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A NEW ESTIMATOR OF THE DECELERATION PARAMETER FROM GALAXY ROTATION CURVES

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ABSTRACT

The nature of dark energy can be probed by the derivative $Q = dq(z)/dz|_0$ at redshift $z = 0$ of the deceleration parameter $q(z)$. It is probably static if $Q < 1$ or dynamic if $Q > 2.5$, supporting ΛCDM or $\Lambda = (1 - q)H^2$, respectively, where H denotes the Hubble parameter. We derive $q = 1 - (4\pi a_0/cH)^2$, enabling a determination of $q(z)$ by measuring Milgrom's parameter, $a_0(z)$, in galaxy rotation curves, equivalent to the coefficient A in the Tully–Fisher relation $V_c^4 = AM_b$ between a rotation velocity V_c and a baryonic mass M_b. We infer that dark matter should be extremely light, with clustering limited to the size of galaxy clusters. The associated transition radius to non-Newtonian gravity can conceivably be probed in a freefall Cavendish-type experiment in space.

Key words: cosmological parameters – dark energy – dark matter – galaxies: kinematics and dynamics

1. INTRODUCTION

To leading order, large-scale cosmology is described by a Friedmann–Robertson–Walker line-element

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \tag{1}$$

with a dynamical scale factor $a(t)$. Here, the evolution of $a(t)$ is parameterized by $H = \dot{a}/a$ and the deceleration parameter $q = -H^{-2}\ddot{a}/a$, with the dot referring to differentiation with respect to time. Evolving (1) by general relativity, a dark energy density $\Lambda/8\pi > 0$ is inferred from the observed three-flat cosmology with deceleration

$$q = \frac{1}{2}\Omega_M - \Omega_\Lambda < 0 \tag{2}$$

in SN Ia surveys (Riess et al. 1998; Perlmutter et al. 1999). Here, $\Omega_M = \rho_M/\rho_c$ and $\Omega_\Lambda = \Lambda/8\pi\rho_c$, where $\rho_c = 3H^2/8\pi$ denotes the closure density. Λ is commonly referred to as a cosmological constant. The value $\Omega_\Lambda \simeq 0.7$ suggests that our cosmology is presently approaching a de Sitter state with a cosmological horizon at the Hubble radius $R = c/H_0$. A de Sitter state is fully Lorentz invariant, and in our current universe is broken only by the presence of a minor amount of matter.

A tell-tale signature of dark energy in accelerated cosmological expansion (2) is static or dynamic behavior. Here, we consider the problem of discriminating between ΛCDM and a dynamic dark energy in the form of $\Lambda = (1 - q)H^2$, recently proposed as a back reaction of the thermodynamic properties of the cosmological horizon (van Putten 2015a), motivated by holographic arguments (Bekenstein 1981; 't Hooft 1993; Susskind 1995; van Putten 2012) and a modified Gibbons–Hawking temperature (Unruh 1976; Gibbons & Hawking 1977; Cai & Kim 2005). This dynamical dark energy has the property that it vanishes in the radiation-dominated era, leaving baryon nucleosynthesis unaffected. In this era, the surface gravity of the cosmological horizon vanishes when it touches the light cone of distant inertial observers. These two alternatives predict distinct values of the derivative $Q = dq(z)/dz$ at redshift $z = 0$:

$$Q_{\rm stat} < 1, \quad Q_{\rm dyn} > 2.5 \tag{3}$$

in ΛCDM, and, respectively, $\Lambda = (1 - q)H^2$.

Indicative of a holographic origin of Λ is a dimensional analysis based on $L_0 = c^5/G$ and the associated pressure $p = L_0 c/A_H$ on the cosmological horizon, where $A_H = 4\pi R_H^2$. In a pure de Sitter space ($q = -1$), $\rho_\Lambda = -p$ by Lorentz invariance, whereby $\Omega_\Lambda = 2/3$, in remarkable agreement with observations. See also Easson et al. (2011) for a derivation based on entropic forces.

From (3), $q(z)$ can be used to distinguish between ΛCDM and a dynamical Λ, provided it is resolved sufficiently accurately about $z = 0$. Current data from SN Ia surveys, however, seem inconclusive, which appears to be due to systematic errors that are possibly related to the tension with Planck data on the Hubble parameter (Ade et al. 2014; Planck Collaboration 2014).

Here, we consider a new probe of $q(z)$ in galaxy rotation curves and its implications for the clustering of dark matter. This approach is based on a finite sensitivity of weak gravity to Λ (static or dynamic). Gravitational attraction beyond what is inferred from (luminous) baryonic matter is generally observed in galaxies and galaxy clusters (Famae & McGaugh 2012) at accelerations of 1 Å $\rm s^{-2}$ or less. This apparent non-Newtonian behavior is commonly attributed to dark matter, based on the success of Newton's theory of gravity in the solar system and its extension to strong gravity by embedding in general relativity. Supporting data for the latter are derived from orbital motions at accelerations $a = R_g(c/r)^2 \simeq 10^{-6}$–$10^{-2}$ $\rm m\,s^{-2}$ of planets in the solar system at distances r, where $R_g = GM_\odot/c^2 \simeq 1.5$ km denotes the gravitational radius of the Sun with Newton's constant G. Its extension in general relativity to higher accelerations has been fully vindicated by precession measurements in the Hulse–Taylor binary pulsar PSR 1913+16 ($a = 10^0$–10^2 $\rm m\,s^{-2}$) (Hulse & Taylor 1975). However, our observations of dark matter take us to the opposite limit of extremely weak gravity, not probed by our solar system or strong field counterparts in compact binaries. The parameter regime of about 1 Å $\rm s^{-2}$ takes us away from existing tests of Newtonian gravity by a factor of about 10^4, which is not small. Importantly, this scale is similar to the scale

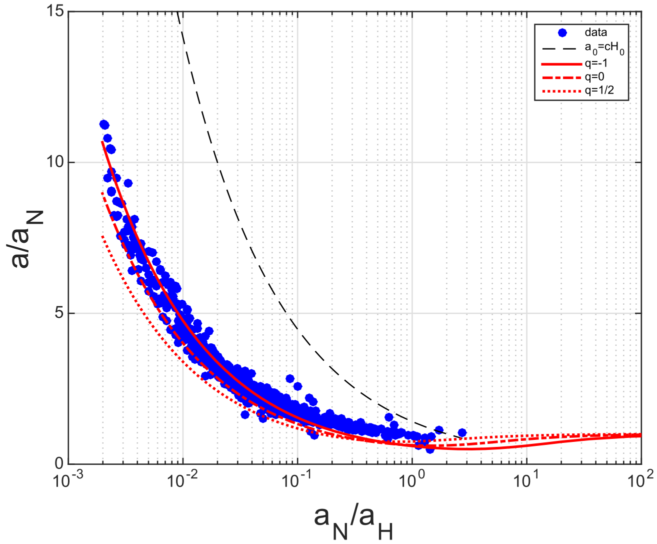

Figure 1. Galaxy rotation curves (blue dots) reveal a transition to a $1/r$ force law at weak accelerations asymptotically in $a \ll a_H$ away from Newtonian forces in $a \gg a_H$ based on the observed baryonic matter. Shown is a theoretical curve (red) in unitary holography with a good match in a cosmological background with a deceleration parameter q in the range $-1 < q < -0.5$. Data are from galaxy curves with essentially zero redshifts from Famae & McGaugh (2012).

of cosmological acceleration $a_H = cH$, where c denotes the velocity of light and H is the Hubble parameter. Currently, $H_0 \simeq 67\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Ade et al. 2014; Planck Collaboration 2014).

To realize our new probe of $q(z)$, we consider weak gravity on the cosmological background (1) parameterized by (H, q) in a recent formulation of unitary holography (van Putten 2015b). In geometrical units with Newton's constant G and the velocity of light c equal to unity, Λ and $\sqrt{\Lambda}$ are of dimensions cm^{-2} and cm^{-1}, respectively, corresponding to dark energy volume, and surface density, respectively. The latter may be recognized as the thermal energy density $\Sigma = \frac{1}{2}T = H/(4\pi)$, defined by a de Sitter temperature $T_{dS} = H/2\pi$ (Gibbons & Hawking 1977). While $\Lambda/8\pi$ is notoriously small, $\Lambda \simeq 1.21 \times 10^{-56}\ \mathrm{cm}^{-2}$, $\Sigma = \sqrt{\Lambda}/(4\sqrt{2}\pi) \simeq 6 \times 10^{-29}\ \mathrm{cm}^{-28}$ is not. An immediate implication is a critical transition radius for gravitational attraction. Around a central mass $M = M_{11} 10^{11} M_\odot$ of a typical galaxy, $A\Sigma = M$ for a two-sphere with area $A = 4\pi r^2$, giving a transition radius

$$r_t = \sqrt{MR_H} = 4.6\, M_{11}^{\frac{1}{2}}\ \mathrm{kpc}. \tag{4}$$

The transition radius (4) is common to galaxy rotation curves and bears out in well in $a_t/a_H \simeq 0.1$ ($a_t = GM/r_t^2$) in a deviation of centripetal accelerations a relative to the Newtonian acceleration a_N expected from the observed baryonic mass (Milgrom 1983; Famae & McGaugh 2012), here shown in Figure 1. (4) defines strong gravitational interactions in $r \ll r_t$ and weak gravitational interactions in $r \gg r_t$ with accelerations, respectively,

$$a \ll a_H, \quad a \gg a_H. \tag{5}$$

In geometrical units, holography hereby identifies a_H as a critical acceleration in galaxy rotation curves.

In Section 2, we express gravitational attraction in terms of a conformal factor, encoding information on particle positions in unitary holography. The Newtonian limit is recovered in Section 3, and extended to non-Newtonian asymptotic behavior in $r \gg r_t$ in (4) on a de Sitter background in Section 4. A further extension to (1) is given in Section 5, in which a finite sensitivity in $r \gg r_t$ to Λ is proposed as a new estimator for $q(z)$, proposed to determine (3). We summarize our theory in Section 6 with an outlook on future tests.

2. CONFORMAL FACTORS FROM DISTANCE INFORMATION

Unitary holography expresses distances of a particle of mass m to time-like holographic two-surfaces in terms of information $I = 2\pi\Delta\varphi$ defined by a total phase difference $\varphi = kr$, derived from its propagator with a Compton wavenumber $k = mc/\hbar$, where $\hbar$ denotes the Planck constant. Holographic imaging is hereby an extension of holographic bounds originally developed for black hole spacetimes (Bekenstein 1981; 't Hooft 1993; Susskind 1995) to spacetimes outside of black hole event horizons. Thus, m is a holographic superposition of A/l_p^2 light modes determined by the hyperbolic structure of spacetime, where $l_p = \sqrt{G\hbar/c^3}$ denotes the Planck length.

This approach has two consequences. First, on macroscopic scales, A/l_p^2 is astronomically large. The holographic modes are extremely light, with an energy scale

$$\epsilon = \frac{mc^2 l_p^2}{A}, \tag{6}$$

which introduces a sensitivity to any similarly low-energy scale in the background vacuum. The latter is described by the elliptic structure of spacetime, which governs gravitational attraction. (In general relativity, the elliptic part embeds Newton's law of gravity in a conformal factor.) Second, holographic imaging is a function of $A\Omega$, where A is the area of the bounding surface and Ω is the projection opening angle of its surface elements. Factorization of $A\Omega$ is hereby an internal symmetry of holography (cf. 't Hooft 2015). Scaling of A and Ω respectively corresponds to curvature lensing. These may be realized by a conformal factor or a deficit angle, which are essentially different manifestations of the same phenomena.

In encoding I in $fA = A - A_E$ or equivalently, $f\Omega = 4\pi - \Omega_E$, $A_E = 8\pi ms$ and $\Omega_E = 8\pi m/s$ are the Einstein area and opening angles, respectively, i.e., $fA\Omega = 4\pi(A - A_E) = 4\pi A(4\pi - \Omega_E) = 16\pi^2 s^2 f$. Here, the factor of four in $A_E = 4Il_p^2$ derives from a counting argument on the minimal number of four bits required to encode matter and fields (van Putten 2015b). A holographic screen hereby attains minimal size with $A = A_E$ or $\Omega_E = 4\pi$ at the Schwarzschild radius $s = R_S$, $R_S = 2m = \sqrt{S/\pi}$, with $S = \min I = 4\pi m^2$ in $I = 2\pi m(s - R_S) + S$ equal to the Bekenstein–Hawking entropy. Accordingly, we have

$$f = 1 - \frac{2m}{s}. \tag{7}$$

In general relativity, the gravitational field about a point mass can be described by a conformal factor Φ in an isotropic line-element

$$ds^2 = -N^2dt^2 + \Phi^4(dx^2 + dy^2 + dz^2), \tag{8}$$

where $N = N(\Phi)$ denotes the gravitational redshift, i.e., the ratio of energy-at-infinity to locally measured energy. According to the above, $R_S = \sqrt{4S/\pi}$ expresses the mass energy of a particle by its linear size, locally measured by the minimal surface area $4S$ of an enveloping holographic screen. We are at liberty to choose a gauge

$$N\Phi^2 \simeq \text{const.}, \tag{9}$$

defined by a constant total mass energy-at-infinity in the approximation of small perturbations to the spherically symmetric line-element (8). For a detailed consideration of such time-symmetric data, see van Putten (2012), where it serves as a condition in the application of Gibbs' principle in entropic forces in black hole binaries. According to the equations of geodesic motion, Newton's law then derives from $N \simeq 1 - m/\rho$ in the large distance limit. With $dx^2 + dy^2 + dz^2 = d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2$ expressed in spherical coordinates (ρ, θ, ϕ), ρ reduces to the ordinary radial distance r at large separations, and (8)–(9) embed Newton's law in

$$\Phi \simeq f^{-\frac{1}{4}} \simeq 1 + \frac{m}{2r} \ (r \gg 2m). \tag{10}$$

3. THE NEWTONIAN LIMIT IN AN ORDINARY VACUUM

In what follows, unless otherwise specified, A shall refer to surface area, as well as the number of Planck-sized surface elements A/l_p^2.

In holography, the wavefunction of a particle m results from A Planck-sized harmonic oscillators of low energy (6). Ordinarily, one mode in the image appears for each mode in the screen. (The dimension of the phase space in the image equals the number of degrees of freedom in the screen.) Quantum-mechanically, m is the time rate-of-change of the total phase as measured at infinity,

$$m = \frac{1}{2}A\omega, \tag{11}$$

of the ground state energies $(1/2)\omega$ of each harmonic oscillator in the screen. Distance encoding derives from the aforementioned $\Delta\varphi = kr$, with the total wavenumber k given by the superposition of these massless modes,

$$k = \frac{1}{2}A\kappa. \tag{12}$$

The trivial dispersion relation $\kappa = \omega$ of an ordinary vacuum recovers the Compton wavenumber $k = k_C$, $k_C = m$, with low-energy frequencies

$$\omega_N = \frac{2m}{A} = \frac{a_N}{2\pi} \tag{13}$$

defined by the Newtonian acceleration $a_N = m/r^2$.

The Compton relation $k = m$ recovered by the trivial dispersion $\kappa_N = \omega_N$ associates κ_N with the Unruh temperature of Newtonian acceleration (13).

In entropic gravity (Verlinde 2011), the above implies entropic forces on a test particle of mass m' at screen temperature $T = m/2\pi r^2$ by $dS = -dI = -2\pi m' dr$, giving $F = -dU/dr = TdS/dr = -mm'/r^2$. In keeping with (9)–(10), however, we shall not pursue these arguments here.

4. SENSITIVITY TO $\sqrt{\Lambda}$ IN A DE SITTER BACKGROUND

From (6), (13) is susceptible to the low-energy de Sitter temperatures of the cosmological horizon. Screen modes satisfy the dispersion relation

$$\omega = \sqrt{\kappa^2 + \omega_H^2}, \tag{14}$$

representing an incoherent sum of a momenta κ and a background de Sitter temperature, $\omega_H = T_{dS}$ (Narnhofer et al. 1996; Deser & Levin 1997; Jacobson 1998). A spherical screen imaging a mass m at its center hereby assumes

$$\omega_N = \omega - \kappa_H\colon\ \ \kappa = \sqrt{\omega_N^2 + 2\omega_N\omega_H}, \tag{15}$$

giving $\kappa \simeq \omega_N$ $(r \gg r_t)$ and $\kappa \simeq \sqrt{a_0 a_N}$ $(r \ll r_t)$, with $a_0 = 2a_H$ as proposed in Klinkhamer & Kopp (2011). However, $\kappa(\omega_N)$ from (15) overestimates the Milgrom parameter a_0 (Milgrom 1983) by about one order of magnitude according to the data shown in (Figure 1). Here, Milgrom's parameter is equivalent to the coefficient A in the Tully–Fisher relation $V_c^4 = AM_b$, where V_c denotes the rotation velocity in a galaxy of baryonic mass M_b (McGaugh 2011a, 2011b). The wavenumber κ_N in (15) is *not* representative for the κ of the image within.

On a cosmological background (1) with $\Lambda > 0$, the image modes satisfy the dispersion relation

$$\omega' = \sqrt{\kappa^2 + \Lambda}, \tag{16}$$

defined by the waveequation of a vector field in curved spacetime by coupling to the Ricci tensor $R_{ab} = \Lambda g_{ab}$. This applies to the electromagnetic vector potential (e.g., Wald 1984), as well as the Riemann–Cartan connections in SO(3,1) in a Lorenz gauge (van Putten & Eardley 1996). It implies an effective rest mass energy $\sqrt{\Lambda}$ of the photon and graviton and photon. Effective mass is not the same as true mass. Even so, we mention in passing that the problem of consistent general relativity with massive gravitons has recently received considerable attention (de Rham et al. 2011; Bernard et al. 2014). With $q_0 H^2 = H^2 + \dot{H}$, the generalized Higuchi constraint $m^2 \geqslant 2(H^2 + \dot{H})$ (Higuchi 1987; Deser & Waldron 2001; Grisa & Sorbo 2010) reduces to $\Omega_\Lambda \geqslant 2q_0$. Based on observations, $1 < q_0 < 0.5$ (Riess et al. 2004; Wu & Yu 2008; Giostri et al. 2012), whereby $q_0 > -1$ appears to be secure at any rate.

From (14) and (16), distinct effective masses appear in the kinetic energies $E = \omega - \omega_H$ and $E' = \omega' - \sqrt{\Lambda}$ of low-energy modes in the screen and image, namely

$$E \simeq \frac{\kappa^2}{2\kappa_H},\ \ E' \simeq \frac{\kappa^2}{2\sqrt{\Lambda}} \tag{17}$$

$(\kappa \ll \kappa_H, \sqrt{\Lambda})$. Therefore, in weak gravitation in de Sitter space, a direct correspondence between screen and image modes is lost, a striking departure from the above Newtonian limit in $r \ll r_t$ in the previous section.

Specifically, (17) shows a discrepancy by a factor of $2\sqrt{2}\pi$ in effective mass $\sqrt{\Lambda}$ over that in κ_H. A given $\kappa = \kappa(\omega_N)$ of

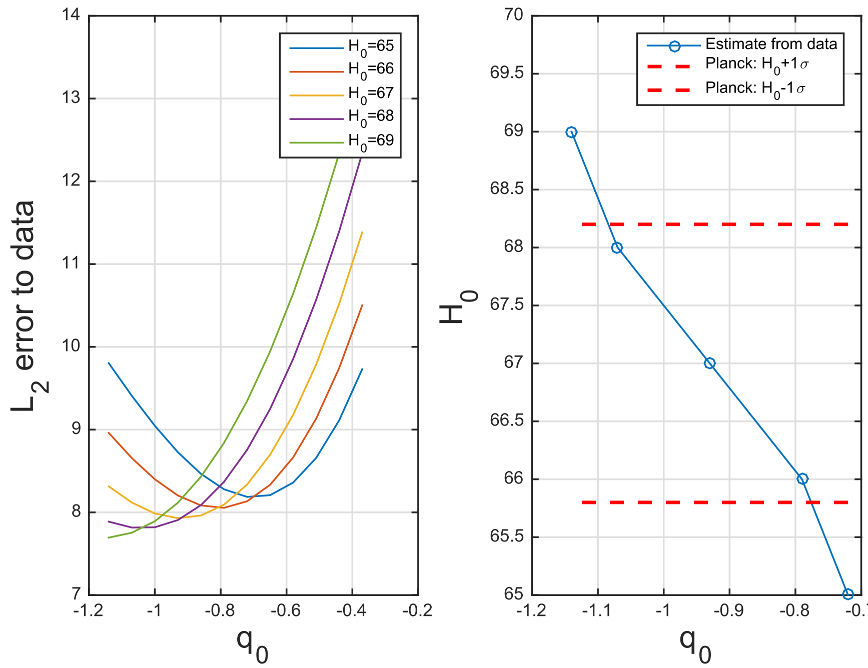

Figure 2. Estimation of q_0 by a least-squares fit to the Famae & McGaugh (2012) sample of low-redshift galaxies shown in Figure 1 following a rescaling to various H_0 in units of km s^{-1} Mpc^{-1}. The resulting correlation (q_0, H_0) agrees with Planck data on a relatively low Hubble parameter of about 67 km s^{-1} Mpc^{-1}.

screen modes has an associated reduced energy in the relatively more heavy image modes, satisfying

$$\omega_N' = \omega' - \sqrt{\Lambda} = \sqrt{\kappa^2 + \Lambda} - \sqrt{\Lambda}, \tag{18}$$

with corresponding reduced screen momenta $\kappa' = \sqrt{\omega_N'^2 + 2\omega_N'\omega_H}$. Figure 1 shows the graph $\kappa'(\omega_N)$ to be in agreement with the data. Specifically, we arrive at Milgrom's constant on a de Sitter background

$$a_0 = \left(\frac{\kappa_H}{\sqrt{\Lambda}}\right) 2cH_0 = \frac{cH_0}{\sqrt{2}\,\pi} \simeq 1.5 \times 10^{-8} \text{ cm s}^{-2}, \tag{19}$$

where we restored dimensions in cgs units.

5. SENSITIVITY TO $q(z)$ IN A FRIEDMANN–ROBERTSON–WALKER (FRW) BACKGROUND

The above generalizes to general FRW universes with a modified de Sitter temperature (Cai & Kim 2005; van Putten 2015a)

$$T_{dS} = \frac{1-q}{2}\frac{H}{2\pi}. \tag{20}$$

A key feature of (20) is that $T_{dS} = 0$ in the radiation-dominated era $q = 1$, whereby it pertains only to relatively late time cosmologies, satisfying

$$\Omega_\Lambda = \frac{1}{3}(1-q), \quad \Omega_M = \frac{1}{3}(2+q). \tag{21}$$

As a consequence of (20), Milgrom's constant attains the explicit expression

$$a_0 = \frac{\sqrt{1-q}}{4\pi} cH, \tag{22}$$

allowing the measurement of q from a_0 as a function of redshift:

$$q(z) = 1 - \left(\frac{4\pi a_0(z)}{cH(z)}\right)^2. \tag{23}$$

The existing low-redshift sample of galaxies of Famae & McGaugh (2012) recovers the value $-1 < q_0 < -0.8$ for the Planck estimate of H_0 (Figure 2) and is broadly consistent with SN Ia surveys (Riess et al. 2004; Wu & Yu 2008; Giostri et al. 2012).

More detailed future observations of $a_0(z)$ about $0 \leqslant z \ll 1$ will offer a new venue for determining

$$Q = 2(1-q_0^2) - 2(1-q_0)a_0^{-1}\frac{da_0(z)}{dz}\bigg|_{z=0} \tag{24}$$

from a sample of galaxy rotation curves covering a finite range of low-redshift $0 \leqslant z \ll 1$.

In van Putten (2015a) we considered the problem of discriminating between a dynamical and static Λ parameter $(1-q)H^2$ versus ΛCDM. It shows the disjoint ranges

$$\begin{aligned} Q_{\rm stat} &= (1+q_0)(1-2q_0) < 1 \\ Q_{\rm dyn} &= (2+q_0)(1-2q_0) > 2.5 \end{aligned} \tag{25}$$

associated with ΛCDM satisfying (2) with $\Omega_M = \frac{2}{3}(1+q_0)$ and, respectively, $\Lambda = (1-q)H^2$ satisfying (21). Figure 3 shows the correlation of (24) with $a_0^{-1}da_0(z)dz$ at $z=0$ in these two cases.

6. CONCLUSIONS

In a unitary holography of matter, conformal factors encoding positions and gravitational attraction have a hidden low-energy scale, (6), that introduces a finite sensitivity to low-energy scales in the cosmological background, (1),

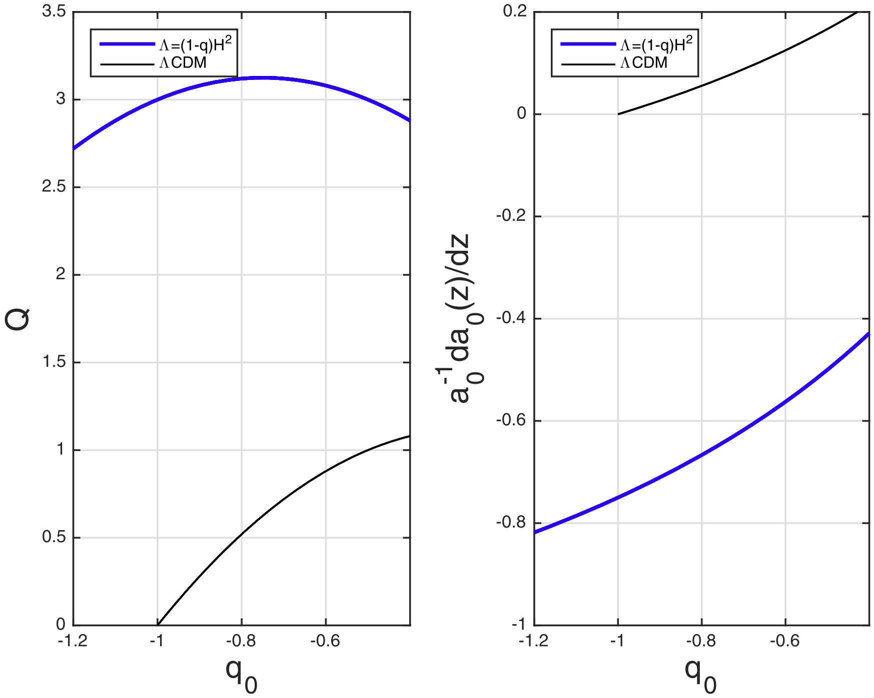

Figure 3. Correlations of $a_0^{-1}da_0(z)/dz$ at $z = 0$ with q_0 for a dynamical dark energy $\Lambda = (1 - q)H^2$ and a static dark energy in ΛCDM. In $-1 < q_0 < -0.4$, these correlations are sufficiently distinct to be discriminated observationally, provided that $a_0(z)$ is measured accurately over a small range of low redshifts.

parameterized by (H,q). This sensitivity is manifest in a transition to non-Newtonian gravitational attraction, which scales with inverse distance beyond a critical radius r_t at accelerations on the scale of the surface gravity of the cosmological horizon. It produces Milgrom's law with a specific expression for the a_0 as a function of (H, q). This result is due to anomalous behavior in unitary holography on an FRW background in the limit of weak gravity. This a_0 sensitivity to (H, q) may be probed observationally in low-redshift galaxy rotation curves.

From agreement with data shown in Figure 1, we see that there is no apparent need for the clustering of dark matter on the scale of galactic disks. Even so, there exists a cosmological distribution of dark matter (van Putten 2015a). A major conclusion of the present work, therefore, is that dark matter must be extremely light, producing clustering on the scale of galaxy clusters (Vikram et al. 2015) but not down to a much smaller scale of galaxies. Conceivably, the putative dark matter particle is the lightest element in the universe and may not be readily detectable in a laboratory experiment based on interactions with ordinary matter.

We propose probing the static or dynamic nature of dark energy by $dq(z)/dz$. Values at $z = 0$ less than 1 or greater than 2.5 support ΛCDM, or $\Lambda = (1 - q)H^2$, respectively. Here, we formulate this in terms of $a_0^{-1}da_0(z)/dz$ being greater or less than 0, respectively. These data may be obtained from an extended sample of low-redshift galaxy rotation curves.

Finally, scaling the transition radius, (4), to laboratory test masses, $r_t \simeq 1$ cm $M_0^{\frac{1}{2}}$, with $M = M_0$ g, suggests a possible laboratory test, probing the proposed sensitivity to the cosmological background by a space-based freefall Cavendish experiment.

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