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ADDENDUM

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Addendum: A geodesic principle for strong coupling gravity (2015 *Class. Quantum Grav.* 32 215022)

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The derivation of the geodesic congruence equations $\mathcal{E}_0 = 0 = \mathcal{E}_a$ from equation (3.1), while correct, entails the undesirable restriction $\mathcal{D} = 0$ as defined in (3.6). Since $\mathcal{D} = \sqrt{g}\,\nabla^\mu P_\mu$, $g = \det g_{\mu\nu}$, measures the expansion of the congruence the background geometry would be constrained by requiring vanishing expansion, and correspondingly for strong coupling gravity. A derivation without this limitation proceeds as follows. Defining the acceleration by $\dot{P}_\mu := P_\nu \nabla^\nu P_\mu$ its components can be read off from $\int \mathrm{d}t\mathrm{d}x\sqrt{g}\;\xi^\mu \dot{P}_\mu = \int \mathrm{d}t\mathrm{d}x\sqrt{g}\;\left\{\frac{1}{2}\mathcal{L}_\xi g^{\mu\nu}P_\mu P_\nu - \xi^\mu P_\mu \nabla^\nu P_\nu\right\}$. Inserting (3.3) and (3.4) one finds that in the $1+d$ components $\dot{P}_\mu = (\dot{P}_0, \dot{P}_a)$ the acceleration $\mathcal{D}$ drops out

$$\dot{P}_0 = \mathcal{E}_0 + \frac{\epsilon_g}{N} N^a \mathcal{E}_a, \qquad \dot{P}_a = \frac{\epsilon_g}{N}\mathcal{E}_a, \tag{1}$$

showing that $\dot{P}_0 = 0 = \dot{P}_a$ is equivalent to $\mathcal{E}_0 = 0 = \mathcal{E}_a$ alone. Since (3.3) and (3.4) have well-defined strong coupling limits one can define $\dot{P}_0^\infty$, $\dot{P}_a^\infty$ as the coefficients of ξ^0, ξ^a in the analogous computation, with the result

$$\dot{P}_0 = \epsilon_g \phi e_0(\phi) + N^a \dot{P}_a^\infty, \qquad \dot{P}_a^\infty = \frac{\epsilon_g \phi}{N}[e_0(P_a) - \phi \partial_a N]. \tag{2}$$

Here $\mathcal{D}^\infty := \epsilon_g e_0(\sqrt{q}\,\phi)$ cancels and (2) coincides with the scaling limit (A.2) of (1).

In the transcription of the $\partial_a \xi^0$ coefficient from equations (3.22) to (3.23) a term $N^b \mathcal{E}_b^\infty$ has been dropped by mistake and the $\partial_a \mathcal{E}_a^\infty$ term should read $\partial_t \mathcal{E}_a^\infty$. The corrected equation (3.23), i.e. $\delta_\xi^\infty \mathcal{E}_a^\infty = \xi^0 \partial_t \mathcal{E}_a^\infty + \mathcal{L}_{\vec{\xi}} \mathcal{E}_a^\infty + \partial_a \xi^0 [e_0(\phi) + N^b \mathcal{E}_b^\infty]$, in combination with (3.19) is equivalent to

$$\delta_\xi^\infty \dot{P}_0^\infty = \partial_t(\xi^0 \dot{P}_0^\infty) + \xi^a \partial_a \dot{P}_0^\infty + \partial_t \xi^a \dot{P}_a^\infty,$$
$$\delta_\xi^\infty \dot{P}_a^\infty = \xi^0 \partial_t \dot{P}_a^\infty + \mathcal{L}_{\vec{\xi}} \dot{P}_a^\infty + \partial_a \xi^0 \dot{P}_0^\infty. \tag{3}$$

Under the strong coupling realization of the diffeomorphism group the acceleration $(\dot{P}_0^\infty, \dot{P}_a^\infty)$ thus transforms like (P_0, P_a) in (3.18) and the conditions $\dot{P}_0^\infty - \alpha \dot{P}_0 = 0$, $\dot{P}_a^\infty - \alpha \dot{P}_a = 0$, are manifestly gauge invariant. Using $\alpha = \epsilon_g e_0(\phi)/N$ the non-redundant condition on a geodesic congruence in strong coupling gravity is

$$e_0(\phi) P_a - \phi[e_0(\phi) - \phi \partial_a N] = 0. \tag{4}$$

Imposing $\alpha = 0$ reproduces (3.17) but without the undesirable constraint $e_0(\sqrt{q}\,\phi) = 0$. Note that $\alpha = 0$ does not restrict the 'norm' $\epsilon_g \phi^2$ of (P_0, P_a) to be a spatiotemporal constant.