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An issue encountered in solving problems in electricity and magnetism: curvilinear coordinates*

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Abstract

In physics lectures on electromagnetic theory and mathematical methods, physics teacher candidates have some difficulties with curvilinear coordinate systems. According to our experience, based on both in-class interactions and teacher candidates' answers in test papers, they do not seem to have understood the variables in curvilinear coordinate systems very well. For this reason, the problems that physics teacher candidates have with variables in curvilinear coordinate systems have been selected as a study subject. The aim of this study is to find the physics teacher candidates' problems with determining the variables of drawn shapes, and problems with drawing shapes based on given variables in curvilinear coordinate systems. Two different assessment tests were used in the study to achieve this aim. The curvilinear coordinates drawing test (CCDrT) was used to discover their problems related to drawing shapes, and the curvilinear coordinates detection test (CCDeT) was used to find out about problems related to determining variables. According to the findings obtained from both tests, most physics teacher candidates have problems with the ϕ variable, while they have limited problems with the r variable. Questions that are mostly answered wrongly have some common properties, such as value. According to inferential statistics, there is no significant difference between the means of the CCDeT and CCDrT scores. The mean of the CCDeT scores is only 4.63 and the mean of the CCDrT is only 4.66. Briefly, we can say that most physics teacher candidates have problems with drawing a shape using the variables of curvilinear coordinate systems or in determining the variables of drawn shapes.

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Keywords: curvilinear coordinates, determining variables, drawing figures

Introduction

Cylindrical and spherical coordinate systems are used for different purposes in various scientific disciplines. In electricity, magnetism and electromagnetic theory, solutions of some problems can be closely related to symmetrical shapes like the sphere and the cylinder. A symmetrical structure, based on the appropriate coordinate system, provides a reduction of equations from three dimensions to one or two dimensions and thereby helps with the elimination of calculation complexity (Nelson 2014). Schermerhorn and Thompson (2016) highlight that working in cylindrical and spherical coordinate systems in accordance with the symmetry of the physical system has a key role in the understanding of some concepts and implementation of some operations about electricity and magnetism. Besides, in problem solving, line, surface or volume elements should be written correctly in order to create integral expressions for systems having different charge distributions (Schermerhorn and Thompson 2016). For example, the electric flux passing through a closed surface surrounding a q point charge can be found easily using the spherical coordinate system. If the closed surface is chosen as a spherical shell with a radius R, the electric field which will be produced by the point charge from a distance R is $\vec{E} = k\,\dfrac{q}{R^2}\hat{r}$ and the electric flux is $\varnothing_E = \oiint \vec{E}\mathrm{d}\vec{a}$. In the spherical coordinate system, the expression for any surface element chosen on the spherical shell is written as $\mathrm{d}\vec{a} = r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi \hat{r}$. Accordingly, since $\varnothing_E = \oiint \vec{E}\mathrm{d}\vec{a} = \oiint k\,\dfrac{q}{R^2} r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$ and $r = R$ (constant), $\theta = (0,\ \pi)$, $\varphi = (0,\ 2\pi)$, $\varnothing_E$ it is found that $4\pi kq = \dfrac{q}{\varepsilon_0}$. As is known, this mathematical expression is the Gaussian law in an electric field. Physics teacher candidates may have some problems in determining variables defining the surface element in the spherical coordinate system and limits of these variables during the deduction of the Gaussian law.

All problems may be solved using the Cartesian coordinate system but the results would be unduly complicated. Solving the problems in this way helps to reveal mathematical skills, but has little contribution to understanding of the basic concepts (Nelson 2014). Brannon (2004) specifies that learning about the curvilinear coordinate systems helps with better understanding of tensor analysis even though it is never used. Koss (2011) specifies that some students in mathematics classes have great difficulties in understanding three-dimensional coordinate systems represented on the board or by a computer animation.

Students have apparent choices towards using the Cartesian coordinate system in accordance with previous mathematics and physics lectures (Sayre and Wittmann 2008, Wilcox *et al* 2013). There may be various reasons for students choosing the Cartesian coordinate system. The first reason may be the fact that they are more familiar with the Cartesian coordinate system. The previous experiences of the students about axes start with using the number line and continue with transfer of the number pairs to a graph using two number lines perpendicular to each other (Nelson 2014). The other reason may be the fact that in the cylindrical and spherical coordinate systems, the direction of unit vectors changes according to their positions (Brannon 2004, Nelson 2014). Furthermore, the difference between the mathematics used in mathematics and physics lessons causes serious problems, particularly in physics subjects that require complex mathematical skills. As highlighted by Manogue *et al* (2006), mathematics teachers set different goals for their lessons compared to physics teachers depending on their different thoughts about mathematics. Another difficulty is that curvilinear coordinates are taught but they are not much used. Accordingly, many students prefer to do spherical integrals in the Cartesian coordinate system instead of the spherical coordinate system (see Student difficulties).

In Europe, there are two ways to become a physics teacher (Sauer 2011). In one of these ways, physics teacher candidates take lectures about education after they finish lectures related to physics (in France, Malta, Netherlands, Spain, Turkey, United Kingdom, etc). In the other way, physics teacher candidates take lectures about education and physics together (In Finland, Germany, Ireland, Malta, Sweden, Turkey, etc). The subject of curvilinear coordinates is taught to physics teacher candidates within related physics lectures (calculus III, mathematical methods in physics, electromagnetic theory, etc). Nevertheless, our experiences that we get from the practice and evaluation steps of electromagnetic theory lectures and the theoretical/practical/evaluation steps of mathematical methods in physics lectures, suggest that physics teacher candidates have some problems with curvilinear coordinate systems. Consequently, the problems that physics teacher candidates have concerning variables in curvilinear coordinate systems have been selected as the subject of this study. The aim of this study is to find out physics teacher candidates' problems with determining the variables of drawn shapes and problems with drawing shapes based on given variables.

Methods

Research instruments

Two different assessment tests were used in the study.

1. The curvilinear coordinates drawing test (CCDrT): the physics teacher candidates first answered this open-ended (drawing-based) test. In the CCDrT, the teacher candidates were asked to:
   a. draw shapes in curvilinear (spherical or cylindrical) coordinates,
   b. specify the shape type (line/surface/volume) that they draw,
   c. provide a short explanation for defining their shapes,

using the variables (r, θ, ϕ, z), and the values and/or the limits of which are given, in each question.

2. The curvilinear coordinates detection test (CCDeT): this test is a multiple-choice test. In the CCDeT, the teacher candidates were asked to choose:
   a. number,
   b. kind, and
   c. value/limits

of the variables among the options required to draw the shown shape in each question, and to provide explanations for their answers, when needed.

When preparing the tests, the following factors were considered: (a) choosing shapes that may be drawn in curvilinear coordinate systems, and (b) increasing the diversity in the value/limits of the variables. In the first step, 22 questions were prepared in each test; 12 of them involved spherical coordinates and 10 of them involved cylindrical coordinates.

Studies about spatial intelligence show that slight changes in the shape orientations affect the visualization of the shapes (Velez *et al* 2005). Therefore, the perspective for the coordinate system remained the same in all questions. It is known that 'hidden geometrical properties' also affect the visualization (Velez *et al* 2005). Therefore, nine questions in which the teacher candidates might not to see the geometrical properties of the shapes were removed from the test.

Two questions were added to the CCDeT, which included circle and disc shapes that may be drawn in any of the curvilinear coordinate systems in order to determine to which of the coordinate systems the teacher candidates have more predisposition. In contrast to the CCDeT, the disc shape was included in the spherical coordinate system and the circle shape was included in the cylindrical coordinate system as the values/limits of the variables are given in the CCDrT.

In the final version of the CCDeT there were 15 questions in total. Six were related to the spherical coordinate system, seven were related to the cylindrical coordinate system and two were related to both coordinate systems. In the tests having binary results (correct—1; wrong—0), KR-20 and Cronbach's alpha had the same values (Bademci 2011); the value of Cronbach's alpha must be above 0.70 (Pallant 2001). The Cronbach alpha value of the CCDeT was calculated as 0.91. In the final version of the CCDrT, there were also 15 questions including shapes which may be drawn using the variables in the coordinate system, seven of which were spherical and eight of which were cylindrical. The Cronbach alpha value of the CCDrT was calculated as 0.81.

The questions in the final versions of the CCDeT and the CCDrT having different structures have become conjugate with each other. Therefore, the answer key of the CCDrT was formed by the shapes in each question of the CCDeT, whereas the answer key of the CCDeT was formed by the values of the variables which are given in the CCDrT.

The teacher candidates first answered the CCDrT and after one month (to make it difficult to remember the questions) the teacher candidates answered the CCDeT, in which the order of the questions was changed.

Evaluation steps of the curvilinear coordinates detection test (CCDeT)

The evaluation of the properties of the variables was made according to common scientific principles. As shown in figure 1(a), the rotation direction of the angle ϕ is counter clockwise and the limits of angle ϕ are defined as $0 \leqslant \phi \leqslant 2\pi$. The direction of the angle θ is in the downward direction such that it will move the r radius vector away from the positive z-axis (figure 1(b)). The limits for the angle θ are defined as $0 \leqslant \theta \leqslant \pi$.

The answers of the teacher candidates to the CCDeT were scored according to the following steps.

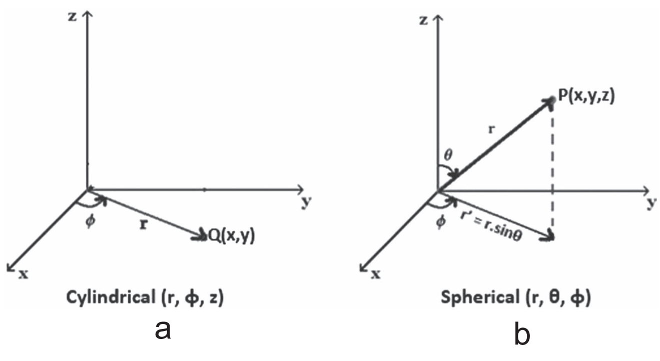


Figure 1. Connecting the Cartesian coordinate variables to curvilinear coordinate variables.

1st step: correct determination of the number of variables which are constant or have limits.

2nd step: correct determination of the number and kind of variables which are constant or have limits.

3rd step: correct determination of the number, kind and value of variables which are constant or have limits.

The explanations of the teacher candidates in the CCDeT facilitated the evaluation of the situations that could not be decided. Step by step score calculations were provided by an example in the following.

While the 1st step score was calculated according to the fourth question (figure 2), the number of variables in the variables column that the teacher candidates marked was considered. Three variables are needed (variables which are constant or have limits) in order to allow any shape to be drawn. The answers of the teacher candidates who marked any three variables were accepted as correct.

In the 2nd step, the number of variables that the teacher candidates marked was also considered. However, while the 2nd step score was calculated, it was considered whether the correct variables were selected or not, as well as the number of the variables. In other words, the answers of the teacher candidates to the fourth question in which only the choices (A) r, (C) θ, (D) ϕ were marked, were coded as correct.

While the 3rd step scores were calculated, the value column was considered in addition to the previous steps. The teacher candidates filled the cells in this column for each question using the table (figure 3) given in the front part of the CCDeT. For the fourth question, only the answers that the teacher candidates marked (A) r, (C) θ, (D) ϕ in the variables column and wrote **b** for **r** ($0 \leqslant r \leqslant$ R), **c** for $\boldsymbol{\theta}$ ($\pi/2 \leqslant \theta \leqslant \pi$), **e** for $\boldsymbol{\phi}$ ($-\pi/2 \leqslant \phi \leqslant \pi/2$) and **null** for **z** in the value column, were coded as correct. Besides, the teacher candidates' determination status of the variables was evaluated in detail for each shape.

Evaluation steps of the curvilinear coordinates drawing test (CCDrT)

The answers of the teacher candidates to the CCDrT were scored according to the following steps.

1st step: correct drawing of the shape defined by the variables which were given.

2nd step: correct drawing of the shape and positioning the shape appropriately to the coordinate system.

3rd step: correct drawing of the shape, positioning the shape appropriately to the coordinate system and specifying the shape type (line/surface/volume).

The explanations of the teacher candidates facilitated the evaluation of the drawings of the teacher candidates whose drawing abilities were not good. Score calculations were provided step by step by an example in the following.

While the 1st step scores of the eighth question in the CCDrT (figure 4(a)) were calculated, drawing the appropriate shape in the area shown in the shape column was accepted as correct. In this step, however, the explanations of the teacher candidates in the shape type and explanation columns were considered in the cases that the drawings were not very well understood (line, surface, volume or on which plane it was drawn, etc). In this regard, when only the shape drawn in figure 4(c) is considered, it is seen that the circle, which should be drawn in the plane x–y, was drawn in the plane y–z. However, in the explanation part, as the plane was correctly specified as x–y, the drawing was accepted as correct according to the first step scoring.

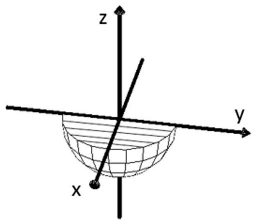
No	Shapes	Variables	Value	Explanation
4		A) r		
		B) z		
		C) θ		
		D) Ø		

Figure 2. CCDeT question sample.

Dimension in choice A *Values of r dimension*	Dimension in choice B *Values of z dimension*	Dimension in choice C *Values of r dimension*	Dimension in choice D *Values of ϕ dimension*
a) $r = 0$	a) $z = 0$	a) $0 \le \theta \le \pi$	a) $0 \le \phi \le 2\pi$
b) $r : 0 \le r \le R$	b) $z : 0 \le z \le Z$	b) $0 \le \theta \le \pi/2$	b) $\pi/2 \le \phi \le 3\pi/2$
c) $r : R_1 \le r \le R_2$ and $R_1 < R_2$	c) $z = Z$(constant and $Z \neq 0$)	c) $\pi/2 \le \theta \le \pi$	c) $\pi \le \phi \le 2\pi$
d) $r = R$(constant and $R \neq 0$)	d) $z : -Z \le z \le Z$	d) $-\pi/2 \le \theta \le \pi/2$	d) $\pi/2 \le \phi \le \pi$
		e) $\pi \le \theta \le 2\pi$	e) $-\pi/2 \le \phi \le \pi/2$
		f) $\pi \le \theta \le 3\pi/2$	f) $3\pi/2 \le \phi \le \pi/2$
		g) $\pi/2 \le \theta \le 3\pi/2$	g) $3\pi/2 \le \phi \le 2\pi$
		h) $0 \le \theta \le 2\pi$	h) $\pi \le \phi \le 3/2\pi$
		i) $3\pi/2 \le \theta \le 2\pi$	i) $0 \le \phi \le \pi$
		j) $0 \le \theta \le \pi/4$	j) $0 \le \phi \le \pi/2$
		k) $\theta = 0$	k) $\pi/2 \le \phi \le 3\pi/2$
		l) $\theta = \pi/2$	l) $\pi/2 \le \phi \le 2\pi$
		m) $\theta = \pi$	m) $\phi = \pi/2$
		n) $\theta = 3\pi/2$	n) $\phi = \pi$
		o) $\theta = 2\pi$	o) $\phi = 3\pi/2$
			p) $\phi = 2\pi$

Figure 3. CCDeT 'value' column choices.

While the 2nd step scores were calculated, only the fact whether the shape was drawn correctly or not was considered without taking the explanations into consideration. For this reason, the shape shown in figure 4(b) was accepted as correct since it was appropriately positioned to the coordinate system and appeared on the plane x–y although it did not contain any explanation. On the other hand, the shape shown in figure 4(c) was considered wrong because the shape was not positioned appropriately on the x–y plane.

In the 3rd step, the data in the shape type column were considered in addition to the previous steps. For example, for the type of shape shown in figure 4(b) the 'line' answers in the shape type column were accepted as correct. For that shape, the 'surface' or 'volume' answers were considered wrong according to the third step. The findings were analysed in detail to determine the correct usage of each variable by the teacher candidates.

Sample

The sample of this study consisted of physics teacher candidates who had attended at least one compulsory lecture including the curvilinear coordinate subject during their undergraduate education. The distribution of the teacher candidates in the sample is provided in table 1.

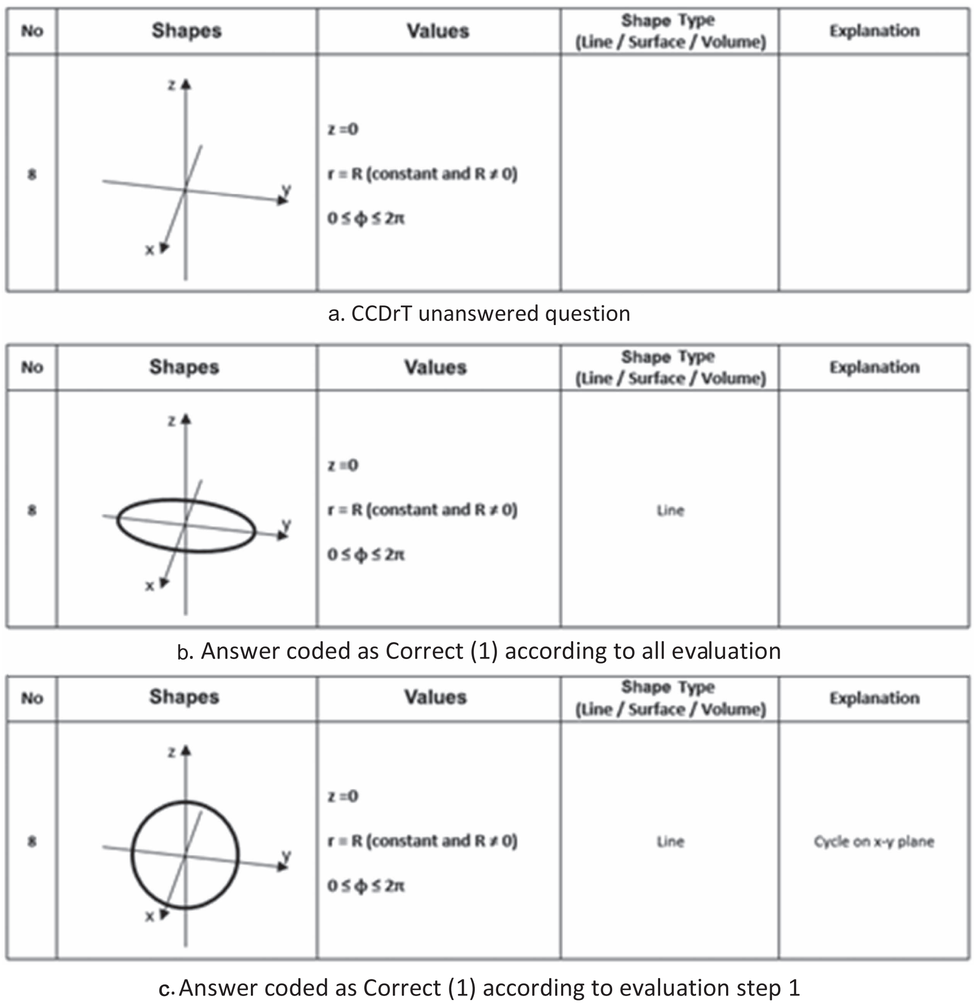


Figure 4. CCDrT question sample.

The findings obtained from the CCDeT and the CCDrT were evaluated individually. However, data of the 54 candidates who took both tests were considered when the results of the CCDeT and the CCDrT were compared.

Findings

This section includes findings related to the CCDeT and the CCDrT.

Findings related to the CCDeT

Findings about the 1st step scores: determination of the number of variables. The findings about the determination of the number of variables are provided in table 2. A total of 63.6% of the teacher candidates were of the opinion that the number of variables should be three in order to draw the shapes. Of the teacher candidates, 6.1% claimed that they could draw the

Table 1. Distribution of the sample.

Group	Gender	Answered test type CCDeT	CCDrT	Both
Physics teacher candidates	Female	52	38	31
	Male	31	26	23
	Total	83	64	54

shapes using one variable, whereas 11.6% claimed that they could draw the shapes using two variables and 18.2% claimed that that they could draw the shapes using four variables. In other words, 35.9% of the teacher candidates had inadequate knowledge of the number of variables in the curvilinear coordinate systems. When the null values are accepted as wrong this value rises to 36.4%. The teacher candidates who specified the number of variables as four marked all variables (r, θ, ϕ and z) without distinguishing between the spherical and cylindrical coordinate systems (18.2%). Two shapes, one of which is a disc (SC1) and the other is a circle (SC2), may be drawn using two variables in any curvilinear coordinate system. However, the answers of the candidates who specified the number of the variables required to draw these shapes as two were considered as wrong. The candidates had been warned previously about the practice of giving answers by taking into consideration the variables which are constant for drawing the shapes. Nevertheless, only 50% of the candidates could correctly specify the number of the variables belonging to the disc and circle shapes. It was determined that the candidates were more successful in distinguishing the variable number of the shapes drawn in the cylindrical coordinate system (70.2%). The candidates exhibited limited success in the shapes that could be drawn in any of the curvilinear coordinate systems (50%).

Findings about the 2nd step scores: determination of the number and kind of variables. In this section, the findings about the determination of the number and kind of variables were examined. According to the findings in table 2, the kind of variables was correctly determined by 58.2% of the teacher candidates and incorrectly determined by the rest of the candidates. As noted, the correct percentage answer is 63.6% in the 1st step scores, whereas this value decreases to 58.2% in the 2nd step scores. Only 5.4% of the candidates correctly determined the number of variables, but did not correctly determine the kind of variables. According to the analyses related to each coordinate system, the maximum decrease in the correct answer was seen in the cylindrical coordinate system (from 70.2 to 63.5%). The minimum decrease (from 50% to 48.8%) was seen in determining the kind of variables of the shapes which may be drawn in any of the curvilinear coordinate systems. When the findings in table 2 were studied in detail, it was seen that the maximum decrease (9.6%) in the correct answer in determining the variables in S5 (figure 5(a)) in the spherical coordinate system. This was followed by C1 (figure 5(b)) with 8.4% in the cylindrical coordinate system.

Findings related to the 3rd step scores: determination of the number, kind and value of variables. In this section, the findings about the determination of the number, kind and value of variables were examined. These findings revealed that the teacher candidates gave correct answers for all steps. Therefore, the findings in the 3rd step scores are important in terms of reflecting the exact correct answers.

As expected, a much higher decrease was seen in the correct answer of the 3rd step scores in table 2. The correct answer was given by 58.2% in the 2nd step scores, whereas it

Table 2. Findings about CCDeT to the step scores.

			Coordinate Systems															
			Spherical						Spherical or Cylindrical		Cylindrical							
			%						%		%							
		Answers	S1	S2	S3	S4	S5	S6	SC1	SC2	C1	C2	C3	C4	C5	C6	C7	Mean
1^st Step (Number of var.)	Distribution of variable's number	**1V**	7,2	3,6	3,6	4,8	8,4	7,2	7,2	14,5	7,2	3,6	6,0	6,0	4,8	2,4	4,8	6,1
		2V	13,3	8,4	10,8	9,6	9,6	8,4	25,3	19,3	15,7	7,2	16,9	3,6	9,6	8,4	7,2	11,6
		3V	55,4	62,7	62,7	61,4	59,0	61,4	51,8	48,2	61,4	73,5	63,9	75,9	69,9	73,5	73,5	63,6
		4V	24,1	25,3	22,9	24,1	22,9	21,7	15,7	16,9	14,5	15,7	12,0	14,5	15,7	14,5	13,3	18,2
		Null	0,0	0,0	0,0	0,0	0,0	1,2	0,0	1,2	1,2	0,0	1,2	0,0	0,0	1,2	1,2	0,5
		Tot.	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
	According to each questions	**C**	55,4	62,7	62,7	61,4	59,0	61,4	51,8	48,2	61,4	73,5	63,9	75,9	69,9	73,5	73,5	63,6
		W	44,6	37,3	37,3	38,6	41,0	38,6	48,2	51,8	38,6	26,5	36,1	24,1	30,1	26,5	26,5	36,4
		Tot.	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
	According to coordinate systems	**C**	60,4						50,0		70,2							63,6
		W	39,6						50,0		29,8							36,4
		Tot.	100,0						100,0		100,0							100,0
2nd Step (Number and kind of var.)	According to each questions	**C**	54,2	59	56,6	55,4	49,4	55,4	50,6	47	53	67,5	57,8	69,9	62,7	67,5	66,3	58,2
		W	45,8	41	43,4	44,6	50,6	44,6	49,4	53	47	32,5	42,2	30,1	37,3	32,5	33,7	41,8
		Tot.	54,2	59	56,6	55,4	49,4	55,4	50,6	47	53	67,5	57,8	69,9	62,7	67,5	66,3	100,0
	According to coordinate systems	**C**	55,0						48,8		63,5							58,2
		W	45,0						51,2		36,5							41,8
		Tot.	100,0						100,0		100,0							100,0
3rd Step (Number, kind and value of var.)	According to each questions	**C**	33,7	39,8	32,5	2,4	10,8	31,3	20,5	20,5	31,3	44,6	8,4	7,2	21,7	49,4	21,7	25,1
		W	66,3	60,2	67,5	97,6	89,2	68,7	79,5	79,5	68,7	55,4	91,6	92,8	78,3	50,6	78,3	74,9
		Tot.	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
	According to coordinate systems	**C**	25,1						20,5		26,3							25,1
		W	74,9						79,5		73,7							74,9
		Tot.	100,0						100,0		100,0							100,0

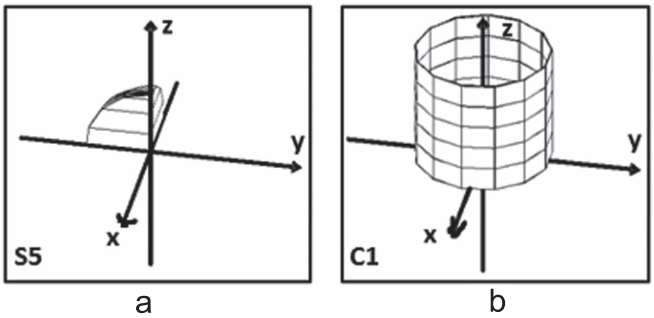


Figure 5. Shapes in S5 and C1.

was 25.1% in the 3rd step score. In other words, a decrease of 33.1% was seen. The correct answer in the spherical coordinate system decreased from 55.0% to 25.1%; a reduction of 29.9%. Maximum decreases in the correct answers in the spherical coordinate system (53.0% and 38.6%) were seen in S4 and S5 (figures 6 and 5(a)). The correct answer of S4 was very low (2.4%).

The correct percentage in the cylindrical coordinate system decreased from 63.5% to 26.3%; a reduction of 37.2%. The decrease in the correct answer in the cylindrical coordinate system was more than in the spherical coordinate system. The maximum correct answer decreased from 62.7% to 49.4% in C4 and C3 (figures 7(a) and (b)) in the cylindrical coordinate system. The shape in C4 was answered correctly by only 7.2% of the candidates.

C6 and S2 (figures 8(a) and (b)) had maximum correct answers (49.4%, 39.8%). The correct answer of the shapes, which may be drawn in any of the curvilinear coordinate systems, decreased from 48.8% to 20.5%.

Detailed findings related to the value of variables. The spherical part of table 3 is related to the determination of the values of variables in the spherical coordinate system. The teacher candidates had fewer problems with this determination compared to the others while determining the value of r in the spherical coordinate system. A total of 67.5% of the teacher candidates correctly determined the values of r. The correct determination of θ and ϕ was by 43.6% and 48.2%, respectively, of the candidates. The correct answers of all three values of the variables (r: 51,8%, θ: 30,1% and ϕ: 15,7%) in the spherical coordinate system was minimum in S4 (figure 6).

The cylindrical part of table 3 is related to the determination of the values of variables in the cylindrical coordinate system. The correct determination of r and z in the cylindrical coordinate system were close to each other and were 68.8% and 67.5%, respectively. Correct determination of the values of ϕ was obtained by 41.1% of the teacher candidates. It was noted that in C3 and C4 (figures 7(a) and (b)), the correct answers were 9.6% and 15.7%, respectively; these answers related to the determination of the values of ϕ are low.

The spherical/cylindrical part of table 3 is related to the shapes which may be drawn using any of the curvilinear coordinate systems. It was found that for the correct determination of the values of the variables in SC1 and SC2, the candidates who chose the spherical coordinate system had significant problems in determining θ, whereas the candidates who chose the cylindrical coordinate system had significant problems in determining z. In SC1, the correct answers of θ and z were provided by 16.9% and 22.9%,

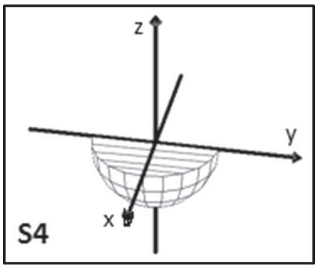


Figure 6. Shape in S4.

respectively, of the candidates. In SC2, the correct answers of θ and z were given by 16.9% and 27.7%, respectively, of the candidates.

The general percentages were also given by taking the variables of both coordinate systems into consideration in table 3. Accordingly, the teacher candidates had most problems with θ (63.1%) and minimal problems (32.0%) with r while determining the values of the variables.

Findings related to the CCDrT

Findings about the 1st step scores: drawing of the shape defined by the variables. The findings about drawing of the shapes defined by the variables are provided in table 4. According to the findings in table 4, the percentage of candidates who correctly drew the shapes was 35.7%. The correct drawing of the shapes was 40.2% in the spherical coordinate system and 31.8% in the cylindrical coordinate system.

Accordingly, in the spherical coordinate system the mostly correctly drawn (70.3%) shape was in S2 (figure 8(b)). The mostly wrongly drawn (96.9%) shape was in S4 (figure 6).

In the cylindrical coordinate system, the mostly correctly drawn (53.1%) shapes were in C1 (figure 5(b)) and C8 (figure 9). In the cylindrical coordinate system, the mostly wrongly drawn shapes were in C3 and C4 (figures 7(a) and (b)) by 92.2% of the teacher candidates.

Findings about the 2nd step scores: drawing of the shape and positioning the shape appropriately to the coordinate system. As stated previously, it was observed that although some candidates correctly drew the shapes, these candidates could not position their drawings appropriately (53.0% and 38.6%) to the coordinate system. These drawings were accepted as correct as 1st step scores, but they were considered wrong as 2nd step scores (table 4).

The percentage of candidates who made correct drawings decreased from 35.7% to 30.2% according to the 2nd step scores. Decreases in the shapes that were correctly drawn in each of the spherical and cylindrical coordinates were detected among 6.9% and 4.6% of the candidates, respectively. The maximum decrease (17.1%) was seen in S6 (figure 10(a)) in the spherical coordinate system and in both C1 (figure 5(b)) and C6 (figure 8(a)) in the cylindrical coordinate system with an equal rate of 7.8%. There was no change in S2 (figure 8(b)) and S4 (figure 6) in the spherical coordinate system. In the cylindrical coordinate system there was no change in C7 (figure 10(b)).

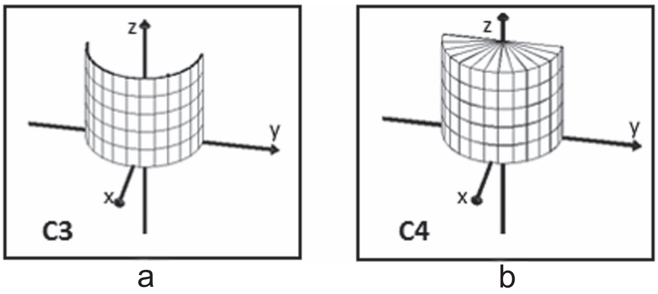


Figure 7. Shapes in C3 and C4.

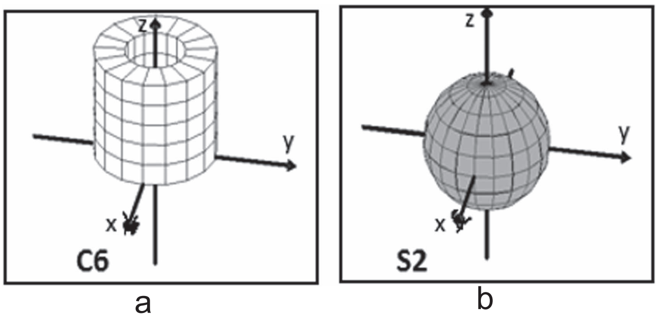


Figure 8. Shapes in C6 and S2.

Findings about the 3rd step scores: drawing of the shape, positioning the shape appropriately to the coordinate system and specifying the shape type. The evaluation in this section included specifying the shape type suggested by the candidates in addition to the previous steps (table 4). In other words, these findings revealed the teacher candidates who gave correct answers for all steps. So, the findings in the 3rd step scores are important in terms of reflecting the exact correct answers.

The correct percentage of answers decreased from 30.2% to 28.5% for the 3rd step scores. The decrease in the correct answers is as follows: for the spherical coordinate system from 33.3% to 31.7% and for the cylindrical coordinate system from 27.2% to 25.4%. The maximum decrease was seen in S7 (figure 11) in the spherical coordinate system with 4.7% and in C7 (figure 10(b)) in the cylindrical coordinate system with 4.7%. There was no change in S3, S4, S5, C1, C3, C4 and C5.

Detailed findings about the use of variables in the drawings. As seen in table 5, the correct usage of the variables by the candidates is different from each other. The drawn shape in which r was mostly used wrongly (25.0%) is in S5 (figure 5(a)) in the spherical coordinate system and in C3 (figure 7(a)) in the cylindrical coordinate system (46.9%). Regarding the usage of ϕ, maximum mistakes (95.3%) were found in S4 (figure 6) in the spherical coordinate system and in C4 (figure 7(b)) in the cylindrical coordinate system (90.6%). In the spherical coordinate system, the shapes in which θ was used wrongly (50%) were seen in S4 (figure 6). In the cylindrical coordinate system, the shape in which z was used wrongly (50%) was seen in C5 (figure 12(b)).

Table 3. Findings about determining the value of each variable.

General percentages				
	Var. (%)	Var. (%)	Var. (%)	Var. (%)
Ans.	**r**	**z**	$\boldsymbol{\theta}$	**Φ**
C	68,0	58,1	36,9	47,6
W	32,0	41,9	63,1	52,4

Spherical					**Cylindrical**					**Spherical/Cylindrical**					
		Var. (%)					**Var. (%)**					**Var. (%)**			
Ques.	**Ans.**	**r**	$\boldsymbol{\theta}$	**Φ**	**Ques.**	**Ans.**	**r**	**z**	**Φ**	**Ques.**	**Ans.**	**r**	**z**	$\boldsymbol{\theta}$	**Φ**
S1	**C**	74.7	50.6	68.7	**C1**	**C**	63.9	61.4	59.0	**SC1**	**C**	61.4	22.9	16.9	73.5
	W	25.3	49.4	31.3		**W**	36.1	38.6	41.0		**W**	38.6	77.1	83.1	26.5
S2	**C**	75.9	51.8	71.1	**C2**	**C**	62.7	69.9	74.7	**SC2**	**C**	72.3	27.7	16.9	63.9
	W	24.1	48.2	28.9		**W**	37.3	30.1	25.3		**W**	27.7	72.3	83.1	36.1
S3	**C**	71.1	51.8	61.4	**C3**	**C**	63.9	65.1	9.6	**Mean**	**C**	66.9	25.3	16.9	68.7
	W	28.9	48.2	38.6		**W**	36.1	34.9	90.4		**W**	33.1	74.7	83.1	31.3
S4	**C**	51.8	30.1	15.7	**C4**	**C**	72.3	69.9	15.7						
	W	48.2	69.9	84.3		**W**	27.7	30.1	84.3						
S5	**C**	54.2	44.6	20.5	**C5**	**C**	71.1	68.7	26.5						
	W	45.8	55.4	79.5		**W**	28.9	31.3	73.5						
S6	**C**	77.1	32.5	51.8	**C6**	**C**	75.9	67.5	75.9						
	W	22.9	67.5	48.2		**W**	24.1	32.5	24.1						
Mean	**C**	67.5	43.6	48.2	**C7**	**C**	72.3	69.9	26.5						
	W	32.5	56.4	51.8		**W**	27.7	30.1	73.5						
					Mean	**C**	68.8	67.5	41.1						
						W	31.2	32.5	58.9						

The correct usage of r, θ and ϕ in the spherical coordinate system was 82.1%, 61.8% and 58.0%, respectively. The correct usage of r, ϕ and z in the cylindrical coordinate system was 69.1%, 51.6% and 60.4%, respectively.

The general percentages related to the usage of the variables are also provided in table 5. When the variables in both coordinate systems are considered, the ordering of the correct usage of r, θ, ϕ and z is as follows: r (75.2%), θ (61.8%), ϕ (54,6%) and z (60.4%).

Findings related to comparison of the CCDeT and CCDrT

Descriptive comparison of findings obtained from the 3rd step scores of the CCDeT and the CCDrT. In table 6, a descriptive comparison of findings obtained from 3rd step scores of the CCDeT and the CCDrT is shown for ease of comparison.

According to table 6 in the spherical coordinate system, physics teacher candidates made maximum mistakes in S4 (figure 6) in both the CCDeT and the CCDrT with percentages of 97.6% and 96.9%, respectively. In the cylindrical coordinate system, the maximum mistakes were made in C4 (figure 7(b)) with 92.8% in the CCDeT and 95.3% in the CCDrT. In the spherical coordinate system, the maximum correct answers were found in S2 (figure 8(b)) with 39.8% in the CCDeT and 68.8% in the CCDrT. In the cylindrical coordinate system, the maximum correct answers were found in C6 (figure 8(a)) with 49.4% in the CCDeT and in C1 (figure 5(b)) and 45.3% in the CCDrT. Mean percentages of the correct answers in the cylindrical coordinate system were close to each other, with 25.6% in the CCDeT and 25.4% in the CCDrT. For correct answers in the spherical coordinate system, the mean percentage in the CCDeT was 24.4% whereas it was 31.7% in the CCDrT. The general mean percentage of the correct answers in each curvilinear coordinate system were calculated as 25.1% in the CCDeT and 28.5% in the CCDrT.

Inferential statistics for the findings obtained from the 3rd step scores of the CCDeT and CCDrT. The number of teacher candidates who answered both the CCDeT and the CCDrT was 54. Data that were taken from the CCDeT and the CCDrT do not have a normal distribution. Therefore, data were compared using the Wilcoxon T-test. Descriptive statistics of the Wilcoxon T-test are provided in table 7 and the Wilcoxon T-test results are provided in table 8.

As seen in table 7, the mean scores of the teacher candidates were very low and close to each other. As is understood from table 8, there was no significant difference between the means of the scores obtained from the CCDeT and the CCDrT ($p > 0.05$).

Results and discussion

A total of 63.6% of the teacher candidates correctly suggested that there were three variables defining a shape in the curvilinear coordinate system. A total of 36.4% of them had incorrect knowledge about the number of the variables in the curvilinear coordinate systems. As many as 18.2% of the candidates marked the four variables r, θ, ϕ and z without distinguishing between the spherical and the cylindrical coordinate systems. In the spherical coordinate system, the variables to define a disc (SC1) on the plane x–y should be r: $0 \rightarrow R$, $\theta = \frac{\pi}{2}$ and ϕ: $0 \rightarrow 2\pi$. If the same disc is defined in the cylindrical coordinate system the following cases should be provided: r: $0 \rightarrow R$ and ϕ: $0 \rightarrow 2\pi$ and $z = 0$. A disc may be defined using variables belonging to any of the curvilinear coordinate systems. However, there was the

Table 4. Findings about CCDrT to the step scores.

			Coordinate Systems															
			Spherical							Cylindrical								
		Answers	S1	S2	S3	S4	S5	S6	S7 (SC1)	C1	C2	C3	C4	C5	C6	C7	C8 (SC2)	Mean
1st Step (drawing)	According to each questions	**C**	50,0	70,3	56,3	3,1	17,2	35,9	48,4	53,1	50,0	7,8	7,8	23,4	46,9	12,5	53,1	35,7
		W	50,0	29,7	43,8	96,9	82,8	64,1	51,6	46,9	50,0	92,2	92,2	76,6	53,1	87,5	46,9	64,3
		Tot.	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
	According to coordinate systems	**C**				40,2							31,8					35,7
		W				59,8							68,2					64,3
		Tot.				100,0							100,0					100,0
2nd Step (drawing and positioning)	According to each questions	**C**	43,8	70,3	45,3	3,1	14,1	18,8	37,5	45,3	43,8	6,3	4,7	17,2	39,1	12,5	48,4	30,2
		W	56,3	29,7	54,7	96,9	85,9	81,3	62,5	54,7	56,3	93,8	95,3	82,8	60,9	87,5	51,6	69,8
		Tot.	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
	According to coordinate systems	**C**				33,3							27,2					30,2
		W				66,7							72,8					69,8
		Tot.				100,0							100,0					100,0
3rd Step (drawing, positioning and type)	According to each questions	**C**	40,6	68,8	45,3	3,1	14,1	17,2	32,8	45,3	42,2	6,3	4,7	17,2	35,9	7,8	43,8	28,5
		W	59,4	31,3	54,7	96,9	85,9	82,8	67,2	54,7	57,8	93,8	95,3	82,8	64,1	92,2	56,3	71,5
		Tot.	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
	According to coordinate systems	**C**				31,7							25,4					28,5
		W				68,3							74,6					71,5
		Tot.				100,0							100,0					100,0

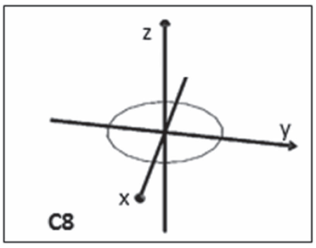


Figure 9. Expected drawing for C8.

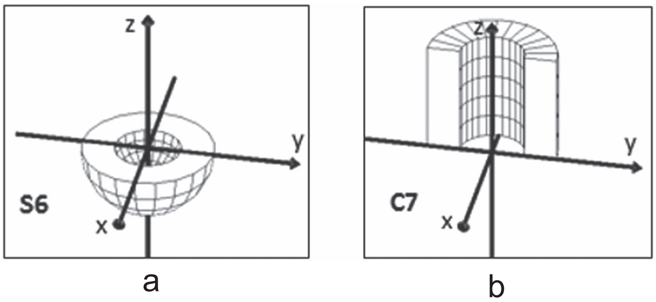


Figure 10. Expected drawings for S6 and C7.

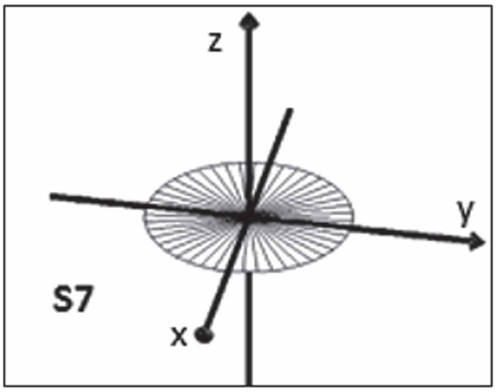


Figure 11. Expected shape for S7.

impression that in the curvilinear coordinate system a disc may be defined by a change in two variables depending on the fact that some candidates ignored the variable with a constant value. A similar impression was observed in determining the number of variables of a circle shape, when some candidates only considered the variables with a change. Therefore, the shapes for which the candidates exhibited minimum success in determining the number of variables are the disc and the circle. Approximately half of the teacher candidates wrongly specified the variables of each curvilinear coordinate system. In determining the values of the variables, over half of the candidates had problems with θ and ϕ in the spherical coordinate system and with ϕ in the cylindrical coordinate system. Minimum problems were experienced

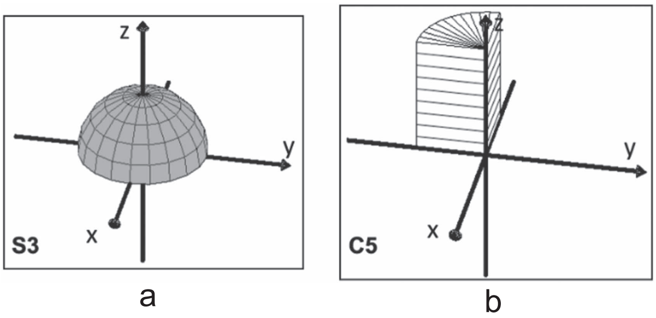


Figure 12. Shapes in S3 and C5.

Table 5. Detailed findings about use of variables in the drawings.

General percentages				
	Var. (%)	Var. (%)	Var. (%)	Var. (%)
Ans.	r	θ	Φ	z
C	75,2	61,8	54,6	60,4
W	24,8	38,2	45,4	39,6

Spherical					**Cylindrical**				
		Var. (%)					**Var. (%)**		
Ques.	**Ans.**	**r**	$\boldsymbol{\theta}$	**Φ**	**Ques.**	**Ans.**	**r**	**Φ**	**z**
S1	**C**	76.6	65.6	85.9	**C1**	**C**	68.8	79.7	64.1
	W	23.4	34.4	14.1		**W**	31.2	20.3	35.9
S2	**C**	89.1	78.1	85.9	**C2**	**C**	76.6	78.1	59.4
	W	10.9	21.9	14.1		**W**	23.4	21.9	40.6
S3	**C**	85.9	79.7	68.8	**C3**	**C**	53.1	10.9	59.4
	W	14.1	20.3	31.2		**W**	46.9	89.1	40.6
S4	**C**	82.8	50.0	4.7	**C4**	**C**	75.0	9.4	53.1
	W	17.2	50.0	95.3		**W**	25.0	90.6	46.9
S5	**C**	75.0	56.3	18.8	**C5**	**C**	71.9	28.1	50.0
	W	25.0	43.7	81.2		**W**	28.1	71.9	50.0
S6	**C**	78.1	51.6	67.2	**C6**	**C**	78.1	93.8	57.8
	W	21.9	48.4	32.8		**W**	21.9	6.2	42.2
S7 (SC1)	**C**	87.5	51.6	75.0	**C7**	**C**	64.1	23.4	59.4
	W	12.5	48.4	25.0		**W**	35.9	76.6	40.6
Mean	**C**	82.1	61.8	58.0	**C8 (SC2)**	**C**	65.6	89.1	79.7
	W	17.9	38.2	42.0		**W**	34.4	10.9	20.3
					Mean	**C**	69.1	51.6	60.4
						W	30.9	48.4	39.6

in determining the value of *r*. According to the findings obtained from the CCDeT it can be said that most of the candidates had problems with determining the number, kind and value of variables in the curvilinear coordinate systems. This problem was observed in nearly equal ratios in both the spherical and cylindrical coordinate systems.

Table 6. Descriptive findings obtained from the 3rd step scores of the CCDeT and the CCDrT.

			CCDeT				CCDrT			
		Ques.	C(%)	W(%)	C(%)	W(%)	C(%)	W(%)	C(%)	W(%)
Coordinate systems	**Spherical**	**S1**	33.7	66.3			40.6	59.4	31.7	68.3
		S2	39.8	60.2			68.8	31.3		
		S3	32.5	67.5			45.3	54.7		
		S4	2.4	97.6	24.4	75.6	3.1	96.9		
		S5	10.8	89.2			14.1	85.9		
		S6	31.3	68.7			17.2	82.8		
		S7(**SC1**)	20.5	79.5			32.8	67.2		
	Cylindrical	**C1**	31.3	68.7			45.3	54.7	25.4	74.6
		C2	44.6	55.4			42.2	57.8		
		C3	8.4	91.6			6.3	93.8		
		C4	7.2	92.8	25.6	74.4	4.7	95.3		
		C5	21.7	78.3			17.2	82.8		
		C6	49.4	50.6			35.9	64.1		
		C7	21.7	78.3			7.8	92.2		
		C8(**SC2**)	20.5	79.5			43.8	56.3		
		Mean	25.1	74.9	25.1	74.9	28.5	71.5	28.5	71.5

Table 7. Descriptive statistics of the Wilcoxon T-test.

	Spherical		Cylindrical		Whole test	
	CCDeT	CCDrT	CCDeT	CCDrT	CCDeT	CCDrT
Number of questions	7	7	8	8	15	15
$\bar{\mathbf{x}}$	2.04	2.31	2.59	2.24	4.63	4.56
σ	0.29	0.26	0.36	0.27	0.60	0.46

Table 8. Wilcoxon T-test results.

Wilcoxon T-applied groups	z	p
CCDeT spherical–CCDrT spherical	−0.149	0.881
CCDrT spherical–CCDrT cylindrical	−1.192	0.233
CCDeT whole test–CCDrT whole test	−1.033	0.302

According to the 3rd step scores of the CCDeT, four shapes, which were mostly answered correctly by about 10% of the candidates, are shown above. If we look closely, the values of the ϕ variable in S4, C3 and C4 are the same $\left(\phi\colon -\frac{\pi}{2} \rightarrow \frac{\pi}{2}\right)$. This case indicates that the candidates had problems particularly with ϕ in these shapes. It was mentioned above that most problems were observed with θ in the spherical coordinate system. Part of the problem experienced with the shape in S4 is related to θ $\left(\theta = \frac{\pi}{2}\right)$. It is possible that the candidates were not familiar with the position of the shape shown in S5. According to our previous experience, understanding of the candidates is facilitated if the same shape is placed entirely to the positive part of the coordinate system. Educators/textbook authors generally place the same or similar shapes in the coordinate system with the same perspective. For that reason, it is likely that the teacher candidates could not develop a correct point of view for different cases.

Similar to the CCDeT, in the CCDrT, the variable with which minimum problem was encountered was r. Although it appeared that most problems were seen in ϕ in the spherical coordinate system, there is no obvious difference between the values related to the incorrect usage of θ and ϕ. In the cylindrical coordinate system, incorrect usage of ϕ was prominent. The findings obtained from both the CCDeT and the CCDrT match with each other. The 3rd step scores of the CCDrT support this match. Namely, four of the five shapes contained in the questions which were mostly answered correctly by about 10% of the candidates were the same as the ones in the CCDrT (figure 13). Moreover, the candidates also experienced problems during drawing the shape related to C7 (figure 10(b)).

Briefly, we can say that most physics teacher candidates had problems during drawing a shape using the variables of the curvilinear coordinate systems or in determining the variables of the drawn shapes. Although studies which are directly associated with this study could not be found in the literature, the results correspond to the studies of Sayre and Wittmann (2008) and Wilcox *et al* (2013). It can be said that physics teacher candidates, like other students, are familiar with the Cartesian coordinate system. For this reason, they have more problems with θ and ϕ than with r in the spherical coordinate system, and with ϕ than with r and z in the cylindrical coordinate system.

The problem may be reduced by using visual models about the subject during the teaching process, giving candidates the opportunity to develop models and using different

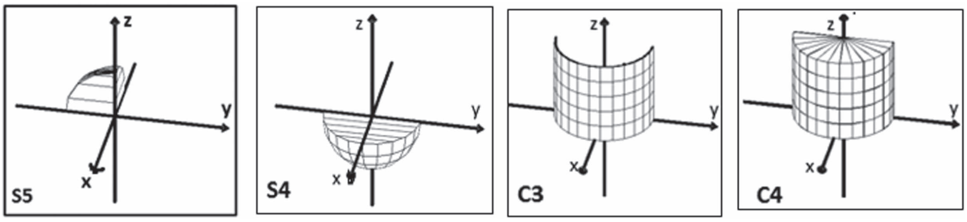


Figure 13. Shapes in S5, S4, C3 and C4.

shapes with different perspectives in applied courses or textbooks. For example, Koss (2011) has developed material that facilitates understanding of the three-dimensional Cartesian, cylindrical and spherical coordinate systems. Koss (2011) highlighted that he struggled to teach three-dimensional coordinate systems in the past as students did not understand the shapes related to this subject. He specified that after using his materials, the students understood the three-dimensional coordinate systems better as compared to previous methods. Esteban *et al* (2004) have used the augmented reality tool in the multi-variate calculus course to integrate 3D virtual objects in a real environment. They concluded that in their study, if the technology is integrated in education, mathematical concepts become better understood, although according to Koss (2011) some students have great difficulties in understanding three-dimensional coordinate systems represented by a computer animation. In another study (Scott *et al* 2003), the researchers have developed the electromagnetic visualization computer application (EM-Viz) to support undergraduate students in gaining an understanding of the theory of electromagnetics, and the results of the study have revealed the efficiency of using EM-Viz. According to Notaroš (2013) if vector analysis is taught using traditional methods in electromagnetic field subjects in which the curvilinear coordinate systems are used, this subject is less internalized by the students as it is an abstract and mathematically complex subject. Therefore, it is important to find different ways that will facilitate the development of three-dimensional perceptions. Improvement of three-dimensional perceptions among students can be the solution to overcoming their problems in understanding curvilinear coordinates.

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