, by a discrete matrix with a step $(\Delta a, \Delta e)$; the smaller the step, the more accurate the results. Furthermore, in the above analysis we did not make a distinction between the nonresonant and the resonant particles. Meanwhile, a particle coming to a resonant orbit will have a smaller probability of subsequent gravitational scattering by the planet with which it is in a resonance. This results in a further accumulation of the particles on the resonant orbits. Such resonant capture will be included in our future work. Another example of a resonance effect to be included is an increase of the eccentricity of a particle captured into the resonance. This process stops when the particle is ejected out of the resonance while being gravitationally scattered on one of the planets.

Neglecting particle-particle collisions.—Each collision results in a jumplike decrease of the particle's eccentricities. This process can be incorporated by introducing the "collision function," like the above scattering function (employed as scattering matrix), that describes the transition of both particles into new positions in the (a, e)-space.

Zero particle inclinations and circular orbits of the planets.—Our two-dimensional approximation is just a convenient simplification: (1) it represents a training ground to serve as a basis for making, in our further work, the next step to the three-dimensional case; (2) it serves as a particular case while testing future three-dimensional approximations; and (3) (last but not least) it serves as a simple and convenient pedagogical illustration to be easily understood. The nonzero inclinations of particles and the orbits of the planets as well as eccentricities of the planetary orbits will be accounted for in our future work. The proposed approach easily allows for inclusion of all the planets with important effects.

In summary, there are serious reasons to believe that our method has considerable promise to cut through the complexity of previous work, both analytic and Monte Carlo. The continuity equation with integration over some of the variables yields a statistical description of the dynamical evolution of the interplanetary dust. The complications are included in scattering matrices and drag terms, which can be calculated quickly with an interesting level of accuracy. The method is well suited to the study of large-scale properties of the dust cloud.

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