AN OBSERVATIONAL IMPRINT OF THE COLLAPSAR MODEL OF LONG GAMMA-RAY BURSTS

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ABSTRACT

The Collapsar model provides a theoretical framework for the well-known association between long gamma-ray bursts (GRBs) and collapsing massive stars. A bipolar relativistic jet, launched at the core of a collapsing star, drills its way through the stellar envelope and breaks out of the surface before producing the observed gamma rays. While a wealth of observations associate GRBs with the death of massive stars, as yet there is no direct evidence for the Collapsar model itself. Here we show that a distinct signature of the Collapsar model is the appearance of a plateau in the duration distribution of the prompt GRB emission at times much shorter than the typical breakout time of the jet. This plateau is evident in the data of all three major satellites. Our findings provide evidence that directly supports the Collapsar model. In addition, the model suggests the existence of a large population of choked (failed) GRBs, and implies that the 2 s duration commonly used to separate Collapsars and non-Collapsars is inconsistent with the duration distributions of *Swift* and *Fermi* GRBs and only holds for BATSE GRBs.

Key words: gamma-ray burst: general – gamma rays: stars – stars: Wolf-Rayet

Online-only material: color figure

1. INTRODUCTION

There is a long line of evidence connecting long gamma-ray bursts (GRBs) to collapsing massive stars (for recent reviews see Woosley & Heger 2006; Hjorth & Bloom 2011). Among it is the association of half a dozen GRBs with spectroscopically confirmed broad-line Ic supernovae (SNe), and the identification of "red bumps" in the afterglows of about two dozen more, which shows photometric evidence of underlying SNe. On top of that, the identification of long gamma-ray burst (LGRB) host galaxies as intensively star-forming galaxies and the localization of the LGRBs in the most active star-forming regions within those galaxies (Bloom et al. 2002; Le Floc'h et al. 2003; Christensen et al. 2004; Fruchter et al. 2006) provides indirect evidence for the connection of LGRBs with massive stars. The model that provides the theoretical framework of the LGRB-SN association is known as the Collapsar model (MacFadyen & Woosley 1999; MacFadyen et al. 2001). According to this model, following the core collapse of a massive star, a bipolar jet is launched at the center of the star. The jet drills through the stellar envelope and breaks out of the surface before producing the observed gamma rays. However, although this model is supported indirectly by the LGRB-SN association, to date we could not identify a clear direct observational imprint of the jet-envelope interaction; thus, there is no direct confirmation yet of the Collapsar model.

In this paper, we analyze the expected duration distribution of the prompt GRB emission from Collapsars. We show that under very general conditions the time that the jet spends drilling through the star leads to a plateau in the duration distribution at times much shorter than the breakout time of the jet. We examine the duration distribution of all major GRB satellites and find an extended plateau in all of them over a duration range that is expected for reasonable jet-star parameters. We interpret these plateaus as the first identified imprint of the Collapsar model. Under this interpretation, these findings (1) support the hypothesis of compact stellar progenitors, (2) imply the existence of a large population of choked jets that fail to break

out of the progenitor star, and (3) enable us to determine the transition duration that statistically separates Collapsar from non-Collapsar bursts and to show that this time is individual for each satellite. The separation between Collapsars and non-Collapsars is quantified and discussed in length in O. Bromberg et al. (in preparation). The present paper is structured as follows. In Section 2, we briefly review the propagation of a jet inside a star and the duration of the prompt GRB emission. In Section 3, we analyze the expected duration distribution of the Collapsars. We compare the expected distribution with the observed one in Section 4, and, finally, we discuss our findings in Section 5.

2. THE PROPAGATION OF A JET IN THE STELLAR ENVELOPE

As the jet propagates in the stellar envelope, it pushes the matter surrounding it, creating a "head" of shocked matter at its front. For typical luminosities, the head remains subrelativistic across most of the star (Matzner 2003; Zhang et al. 2003; Morsony et al. 2007; Mizuta & Aloy 2009; Bromberg et al. 2011b), even though the jet is ejected at relativistic velocities. The jet breaks out of the stellar surface after (Bromberg et al. 2011a)

$$t_b \simeq 15 \text{ s} \cdot \left(\frac{L_{\text{iso}}}{10^{51} \text{ erg s}^{-1}}\right)^{-1/3} \left(\frac{\theta}{10^{\circ}}\right)^{2/3} \left(\frac{R_*}{5 R_{\odot}}\right)^{2/3} \times \left(\frac{M_*}{15 M_{\odot}}\right)^{1/3},$$
 (1)

where $L_{\rm iso}$ is the isotropic equivalent jet luminosity and θ is the jet half-opening angle, and we have used typical values for an LGRB. R_* and M_* are the radius and the mass of the progenitor star, where we normalize their values according to the typical radius and mass inferred from observations of the few SNe associated with LGRBs. While the jet propagates inside the star, the head dissipates most of its energy and the engine must continuously power the jet to support its propagation. For a successful jet breakout, the engine working time, t_e , must be

larger than the breakout time, t_b . If $t_e < t_b$, the jet fails to escape and a regular LGRB is not observed.

Once the jet breaks out from the star, it produces the observed emission at large distances from the stellar surface. Because of relativistic effects (Sari & Piran 1997), the observed duration of the prompt γ -ray emission, t_{γ} , reflects the time that the engine operates after the jet breaks out:³

$$t_{\gamma} = t_e - t_b. \tag{2}$$

The distribution of t_{γ} is therefore a convolution of the distributions of the engine activity time and the breakout time combined with cosmological redshift effects.

3. THE DURATION DISTRIBUTION OF COLLAPSARS

Under very general conditions, Equation (2) results in a flat distribution of t_{γ} for durations significantly shorter than the typical breakout time. To see it, consider first a single value of t_b and ignore, for simplicity, cosmological redshift and detector threshold effects. The probability that a GRB has a duration t_{γ} equals, in this case, to the probability that the engine work time is $t_{\gamma} + t_b$, namely,

$$p_{\gamma}(t_{\gamma})dt_{\gamma} = p_{e}(t_{b} + t_{\gamma})dt_{\gamma}, \qquad (3)$$

where p_{γ} is the probability distribution of observed durations and p_e is the probability distribution of engine working times. Expanding $p_e(t_b + t_{\gamma})$ around $t_e = t_b$ gives

$$p_e(t_b + t_\gamma) = p_e(t_b) + \mathcal{O}\left(\frac{t_\gamma}{t_b}\right). \tag{4}$$

Thus, $p_e(t_b + t_\gamma) \approx p_e(t_b) = \text{constant}$ for any t_γ that is sufficiently smaller than t_b . Moreover, if $p_e(t_e)$ is a smooth function that does not rapidly vary in the vicinity of $t_e \approx t_b$ over a duration of the scale of t_b , then the constant distribution is extended up to times $t_\gamma \lesssim t_b$. In the case of interest t_b and t_e are determined by different regions of the star: the breakout time is set by the density and radius of the stellar envelope at radii $> 10^{10}$ cm, while t_e is determined by the stellar core properties at radii $< 10^8$ cm. The core and the envelope are weakly coupled (Crowther 2007) and the engine is unaware of whether the jet has broken out or not. Therefore, it is reasonable to expect $p_e(t_e)$ to be smooth in the vicinity of $t_e \approx t_b$ and $p_\gamma(t_\gamma) \approx \text{constant}$ for $t_\gamma \lesssim t_b$. In the opposite limit, where $t_b \ll t_\gamma$, then $t_\gamma \approx t_e$, and Equation (3) reads

$$p_{\gamma}(t_{\gamma}) \approx \begin{cases} p_{e}(t_{b}) & t_{\gamma} \lesssim t_{b} \\ p_{e}(t_{\gamma}) & t_{\gamma} \gg t_{b} \end{cases} . \tag{5}$$

In reality, we expect t_b to vary from one burst to another due to a variety of progenitor sizes and masses and a scatter in the jet properties. Moreover, effects such as the dependence of the break time on the jet luminosity may introduce correlations between t_e and t_b . Therefore, we consider next a distribution of breakout times, $p_b(t_b)$, which can generally be correlated with $p_e(t_e)$ (we still ignore cosmological redshift and detector threshold effects). Since the stellar and jet properties are bounded (e.g., there is a minimal progenitor size and maximal jet luminosity), it implies that, within the entire population

of bursts, there is a minimal breakout time, $t_{b,min}$. Then, Equation (3) is modified to

$$p_{\gamma}(t_{\gamma})dt_{\gamma} = dt_{\gamma} \int_{t_{b,\min}} p_b(t_b) p_e(t_b + t_{\gamma}|t_b) dt_b, \tag{6}$$

where $p_e(t|t_b)$ is the density distribution of t_e given that the breakout time is t_b . In this case, all the arguments presented above for a single breakout time hold for each value of t_b independently. Therefore, each distribution $p_e(t|t_b)$ has a plateau at times $t \lesssim t_b$ with a normalization $p_b(t_b)p_e(t_b|t_b)dt_b$. It can readily be seen that, for $t_\gamma \lesssim t_{b,\text{min}}$, a plateau exists in all the distributions of engine work times, implying that the right-hand side of Equation (6) can be written as an integration over plateaus with different normalization:

$$p_{\gamma}(t_{\gamma} \lesssim t_{b, \min}) \approx \int_{t_{b, \min}} p_b(t_b) p_e(t_b|t_b) dt_b = \text{constant.}$$
 (7)

Thus, p_{γ} has a plateau at times $t_{\gamma} \lesssim t_{b,\,\mathrm{min}}$. For any reasonable distribution of p_b there is a typical value \hat{t}_b , which dominates the contribution to the integral, namely, bursts with $t_{\gamma} < t_{b,\,\mathrm{min}}$ are dominated by events with $t_b \sim \hat{t}_b$. When p_{γ} is a monotonically decreasing function, as seen in the case of GRBs, then bursts with $t_b \sim \hat{t}_b$ dominate p_{γ} for any $t_{\gamma} \lesssim \hat{t}_b$ and the plateau extends up to \hat{t}_b .

Finally, we also take into consideration the effects of cosmological redshift and detector threshold. All the quantities that we consider hereafter are observed quantities. Namely, for bursts at a redshift z, all durations are stretched by a factor of (1+z); furthermore, all distributions are the observed ones and are affected by the detector thresholds. For abbreviation, we use the same notations as above. Let the observed redshift distribution be $p_z(z)dz$, which may be correlated with any other observed quantity. The observed burst duration distribution is then

$$p_{\gamma}(t_{\gamma}) = \iint p_{z}(z)p_{b}(t_{b}|z)p_{e}(t_{b}+t_{\gamma}|t_{b},z)dzdt_{b}, \qquad (8)$$

where $p_e(t|t_b,z)$ is the density distribution of t_e given a breakout time t_b and redshift z. Following the same considerations as above, if $p_e(t_b|t_b,z)$ does not vary much in the vicinity of t_b (for every t_b and z), then p_γ has a plateau. Similar reasoning to the one that follows Equation (7) implies that, if p_γ is a monotonically decreasing function, then a flat distribution is observed up to \hat{t}_b , which is the breakout time whose contribution to the integral in Equation (8) dominates its value at $t_\gamma < \hat{t}_b$. The integral is also expected to be dominated by bursts from a typical redshift \hat{z} , implying that the typical intrinsic break time is $\hat{t}_b/(1+\hat{z})$.

At long observed durations, $t_{\gamma} > \hat{t}_b$, the distribution p_{γ} can in general depend on all three observed probability distributions, p_b , p_e , and p_z , and their coupling. However, if bursts with the same breakout time, $t_b \approx \hat{t}_b$, and at the same redshift, $z \approx \hat{z}$, dominate the observed distribution both at $t_{\gamma} < \hat{t}_b$ and at $t_{\gamma} > \hat{t}_b$, then $p_{\gamma}(t_{\gamma} \gg \hat{t}_b) \propto p_e(t_{\gamma}|\hat{t}_b,\hat{z})$. Therefore, an extrapolation of $p_{\gamma}(t_{\gamma} \gg \hat{t}_b)$ to durations shorter than \hat{t}_b is similar to an extrapolation of $p_e(t_e \gg \hat{t}_b|\hat{t}_b,\hat{z})$ to a duration $t_e < \hat{t}_b$. Namely, it is an estimate of the number of choked bursts. Now, if bursts with \hat{t}_b and \hat{z} do not dominate the observed distribution at $t_{\gamma} > \hat{t}_b$, then, for any reasonable conditions, the monotonically decreasing $p_{\gamma}(t_{\gamma} > \hat{t}_b)$ is decreasing less rapidly with t_{γ} than $p_e(t_e \gg \hat{t}_b|\hat{t}_b,\hat{z})$. In that case, extrapolation of p_{γ} to

³ Clearly, the engine cannot work in the same mode for a time that is longer than $t_{\gamma} + t_{b}$. On the other hand, if $t_{\gamma} \gg t_{e} - t_{b}$, it is expected to leave a clear signature on the temporal evolution of the prompt emission, which is not seen (Lazar et al. 2009).

short durations $(<\hat{t}_b)$ again provides a reasonable estimate for the minimal number of choked GRBs.

To conclude, under very general conditions the jet-envelope interaction in the Collapsar model is predicted to produce a plateau in the duration distribution of GRBs at short observed durations. This is true for any breakout time, engine working time, and redshift distributions (including cases where the various distributions are correlated) as long as the engine working time distribution is smooth enough. The bursts in the constant section of p_{γ} are dominated by a population with an observed breakout time \hat{t}_b , and the plateau is extended up to $t_{\nu} \approx \hat{t}_{b}$. Based on the typical observed GRB parameters (see Equation (1)) and given that a typical GRB is observed at redshift ≈ 2 , we expect $\hat{t}_b \approx 50$ s. Finally, it is well established that, at much shorter durations ($\lesssim 1$ s), the duration distribution contains a significant population of bursts not associated with the death of massive stars, which are known as short, hard GRBs (SGRBs; Kouveliotou et al. 1993; Narayan et al. 2001; Matzner 2003; Fox et al. 2005; Berger et al. 2005; Nakar 2007). These bursts are non-Collapsars, and the above arguments do not apply to them. Therefore, when considering the overall burst duration distribution, we expect a flat section for durations significantly lower than 50 s down to the duration where these non-Collapsars dominate.

4. THE OBSERVED DISTRIBUTION OF PROMPT GRB DURATIONS

The observed duration of a GRB is characterized using $T_{90} \approx t_{\gamma}$, during which 90% of the fluence is accumulated. We use the data from the three major GRB detectors: BATSE, Swift, and the Fermi gamma-ray burst monitor (GBM). For BATSE we use the current catalog (1991 April 21–2000 May 26; containing 2041 bursts). The data of *Swift* are taken from its online archive⁴ (2004 December 17–2011 August 27; containing 582 bursts). Fermi data are extracted from GCNs using the GRBox Web site⁵ (2008 August 12–2011 July 21; containing 194 bursts). Each data set is binned into equally spaced logarithmic bins, where the minimal number of events per bin is limited to five (Press et al. 1989). A bin with less than five events is merged with its neighbor. We use a χ^2 minimization to look for the longest logarithmic time interval that is consistent with a flat line within 1σ , where the only free parameter is the normalization. We verify that varying the bin size does not change the length of the plateau by much.

Figure 1 depicts the observed distribution of T_{90} , $p_{\gamma}(T_{90})$, for the three major GRB satellites. Note that here we show the quantity $p_{\gamma}(T_{90}) = dN/dT$ and not $dN/d\log T$, which is traditionally shown in such plots (e.g., Kouveliotou et al. 1993). The best-fitted flat regions are highlighted in a solid bold line on top each distribution. In all of the satellites, these plateaus range about an order of magnitude in duration (BATSE 5–25 s, 3.6/6 χ^2 /dof; *Swift* 0.7–21 s, 8.9/7 χ^2 /dof; *Fermi* 1.2–31 s, 4.1/6 χ^2 /dof). The extent of the plateau varies slightly from one detector to another. Given the different detection threshold sensitivities in different energy windows (see below), this is to be expected. At the high end of the plateau, the T_{90} distribution rapidly decreases and can be fitted at long durations (>100 s) by a power law with an index, α , in the range $-4 < \alpha < -3$.

The existence of the plateaus and their duration range (\sim 2–25 s) agrees well with the expectation of the Collapsar

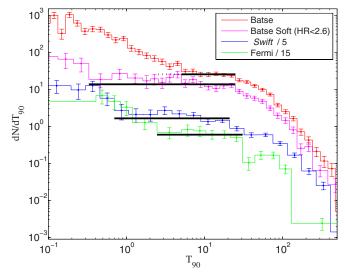


Figure 1. T_{90} distribution, dN/dT_{90} , of BATSE (red), Swift (blue), and Fermi GBM (green) GRBs. Also plotted is the distribution of the soft (hardness ratio < 2.6) BATSE bursts (magenta). For clarity, the Swift values are divided by a factor of 5 and the Fermi GBM by 15. The dotted line that ranges down to \approx 2 s marks the duration range where the Collapsars constitute more than 50% of the total number of BATSE GRBs. At shorter times, the sample is dominated by non-Collapsars. Note that the quantity dN/dT is depicted and not $dN/d\log T$ as traditionally shown in such plots (e.g., Kouveliotou et al. 1993). The black lines show the best-fitted flat interval in each data set: 5–25 s (BATSE), 0.7–21 s (Swift), and 2.5–31 s (Fermi). The upper limits of this range indicate a typical breakout time of a few dozen seconds, which is in agreement with the prediction of the Collapsar model. The distribution at times \gtrsim 100 s can be fitted as a power law with an index $-4 < \alpha < -3$. The soft BATSE bursts show a considerably longer plateau (0.4–25 s), indicating that most of the soft, short bursts are, in fact, Collapsars.

(A color version of this figure is available in the online journal.)

model. However, one cannot exclude the possibility that the origin of the observed flat sections is unrelated to the effect of the jet breakout time that we discuss above. For example, these plateaus may somehow coincidentally arise from the combination of two distributions: one increasing (LGRBs) and one decreasing (SGRBs). To test the hypothesis that the observed plateaus indeed reflect the distribution of LGRBs, which are Collapsars, we use the fact that SGRBs are harder (Kouveliotou et al. 1993). Restricting the analysis only to soft bursts should preferentially remove SGRBs from the sample. Thus, LGRBs should dominate the duration distribution of a sample of soft bursts down to durations that are shorter than in the case of the entire sample. Now, if LGRBs are Collapsars and their duration distribution is flat at short times, then the T_{90} distribution of a sample of soft bursts should exhibit a plateau that extends to shorter durations than the T_{90} distribution of the entire sample. We also present in Figure 1 a distribution of BATSE soft bursts (magenta), which are defined as bursts with a hardness ratio,⁶ HR < 2.6, the median value of bursts with $T_{90} > 5$ s. Remarkably, the plateau in this sample extends from 25 s down to 0.4 s $(15.2/12 \text{ }\chi^2/\text{dof})$, over almost two orders of magnitude in duration, compared with the original 5 s in the complete BATSE sample. This lends strong support to the conclusion that the observed flat distribution is indeed indicating on the Collapsar origin of the population. It also implies that HR is a good indicator that effectively filters out a large number of non-Collapsars from the GRB sample.

⁴ http://swift.gsfc.nasa.gov/docs/swift/archive/grb_table.

http://lyra.berkeley.edu/grbox/grbox.php.

 $^{^6~}$ HR is the fluence ratio between BATSE channels 3 (100–300 keV) and 2 (50–100 keV).

5. DISCUSSION

The observed plateaus in all three duration distributions, and most notably in the distribution of the soft BATSE bursts, provide direct support for the Collapsar model for LGRBs. An inspection of the different regions of the observed temporal distribution (Figure 1), under the interpretation of the plateau as an imprint of the time it takes the jet to break out of the envelope, provides further important information.

- 1. The end of the plateau and the decrease in the number of GRBs at long durations allows us to estimate the typical time it takes a jet to break out of the progenitor's envelope. All three distributions are flat below $\sim 10 \, \mathrm{s}$ in the GRB frame, implying that $\hat{t}_b \sim$ a few dozen seconds. This value fits nicely with the canonical GRB parameters taken in Equation (1) and provides additional support that the stellar progenitors of Collapsars must be compact (Matzner 2003).
- 2. The end of the plateau at long durations is dominated by the distribution of jet breakout times rather than by the engine working time distribution. Thus, as we discuss in Section 3, an extrapolation of the distribution at long durations, $t_{\gamma} \gg \hat{t}_b$, to durations shorter than \hat{t}_b provides a rough estimate of the number of Collapsars with engines that do not work long enough for their jets to break out. If the jet fails to break out, then it dissipates its energy into the stellar envelope, producing a so-called choked or failed GRB. There have been numerous suggestions that such a hypothetical population of hidden "choked GRBs" exists and that these are strong sources of high-energy neutrinos (Eichler & Levinson 1999; Mészáros & Waxman 2001) and possibly gravitational waves (Norris 2003; Daigne & Mochkovitch 2007).

At long durations, $p_{\gamma}(T_{90} \gtrsim 100 \text{ s})$ can be fitted well with a power law, $p_{\gamma} \propto T_{90}^{\alpha}$, with $-4 < \alpha < -3$. Extrapolating to $t_e < \hat{t}_b$, we find that, if p_e continues with this power law to $t_e < \hat{t}_b$, there are many more choked GRBs than successful ones. For example, even if we only extrapolate this distribution down to $t_e = \hat{t}_b/2$, there are still ~ 10 times as many choked GRBs as LGRBs. This prediction is consistent with the suggestion that shock breakout from these choked GRBs produces a low-luminosity smooth and soft GRB (Tan et al. 2001; Wang et al. 2007; Nakar & Sari 2012). Indeed, the rate of such low-luminosity GRBs is far higher than that of regular LGRBs (Soderberg et al. 2006).

3. At short durations, the GRB distribution is dominated by the non-Collapsar SGRBs (Eichler et al. 1989; Fox et al. 2005; Berger et al. 2005; Nakar 2007). They are hard to classify since all their hard energy properties largely overlap with those of the Collapsars (Nakar 2007). The least overlap is in the duration distributions and, hence, traditionally a burst is classified as a non-Collapsar if $T_{90} < 2$ s. Even though this criterion is based on the duration distribution of BATSE, it is widely used for bursts detected by all satellites.

The fraction of Collapsars at short durations can be estimated by extrapolating the plateau in the duration distribution. Since there is also an overlap with the SGRBs at long durations, the real height of the plateaus is somewhat lower than what is shown in Figure 1. A detailed treatment of this issue is presented in O. Bromberg et al. (in preparation). Nevertheless, we can use the observed plateaus to obtain a crude estimation of the duration below which the majority of GRBs are non-Collapsars by extrapolating the plateaus to the bins in which Collapsars constitute 50%

of the bursts. For BATSE, this shows that the transition occurs at \sim 3 s, while it occurs at \sim 0.7 s for *Swift* and at \sim 1.2 s for *Fermi*, although the statistics in the *Fermi* data is quite poor. The shift in the transition time is expected since non-Collapsar bursts are also harder and different detectors have different energy detection windows. BATSE has the hardest detection window, making it relatively more sensitive to non-Collapsar GRBs. *Swift* has the softest detection window, making it relatively more sensitive to Collapsar GRBs.

Thus, placing the dividing line between Collapsars and non-Collapsars at 2 s is statistically reasonable for BATSE bursts. However, our analysis shows that it is clearly wrong to do so for *Swift* (and possibly also for *Fermi*) bursts.

4. While the difference in the lower limit of the flat ranges is qualitatively understood in view of the detection windows of the different detectors, the variance in the upper limit is less obvious. It may reflect various selection effects in triggering algorithms. Unfortunately, Swift's complicated triggering algorithm makes it difficult to quantitatively explore this point. A more interesting possibility is that it reflects a physical origin, e.g., that different satellites probing populations with different \hat{t}_b . This could be explored when the statistical sample of Fermi GBM becomes sufficiently large.

To conclude, we remark that it is intriguing that the unique feature discovered here for the GRB's duration distribution depends just on a simple relation (Equation (2)). This implies that a similar plateau is expected in any transient source whose duration is determined by two unrelated processes in a similar manner, in particular, in any case in which one process emits radiation and another independent one blocks it for a while. For example, this would take place in a source engulfed by dust that will be obscured until the dust is destroyed by a radiation wave. This could be of importance when interpreting numerous transient observations, particularly now at the dawn of astronomy in the time domain.

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