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A Gaussian Model for Anisotropic Strange Quark Stars

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For studying the anisotropic strange quark stars, we assume that the radial pressure inside an anisotropic star can be obtained simply by isotropic pressure plus an additional Gaussian term with three free parameters (A , μ and χ). According to recent observations, a pulsar in a mass range of $1.97 \pm 0.04 M_{\odot}$ has been measured. Hence, we take this opportunity to set the free parameters of our model. We fix χ by applying boundary and stability conditions and then search the $A - \mu$ parameter space for a maximum mass range of $1.9 M_{\odot} < M_{\max} < 2.1 M_{\odot}$. Our results indicate that anisotropy increases the maximum mass $M_{\max}$ and also its corresponding radius R for a typical strange quark star. Furthermore, our model shows magnetic field and electric charge increase the anisotropy factor Δ . In fact, Δ has a maximum on the surface and this maximum goes up in the presence of magnetic field and electric charge. Finally, we show that anisotropy can be more effective than either magnetic field or electric charge in raising maximum mass of strange quark stars.

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Strange quark stars are hypothetical exotic compact objects which were first speculated in Refs. [1–4]. In general, mass M and radius R of stars can be evaluated by solving hydrostatic equilibrium equations. In the case of compact stars, due to high density, general relativity (GR) dominates and Newtonian hydrostatic equilibrium should be replaced by its GR counterpart. Assuming a static and spherical symmetric geometry and an isotropic matter, the Einstein field equations lead to the Tolman–Oppenheimer–Volkoff equation (TOV).^[5] The structure of compact stars has been investigated by solving the TOV equation by using a suitable equation of state.^[6–8] Now in this work, we deal with the strange quark matter (SQM) where its EOS is well described by the MIT bag model.^[6,10] In very high densities, a phase transition to quark matter has been predicted.^[11] Specifically a strange quark matter might be highly stable. Computation of critical density of phase transition to quark matter depends on the choice of the specific theory. While the quantum chromodynamics (QCD) is widely believed as the fundamental theory of strong interactions, it is unknown in what density the phase transition will happen empirically.^[11–13] Furthermore, the existence of strange quark stars has not been confirmed theoretically and even experimentally. Nuclear matter in very high densities can be anisotropic and there is a high interest in studying the effect of anisotropy in the structure of relativistic stars.^[7,14] Additionally, existence of a solid core and a strong magnetic field in neutron stars can be related to anisotropy in the matter of stars.^[7,15] More specifically, according to Ref. [16] the origin of a local anisotropic force in solid quark stars could be from the development of elastic energy as stars spin down, cool down or due to the release of gravitational waves. Solving anisotropic TOV equation requires a physically reasonable assumption. For instance, in Ref. [7] a specific density profile $\rho(r)$ has been chosen and in Ref. [9] a special metric function

$\Lambda(r)$ has been utilized to solve it analytically. However in this study, we try to apply a numerical method for solving the modified TOV equation by considering a perturbation term. Initially, we assume a perfect fluid to obtain a solution for radial pressure $p_r(r)$. Then in the anisotropic case we use the previous $p_r(r)$ as the unperturbed solution to study R , M and especially pressure anisotropy (Δ) in this kind of star. Finally we investigate the effect of magnetic field and electric charge on characteristics and pressure anisotropy of stars. The rest is organized as follows: we first solve the TOV equation for isotropic matter and then compare it with a magnetized, electrically charged and anisotropic one. Then discussion of the result and concluding remarks are represented.

For describing mass and radius of a self gravitating configuration in relativistic terms we have to consider a suitable metric. The line element in the interior of a star, assuming a static and spherically symmetric geometry can be written as

$$ds^2 = -c^2 e^{2\phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\Omega^2, \quad (1)$$

where c is the speed of light in vacuum, and ϕ and Λ are spherically symmetric metric functions. To obtain a GR hydrostatic equilibrium, we have to solve the Einstein equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (2)$$

for the metric given in Eq. (1). Also we must choose a reasonable energy–momentum tensor which satisfies the following conservation law

$$\nabla_{\nu} T^{\mu\nu} = 0. \quad (3)$$

Then we try to obtain the TOV equation for two different cases. First, we consider an uncharged perfect fluid in the interior of the star. The energy–momentum tensor of this matter is well-defined^[17] and

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its gravitational mass is given as follows:^[11]

$$m(r) = \int_0^r 4\pi x^2 \rho(x) dx. \quad (4)$$

Using the above equation, the Einstein Eq. (2) and the conservation law (3) for metric Eq. (1) yield the following hydrostatic equilibrium equation

$$\frac{dp}{dr} = -(\rho c^2 + p) \frac{4\pi G p r^3 + m G c^2}{r c^2 (r c^2 - 2mG)}. \quad (5)$$

Noticeably, gravitational mass m , pressure p , and matter density ρ have spherical symmetry, thus they are functions of radial coordinate r . Equation (4) together with Eq. (5) is called the TOV equation which is unsolvable unless it can be supplemented by an EOS.^[18] In this work, we utilize the MIT bag model which is widely used to describe quark matter,

$$\frac{p}{c^2} = \alpha(\rho - \rho_s). \quad (6)$$

In this model it is assumed that the quarks are free fermions in a bag with a negative pressure.^[19] The constants of the SQM2 model are $\rho_s = 3.056 \times 10^{17} \text{ kg}\cdot\text{m}^{-3}$ and $\alpha = 0.324$.^[20] It is now possible to solve the TOV equation numerically, by using physical boundary conditions.^[17] In addition, in the stellar interior, the following conditions should be satisfied,

$$p > 0, \quad (7a)$$

$$\frac{dp}{dr} < 0, \quad (7b)$$

$$\sqrt{\frac{dp}{d\rho}} \leq c, \quad (7c)$$

$$\frac{dp}{d\rho} \geq 0. \quad (7d)$$

Conditions (7a) and (7b) are trivial for preserving hydrostatic equilibrium and Eq. (7c) refers to causality inside the star which does not permit $\alpha > 1$ in Eq. (6).^[6] Also the EOS should satisfy the microscopic stability condition (7d), otherwise the star collapses spontaneously.^[21] Next, we will use the obtained pressure $p(r)$ as the unperturbed radial pressure ($p_r(r)$) for the anisotropic matter.

In this case, instead of a perfect fluid we are dealing with an anisotropic matter. It means that pressures in radial and tangential directions are not necessarily the same that can cause oblateness in stars. With this assumption, which is predicted in very high density ranges,^[7] the energy momentum tensor in the CGS unit system will be^[9]

$$T_\nu^\mu = - \begin{pmatrix} -(\rho_A c^2 + \frac{B^2}{8\pi}) & 0 & 0 & 0 \\ 0 & p_r - \frac{B^2}{8\pi} & 0 & 0 \\ 0 & 0 & p_t + \frac{B^2}{8\pi} & 0 \\ 0 & 0 & 0 & p_t + \frac{B^2}{8\pi} \end{pmatrix}, \quad (8)$$

where p_t , p_r , ρ_A , and B are tangential pressure, radial pressure, anisotropic matter density and magnetic

field, respectively. The gravitational mass which is the total contribution of the energy density takes the new form as

$$m(r) = \int_0^r 4\pi x^2 \left(\rho_A(x) + \frac{B^2}{8\pi c^2} \right) dx. \quad (9)$$

Defining $\Delta = p_t - p_r$ as the anisotropy factor,^[7] the Einstein Eq. (2) and conservation Eq. (3) lead to

$$e^{-2\Lambda} = 1 - \frac{2mG}{rc^2}, \quad (10)$$

$$\frac{d\phi}{dr} = \frac{4\pi G r^3 (p_r - \frac{B^2}{8\pi}) + m G c^2}{rc^2 (rc^2 - 2mG)}, \quad (11)$$

$$\frac{d\phi}{dr} = \frac{-1}{(\rho_A c^2 + p_r)} \left(\frac{dp_r}{dr} - \frac{2}{r} \Delta \right). \quad (12)$$

Eliminating $\frac{d\phi}{dr}$ between Eqs. (11) and (12) leads to

$$\frac{dp_r}{dr} = -(\rho_A c^2 + p_r) \frac{4\pi G r^3 (p_r - \frac{B^2}{8\pi}) + m G c^2}{rc^2 (rc^2 - 2mG)} + \frac{2}{r} \Delta. \quad (13)$$

It should be noted that if $\Delta > 0$, in Eq. (13) the magnitude of radial pressure derivative will decrease. This causes a softer falling of radial pressure which results in higher maximum masses and their corresponding radii. Equations (9) and (13) represent the modified form of the TOV equation. It is easy to see that putting $B = 0$ and $\Delta = 0$ in Eq. (13), simply returns us to the perfect fluid case. In an anisotropic matter, the MIT bag model is given by^[7]

$$\frac{p_r}{c^2} = \alpha(\rho_A - \rho_s). \quad (14)$$

It is worth mentioning that tangential pressure p_t does not necessarily vanish on the surface of the star (contrary to p_r ^[7,9]). Solving the modified TOV equation requires some extra assumptions. Assuming that anisotropy has a slight effect on p_r , we add a perturbation to isotropic pressure profile to obtain p_r . If we represent anisotropic radial pressure by $p_r(r)$ and the isotropic radial pressure obtained in the perfect fluid case by $p(r)$, our assumption is

$$p_r(r) = p(r) + \delta p(r). \quad (15)$$

The following perturbation function must satisfy the boundary conditions

$$p_r(0) = p_c, \quad (16a)$$

$$p_r(R) = 0, \quad (16b)$$

$$\left. \frac{dp_r}{dr} \right|_{r=0} = 0, \quad (16c)$$

$$\left. \frac{dp_r}{dr} \right|_{r=R} = 0. \quad (16d)$$

Regarding Eq. (15), $p(r)$ and $p_r(r)$ must satisfy the mentioned boundary conditions, thus $\delta p(r)$ must meet those conditions too. The Gaussian function is the simplest choice but not the only one (for instance, the Lorentzian function could be another choice for $\delta p(r)$)

which yields no significant difference) for fulfillment of the conditions (16a)–(16d),

$$\delta p(r) = A \exp\left(\frac{-(r-\mu)^2}{2\chi^2}\right). \quad (17)$$

Considering the assumption, we propose that in Eq. (17), the anisotropy factor will be obtained from Eqs. (13) and (15) as

$$\Delta = \frac{r}{2} \left[\frac{dp}{dr} + \frac{-(r-\mu)}{\chi^2} \delta p + (\rho_A c^2 + p + \delta p) \frac{4\pi G r^3 (p + \delta p - \frac{B^2}{8\pi}) + m G c^2}{rc^2(rc^2 - 2mG)} \right], \quad (18)$$

where the anisotropic perturbed density and mass are, respectively,

$$\rho_A = \frac{p + \delta p}{\alpha c^2} + \rho_s, \quad (19)$$

$$m(r) = \int_0^r 4\pi x^2 \left(\rho_A(x) + \frac{B^2}{8\pi c^2} \right) dx. \quad (20)$$

Gaussian perturbation satisfies the above mentioned boundary conditions physically. In fact, δ and its derivative become nearly zero at the origin and also on the surface by this choice. To obtain a reasonable value for A , χ , and μ , we tune them so that the boundary condition and also the star's stability condition are satisfied. According to Ref. [7] the anisotropy can cause an increase in the radial pressure. Hence we have chosen a positive and small enough value compared with p_r for parameter A . Then we use the predicted maximum magnetic field (10^{17} – 10^{18} G [22]) that can produce anisotropy in the matter to evaluate the maximum effect of magnetization on stars' characteristics. In the anisotropic magnetized case, there are five variables (m , p_r , p_t , ρ and Δ) and five Eqs. (9)–(15). We have used the Runge–Kutta–Fehlberg fourth–fifth order method (RKF45 method) to solve the corresponding coupled ODE systems in each of the cases in the following: (1) perfect fluid ($\Delta = 0$ and $B = 0$), (2) only magnetized ($\Delta = 0$ and $B \neq 0$), (3) only anisotropic ($\Delta \neq 0$ and $B = 0$), and (4) anisotropic and magnetized ($\Delta \neq 0$ and $B \neq 0$).

Now, except for magnetic field, we consider a radial electric field $E(r)$ in energy–momentum tensor as

$$E(r) = \frac{1}{r^2} \int_0^r 4\pi x^2 \rho_{ch} e^A dx, \quad (21)$$

where ρ_{ch} is the charge density. By this assumption for matter, the energy–momentum tensor is

$$T_\nu^\mu = \begin{pmatrix} -(\rho_A c^2 + \frac{E^2}{8\pi}) & 0 & 0 & 0 \\ 0 & p_r - \frac{E^2}{8\pi} & 0 & 0 \\ 0 & 0 & p_t + \frac{E^2}{8\pi} & 0 \\ 0 & 0 & 0 & p_t + \frac{E^2}{8\pi} \end{pmatrix}, \quad (22)$$

and the Einstein field equations lead to

$$e^{-2\Lambda} = 1 - \frac{2mG}{rc^2}, \quad (23)$$

$$\frac{d\phi}{dr} = \frac{4\pi G r^3 \left(p_r - \frac{E^2}{8\pi} \right) + m G c^2}{rc^2(rc^2 - 2mG)}, \quad (24)$$

$$\frac{d\phi}{dr} = \frac{-1}{(\rho_A c^2 + p_r)} \left(\frac{dp_r}{dr} - \frac{2}{r} \Delta - E \rho_{ch} e^{\Lambda(x)} \right). \quad (25)$$

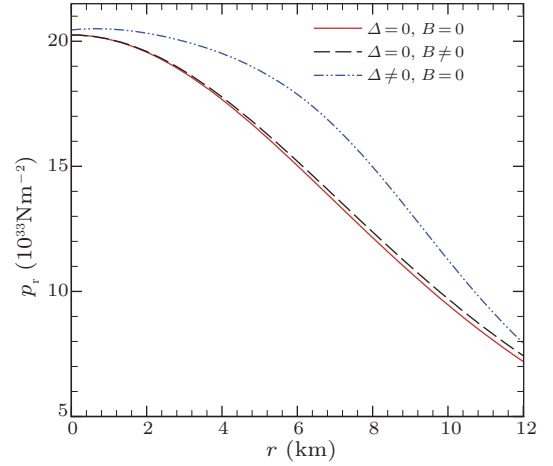


Fig. 1. Radial pressure p_r versus radial coordinate r for various cases: non-magnetized isotropic (perfect fluid), magnetized isotropic, and non-magnetized anisotropic matter with $B = 8 \times 10^{17}$ G, $A = 3 \times 10^{33}$ Nm $^{-2}$, $\chi = 3 \times 10^3$ m and $\mu = \frac{2R}{3}$. In all the cases the central density has the same value of $\rho_c = 1 \times 10^{18}$ kgm $^{-3}$.

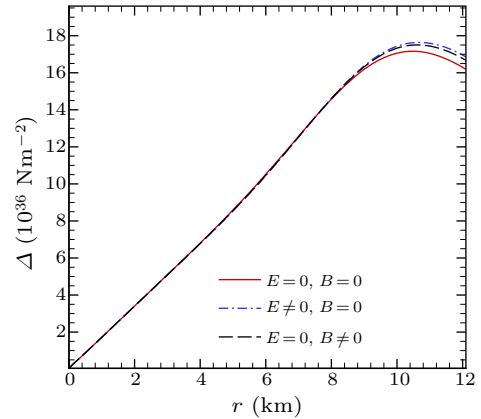


Fig. 2. Anisotropy factor Δ versus radial coordinate r in three cases. Uncharged and non-magnetized anisotropic matter (solid curve), magnetized anisotropic matter with $B = 8 \times 10^{17}$ G (dashed curve), and charged anisotropic matter with $f = 10^{-4}$ esug $^{-1}$ (dotted curve). In all the cases we have used an identical central density of $\rho_c = 1 \times 10^{18}$ kgm $^{-3}$.

In this case, the gravitational mass which is the total contribution of the energy density takes the new form as [23,24]

$$m(r) = \int_0^r 4\pi x^2 \left(\rho_A(x) + \frac{E(x)^2}{8\pi c^2} \right) dx. \quad (26)$$

Similar to the magnetized case, by eliminating $\frac{d\phi}{dr}$ we

obtain

$$\frac{dp_r}{dr} = -(\rho_A c^2 + p_r) \frac{4\pi G r^3 \left(p_r - \frac{E^2}{8\pi}\right) + mGc^2}{rc^2(r^2 - 2mG)} + E\rho_{ch}e^\Lambda + \frac{2}{r}\Delta. \quad (27)$$

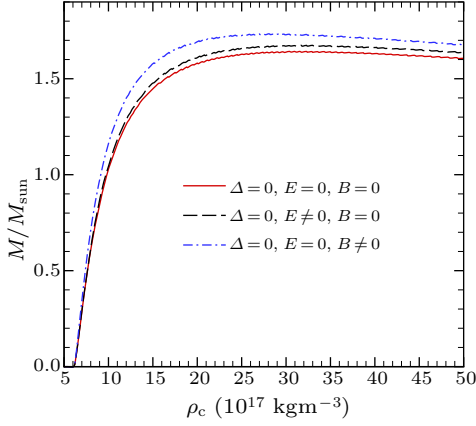


Fig. 3. Gravitational mass M versus central density ρ_c in three cases: perfect fluid (solid curve), magnetized isotropic matter with $B = 8 \times 10^{17}$ G (dashed curve), and charged isotropic matter (dash-dotted curve).

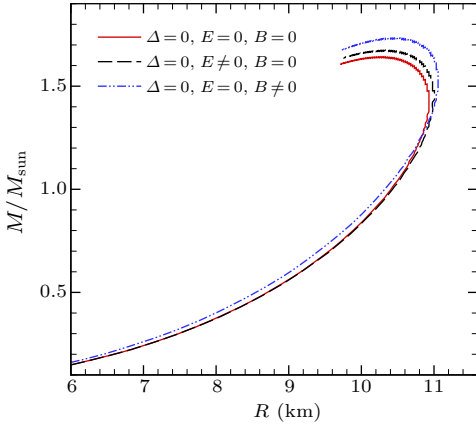


Fig. 4. Gravitational mass M versus radius of star R in three cases: perfect fluid (solid curve), magnetized isotropic matter with $B = 8 \times 10^{17}$ G (dashed curve), and charged isotropic matter (dash-dotted curve).

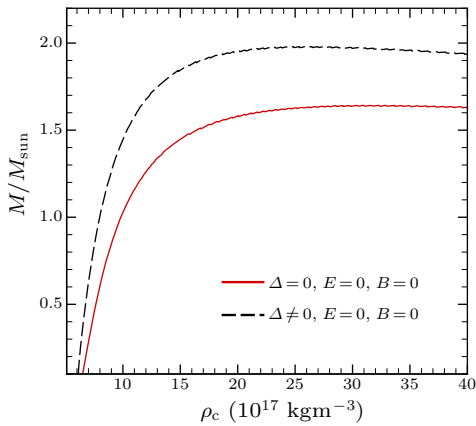


Fig. 5. Gravitational mass M versus central density ρ_c for non-magnetized isotropic (solid curve) and non-magnetized anisotropic matter (dashed curve).

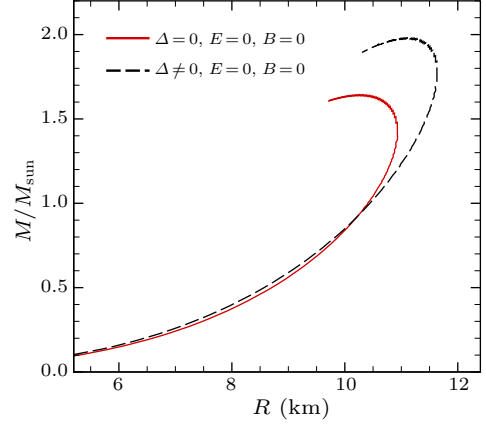


Fig. 6. Gravitational mass M versus radius of star R in two cases: non-magnetized isotropic (solid curve) and non-magnetized anisotropic matter (dashed curve).

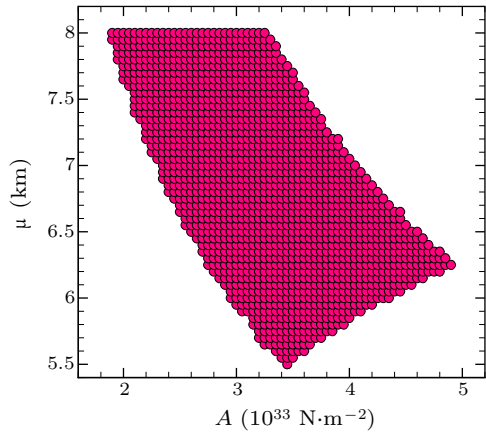


Fig. 7. Values of A and μ causing a strange quark star to have a mass in range of $1.9M_\odot$ – $2.1M_\odot$ by satisfying boundary and stability conditions.

It is easy to see that putting $E = 0$ and $\Delta = 0$ in Eq. (27), it simply transforms to Eq. (5), i.e., uncharged and isotropic matter. Now to solve the modified TOV equations, in addition to one assumption for anisotropic matter (as we carry out in the magnetized case), we need an extra physical assumption. Other steps for solution of anisotropic matter are the same. According to Ref. [23], it is reasonable to consider electric charge density distribution proportional to matter density

$$\rho_{ch} = f \times \rho_A, \quad (28)$$

which is a physically reasonable assumption, and f is the charge fraction that can be considered $f \leq 10^{-3} \text{ esug}^{-1}$ to satisfy causality and stability conditions. In fact, this upper bond for f is keeping the balance among gravitational attraction, nuclear pressure and Coulomb forces.[23] Noticeably, a highly charged compact star needs to be in an isolated system, otherwise the Coulomb force will overwhelm the gravitational one.[23] Putting Eq. (28) in Eq. (27) enables us to solve the charged case numerically. Now we can apply the same procedure as the magnetized case to

find the anisotropy factor Δ ,

$$\Delta = \frac{r}{2} \left[\frac{dp}{dr} - E\rho_{\text{ch}}e^A + \frac{-(r-\mu)}{\chi^2} \delta p + (\rho_A c^2 + p + \delta p) \frac{4\pi G r^3 \left(p + \delta p - \frac{E^2}{8\pi} \right) + mGc^2}{rc^2(rc^2 - 2mG)} \right]. \quad (29)$$

where ρ_A is given in Eq. (19).

Unlike the analytical solutions for anisotropic matter such as Refs. [7,9], in this study we use the numerical result of isotropic solution plus a Gaussian perturbation instead of using an entirely assumed function. In the following lines we discuss the result of adding anisotropy, magnetic field and electrical charge to the TOV equations. Our solution indicates that magnetic field and electric charge cause a slight increase in maximum mass and radius. On the other hand a positive Gaussian anisotropy leads to a considerable increase in the maximum mass of the neutron star and can predict more massive neutron stars which will not be ruled out by recent observations.^[25] Solving the modified TOV equation reveals that adding magnetic field or electrical charge causes an increase in radial pressure p_r . This effect also leads to a rise in the anisotropy factor Δ . In fact, this behavior not only is the direct result of our speculation about p_r , but also comes from the nature of TOV equation. Since the electrical charge and magnetic field effect on p_r and Δ are very small and almost the same, we have only compared the magnetic field effect with anisotropy effect in Figs. 1 and 2. For each particular ρ_c we can obtain a corresponding mass which is plotted in Fig. 3 where the case of perfect fluid (non-magnetized and uncharged isotropic matter) is compared with the magnetized or charged isotropic cases. Figure 4 depicts the well-known mass-radius relation of strange quark stars for perfect fluid, charged and magnetized isotropic cases. As is expected, adding magnetic field and electrical charge to the TOV equation causes an increase in the maximum mass of strange quark star M_{max} and also in the corresponding radius R for obtained M_{max} . Since gravitational mass of the star and total energy in GR are proportional, adding magnetic field increases the total energy and the above result seems plausible. Even though a strong magnetic field can significantly influence the evolution of a star,^[26] the effect of magnetic field and electrical charge on raising the maximum masses and their corresponding radii is insignificant, owing to the high density of matter in these kinds of objects. It is shown in Figs. 5 and 6 that the maximum mass of strange quark star is raised by adding Gaussian perturbation for obtaining p_r . Furthermore, according to Fig. 2 the tangential pressure p_t and anisotropy Δ have a maximum and they do not vanish on the surface. In fact, our solution indicates that if there is any anisotropy in the star, it will have its maximum on the surface. Recent measurements indicate that there exists a pulsar of mass $1.97 \pm 0.04 M_\odot$ and this mass rules out nearly all currently proposed

equations of state.^[25] This observation motivates us to use the mentioned mass for setting our free parameters. We search $A - \mu$ space for a star with a maximum mass range of $1.9 M_\odot < M_{\text{max}} < 2.1 M_\odot$. First, we restrict the free parameters by applying the condition (16a). This condition prevents us from choosing a large A which may cause a deviation in central pressure of the star. Moreover, if we place μ very near to the origin of the star, it would violate the above mentioned condition. On the other hand, a μ located very close to the star's surface is not allowed either, due to the fact that it violates the vanishing radial pressure on the surface. Secondly, the condition (7b) forbids large values of A relative to p_c , small values of χ which cause a bump in pressure profile and finally short distances to origin for μ parameter. The results are illustrated in Fig. 7.

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