LETTER

Fourier transform analysis of irradiated Weiss oscillations

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Fourier transform analysis of irradiated Weiss oscillations

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Abstract – We present a theoretical approach to study the effect of microwave radiation on the magnetoresistance of a one-dimensional superlattice. In our proposal the magnetoresistance of a unidirectional spatial periodic potential (superlattice), is modulated by microwave radiation due to an interference effect between both space- and time-dependent potentials. The final magnetoresistance will mainly depend on the spatial period of the superlattice and the radiation frequency. We consider an approach to study these effects based on the fast Fourier transform of the obtained magnetoresistance profiles in function of the inverse of the magnetic field. Based on this theory we propose the design of a novel radiation sensor for the terahertz band.

Introduction. – Nowadays magnetotransport properties of highly mobile 2DES are a subject of increasing interest. In particular the study of the effects that radiation can produce on these nano-devices is attracting much attention [1]. Besides, the interplay of two different periodic modulations in these physical systems will give rise to interesting features in electron dynamics and transport. For instance, the study of the effect of microwave (MW) radiation on the transport properties of a one-dimensional superlattice in the presence of a perpendicular magnetic field (B) giving rise to Weiss oscillations [2–4] represents a topic of great interest. Especially if we consider that MW radiation gives rise also to magnetoresistance (R_{xx}) oscillations in 2DES [5–7]. Weiss oscillations are a type of R_{xx} oscillations observed in high-mobility 2DES with a lateral periodic modulation (superlattice) imposed in one direction. In this scenario electrons are subjected simultaneously to two different periodic potentials. One is space dependent, *i.e.*, the superlattice, and the other is time dependent, *i.e.*, MW radiation. Thus, interesting properties are expected to turn up due to the joint effect of both potentials.

The origin of Weiss oscillations is a periodically modulated Fermi energy as a direct consequence of the spatial periodic potential $V(x)$ imposed on the 2DES. The spatially modulated Fermi energy (see fig. 1) dramatically changes the scattering conditions between electrons and charged impurities presented in the sample. At a certain B the transport will be more dissipative. This corresponds to a peak in R_{xx} [8]. Meanwhile at other B values,
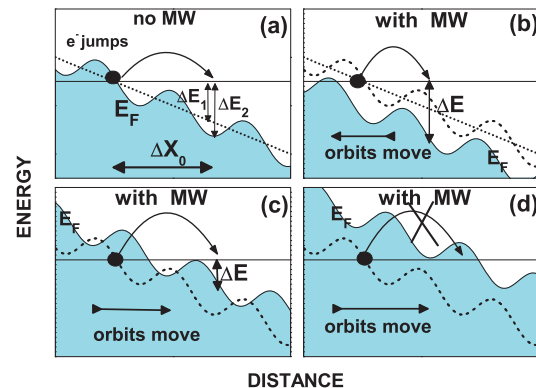

Fig. 1: (Colour on-line) Schematic diagrams of electronic transport of a 2DES with a spatial periodic modulation in one direction. In panel (a) no MW field is present. When the MW field is on, the orbits oscillate with w. In panel (b) the orbits move backwards during the jump. In panel (c) the orbits are moving forwards but still the electronic jump can take place. In panel (d) the orbits are moving forwards but for a higher MW intensity. Thus, the electron movement between orbits cannot take place because the final state is occupied.

the transport will be less dissipative corresponding to a R_{xx} valley [8]. Under this scenario it is expected that the presence of MW radiation will dramatically alter the R_{xx} response of the system. We predict that the combined effect of MW radiation and spatial potential will lead the system to an interference regime with constructive and destructive responses [8]. As a consequence R_{xx}

will present a modulated profile. The theory presented here can have potential applications in other fields and systems like nano-electro-mechanical systems (NEMS) [9] with ac-potentials and surfaces acoustic waves (SAW) [10] in 2DES or quantum dots illuminated with MW radiation. In other words, the physics presented in this letter can be of interest for an audience dealing with the effects that ac or/and dc fields produce on nano-devices.

In this letter we consider at the same footing the effects of both a spatial periodic modulation and MW radiation on the transport properties of a 2DES in the presence of B. We propose an alternative approach to study the effects of both potentials based on the fast Fourier transform (FFT) [11] of the obtained magnetoresistance profiles in function of the inverse of B. We first study the FFT of the system for each potential individually. Then we study jointly the FFT of the system when the two types of potentials are acting simultaneously. We obtain two peaks, corresponding to the two harmonic potentials. An interesting scenario arises when the two FFT peaks coincide. Then we can extract information affecting the two potentials. Also it can be of practical interest, for instance a 2DES with a superlattice of a known spatial period can work as a nano-sensor to detect the radiation of certain frequencies, for instance for the terahertz band.

Our system consists in a 2DES subjected to a perpendicular B (z-direction) and a dc electric field (E_{dc}) (x-direction). We include the unidirectional periodic potential $V(x) = V_0 \cos Kx$ where $K = 2\pi/a$, a being the spatial period of the superlattice. The total Hamiltonian H can be written as

$$H = \frac{P_x^2}{2m^*} + \frac{1}{2}m^* w_c^2 (x - X_0)^2 - eE_{dc}X_0 + \frac{1}{2}m^* \frac{E_{dc}^2}{B^2} + V_0 \cos(Kx) = H_0 + V_0 \cos(Kx). \quad (1)$$

X_0 is the center of the orbit for the electron spiral motion: $X_0 = \frac{\hbar k_y}{eB} - \frac{eE_{dc}}{m^* w_c^2}$, e is the electron charge, w_c is the cyclotron frequency. H_0 is the Hamiltonian of a harmonic quantum oscillator and its wave functions, the well-known oscillator functions (Hermite polynomials). We treat $V_0 \cos(Kx)$ in the first-order perturbation theory and the first-order energy correction is given by $\epsilon_n^{(1)} = V_0 \cos(KX_0)e^{-X/2}L_n(X) = U_n \cos(KX_0)$, where $X = \frac{1}{2}l^2 K^2$, $L_n(X)$ is a Laguerre polynomial and l the characteristic magnetic length. Therefore the total energy for the Landau level n is given by $\epsilon_n = \hbar w_c(n + \frac{1}{2}) - eE_{dc}X_0 + U_n \cos(KX_0)$. This result dramatically affects the Fermi energy as a function of distance (see fig. 1), showing now a periodic and tilted modulation [8].

Now we introduce impurity scattering suffered by the electrons in our model [7,8,12]. When one electron scatters elastically due to charged impurities, its average orbit center position changes, in the electric-field direction, from X_0 to X_0'. Accordingly, the average advanced distance in the x-direction is given by $\Delta X_0 = X_0' - X_0 \simeq 2R_c$, $R_c = \frac{\sqrt{2m^* E_F}}{eB}$ being the orbit radius. There is a direct
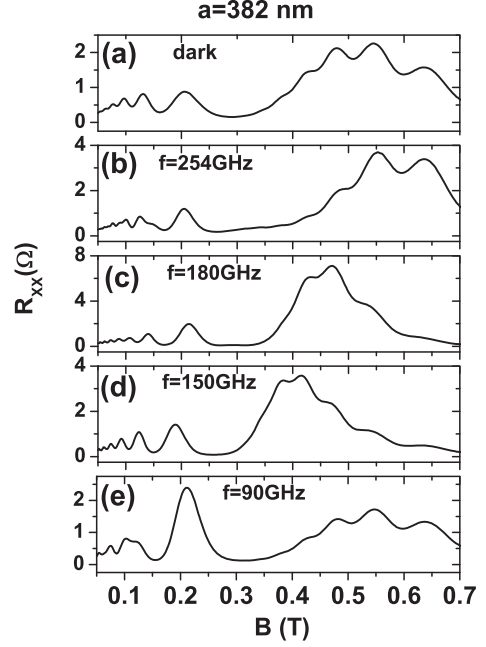

Fig. 2: Calculated R_{xx} as a function of magnetic field B for dark and radiation of different frequencies. The period of the static modulation is $a = 382$ nm and the modulation amplitude is $V_0 \sim 0.1$ meV. ($T = 1$ K.)

relation between advanced distance and dissipated energy (see fig. 1), and when the 2DES is subjected to $V(x)$, the energy variation in the scattering jump has the expression

$$\Delta\epsilon \simeq eE_{dc}2R_c + U_n[\cos(KX_0) - \cos(KX_0')]. \quad (2)$$

After some algebra we can obtain from this expression an averaged energy variation that can be written as [8]

$$\Delta\epsilon \simeq eE_{dc}2R_c - V_0 e^{-X/2}J_0(2\sqrt{X})\cos\left[2\left(R_c K + \frac{\pi}{4}\right)\right], \quad (3)$$

J_0 being the Bessel function of zero order. The advanced distance corresponding to $\Delta\epsilon$ can now be straightforwardly calculated [8]:

$$\Delta X_T = 2R_c - \frac{V_0 e^{-X/2}J_0(2\sqrt{X})}{eE_{dc}}\cos\left[2\left(R_c K + \frac{\pi}{4}\right)\right]. \quad (4)$$

From ΔX_T we calculate a drift velocity for the electron dissipative transport in the x-direction and finally we obtain R_{xx} [7,8,12]. In fig. 2(a) we represent R_{xx} as a function of B. The period of the static modulation is $a = 382$ nm and the modulation amplitude is $V_0 \sim 0.1$ meV. We reproduce Weiss oscillations with reasonable agreement with experiments [2,3].

If now we switch on the MW field, we expect that its effect will alter dramatically the R_{xx} response of the system. In our simulations we consider that the MW field is plane polarized along the x-direction, that is the direction of the
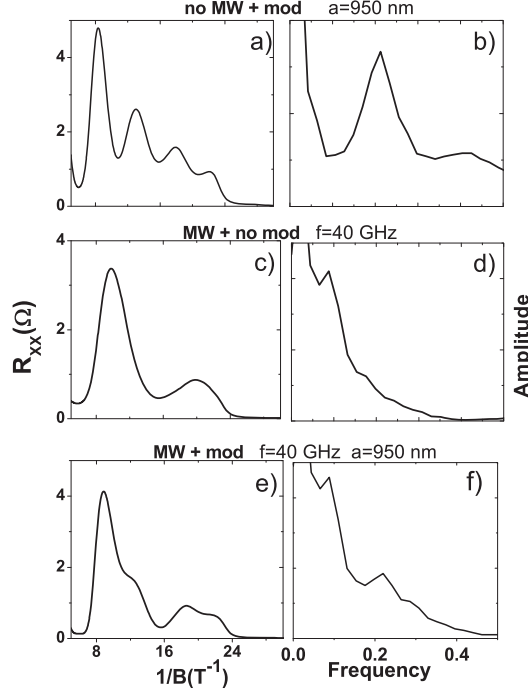
Fig. 3: Left panels: R_{xx} vs. $(1/B)$. In (a) we represent the case of a superlattice of a $= 950\,$nm and no radiation. In (c) we plot the case where the spatial modulation is removed. The frequency of the radiation field is $40\,$GHz. In (e) both spatial and temporal periodic modulations are present. In the right panels we represent the FFT of the corresponding left curves. $(T = 1\,$K.)

Fig. 4: Calculated ρ_{xx} vs. B. Similar panels as in fig. 3. The spatial period is the same but the radiation frequency is $80\,$GHz. $(T = 1\,$K.)

current and the direction of the spatial potential, *i.e.*, of the superlattice. [11] From the MW-driven electron orbits model [7,8], we propose that MW radiation forces the electron orbits move back and forth at the frequency of MW. This affects the scattering conditions of the 2DES increasing or decreasing the distance of the scattering jump giving rise to MW-induced R_{xx} oscillations. Both effects, periodic spatial modulation and MW radiation, alter simultaneously the average advanced distance when the electron scatters (see fig. 1). We obtain an expression for the total average distance in the x-direction [8]:

$$\Delta X_T = 2R_c - S\cos\left[2\left(R_cK + \frac{\pi}{4}\right)\right] + A\cos(w\tau), \quad (5)$$

where $S = \frac{V_0 e^{-X/2} J_0(2\sqrt{X})}{eE_{dc}}$ and $A = \frac{eE_o}{m^*\sqrt{(w_c^2-w^2)^2+\gamma^4}}$. E_o is the MW electric-field amplitude, γ is a phenomenologically introduced [7] damping parameter and τ the scattering time. From ΔX_T we calculate an electron drift velocity [7,8,12] and finally R_{xx} of the system:

$$R_{xx} \propto -S\cos\left[2\left(R_cK + \frac{\pi}{4}\right)\right] + A\cos(w\tau). \quad (6)$$

According to this expression we can predict an interference regime between both periodic potentials which will be reflected on R_{xx} [8,13] showing a modulated profile
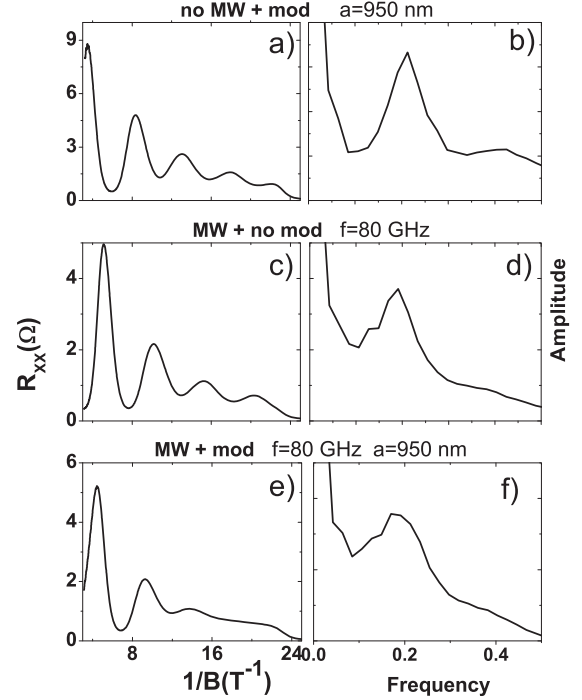
when plotted against B. We can observe this behaviour in figs. 2(b), (c). In these figures we represent magnetoresistance *vs.* B for a one-directional superlattice ($a = 382\,$nm) with MW radiation of different frequencies: 90, 150, 180 and $254\,$GHz. All of them show different modulated profiles due to the interference effect of both potentials. This means that, depending mainly on a, w and for some values of B, R_{xx} will present a constructive response and a reinforced signal. For other values of B the interference will be destructive and the R_{xx} response will be closer to the one of the dark case.

A further approach to analyze this scenario of irradiated Weiss oscillations is based one the FFT of the R_{xx} signal *vs.* the inverse of the applied magnetic field. The goal would be to obtain information from both, superlattice built on the 2DES and radiation. The FFT is carried out on the calculated data of R_{xx} vs. $(1/B)$ because they present harmonic distribution; see left panels of figs. 3 and 4 where we represent R_{xx} *vs.* the inverse of the magnetic field. However, the standard plots of R_{xx} *vs.* B are not harmonic (see figs. 5 and 6). According to eq. (6) the spatial part of R_{xx} oscillations depends on the cosine term: $\cos\left[2\left(R_cK + \frac{\pi}{4}\right)\right]$, where

$$2KR_c = \left[2\frac{2\pi}{a}\frac{\sqrt{2m^*E_F}}{e}\right] \times \frac{1}{B} = k_{esp} \times \frac{1}{B}. \quad (7)$$

Here the *spatial frequency*, k_{esp}, is related with the frequency obtained from the FFT plot, ν_{esp}, through

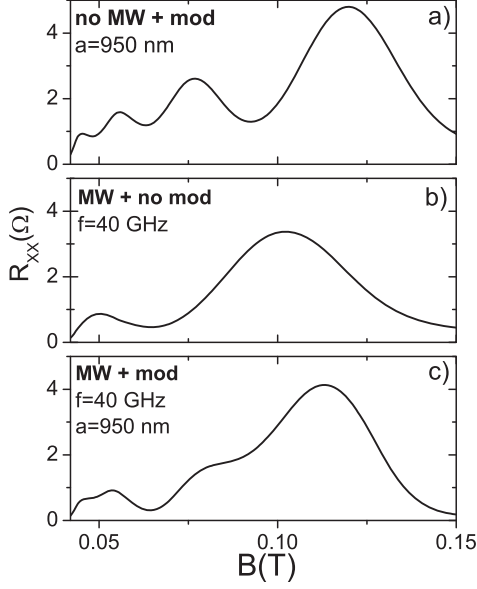$$k_{esp} = 2\pi\nu_{esp} \quad (8)$$

Fig. 5: Calculated R_{xx} vs. B. In panel (a), we plot the case of superlattice with no radiation. Therefore, we represent the Weiss oscillations. Panel (b) represents the case of a 2DES with no superlattice subjected to radiation. Thus, we obtain the well-known radiation-induced resistance oscillations. In panel (c) we represent the joint effect of a one-directional superlattice with radiation. We obtain a modulated R_{xx} profile. The period of the superlattice is 950 nm and the radiation frequency 40 GHz. ($T = 1$ K.)

(see right panels of fig. 3). From this expression we would be able to obtain information about the spatial period of the superlattice a or even the Fermi level (electron density) if we know the frequency from the FFT graph in advance.

We can apply a similar procedure for the temporal part of the R_{xx} oscillations. According to eq. (6) this part depends on $\cos w\tau$, where $\tau = 1/W$ is the scattering time and W the charged impurity scattering rate which in turn can be expressed as [14]

$$W = \left(\frac{e^4 n_i m^*}{4\pi\epsilon^2\hbar^3 q_s}\right)\left[\frac{1 + e^{-\frac{\pi\Gamma}{\hbar w_c}}}{1 - e^{-\frac{\pi\Gamma}{\hbar w_c}}}\right] \simeq \left(\frac{e^4 n_i m^*}{4\pi\epsilon^2\hbar^3 q_s}\right)\left[\frac{2\hbar w_c}{\pi\Gamma}\right],$$
(9)

where n_i is the impurity density, ϵ the dielectric constant, q_s the inverse of the Thomas-Fermi screening length and Γ the Landau level width. Therefore, we can write for $w\tau$, similarly as we did for the spatial part,

$$w\tau = w \times \left(\frac{4\pi\epsilon^2\hbar^3 q_s}{e^4 n_i m^*}\right)\left[\frac{\pi\Gamma m^*}{2\hbar e}\right] \times \frac{1}{B} = w_{time} \times \frac{1}{B}; \quad (10)$$

the angular frequency w_{time} is related to the frequency of the time-dependent FFT ν_{time} through

$$w_{time} = 2\pi\nu_{time} \quad (11)$$

(see right panels of fig. 4). Thus, knowing ν_{time} from the FFT plot, we could obtain information such as the frequency of radiation or material dependent information, for instance the scattering time.
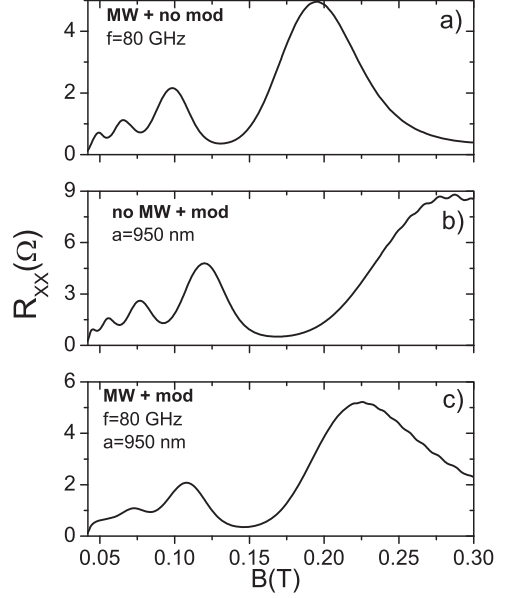

Fig. 6: Calculated ρ_{xx} vs. B. Similar panels as in fig. 5. Now the spatial period of the superlattice is the same but the radiations frequency is 80 GHz. ($T = 1$ K.)

In the above approach we have individually considered the potentials acting on the system. A more interesting scenario arises when we study jointly the FFT of the system when acting the two types of potentials simultaneously (figs. 3(f) and 4(f)). We can have two different situations, in the first one the FFT peaks do not coincide, they show up far apart (see top and middle panels of figs. 3 and 4). In this case the obtained information would be as the one described above. On the other hand, when the two FFT peaks coincide, $\nu_{time} = \nu_{esp}$ and knowing previous information of one the potentials, either radiation or superlattice, we could obtain in turn important information of the other one. An example of application of the proposed theory would be the design of radiation sensors; a 2DES with a superlattice of known spatial period could work as a nano-sensor to detect the radiation of a certain radiation frequency, such as the terahertz band. Using the criteria of coincident peaks, we have calculated that to detect, for instance, radiation of 0.5 THz we would need a superlattice of a spatial period of $a = 122$ nm.

In conclusion, we have further examined the problem of irradiated Weiss oscillations. We have first obtained the interference and modulated profile of R_{xx} vs. B arising from the joint effect of superlattice and radiation. Then we have studied this effect from the perspective of the FFT of R_{xx} response vs. $1/B$. We have obtained two peaks each one corresponding to one potential, time or spatial dependent. Two scenarios have been obtained, when the FFT peaks positions are the same and when they do not coincide. The most interesting is the first one, where the design of a nano-sensor for terahertz radiation based on a unidirectional superlattice of known period has been proposed.

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REFERENCES

[1] IÑARREA J. and PLATERO G., *Europhys. Lett.*, **40** (1997) 417; IÑARREA J., PLATERO G. and MACDONALD A. H., *Phys. Status Solidi. A*, **203** (2006) 1148; IÑARREA JESUS and PLATERO GLORIA, *Nanotechnology*, **21** (2010) 315401.

[2] WEISS D., VON KLITZING K., PLOOG K. and WEIMANN G., *Europhys. Lett.*, **8** (1989) 179; GERHARDSTS R. R., WEISS D. and VON KLITZING K., *Phys. Rev. Lett.*, **62** (1989) 1173; WEISS D., GERHARDTS R. R. and V. KLITZING K., *Phys. Rev. B*, **39** (1989) 13020.

[3] WINKLER R. W., KOTTHAUS J. P. and PLOOG K., *Phys. Rev. Lett.*, **62** (1989) 1177.

[4] VASILOPOULOS P. and PEETERS F. M., *Phys. Rev. B*, **63** (1989) 2120; PEETERS F. M. and VASILOPOULOS P., *Phys. Rev. B*, **47** (1993) 1466; **46** (1992) 4667.

[5] MANI R. G., SMET J. H., VON KLITZING K., NARAYANAMURTI V., JOHNSON W. B. and UMANSKY V., *Nature*, **420** (2002) 646; MANI R. G., SMET J. H., VON KLITZING K., NARAYANAMURTI V., JOHNSON W. B. and UMANSKY V., *Phys. Rev. Lett.*, **92** (2004) 146801; *Phys. Rev. B*, **69** (2004) 193304; MANI R. G., *Appl. Phys. Lett.*, **92** (2008) 102107; MANI R. G., NARAYANAMURTI V., VON KLITZING K., SMET J. H., JOHNSON W. B. and UMANSKY V., *Phys. Rev. B*, **70** (2004) 155310; MANI R. G., RAMANAYAKA A. N. and WEGSCHEIDER W., *Phys. Rev. B*, **84** (2011) 085308; MANI R. G., RAMANAYAKA A. N., YE TIANYU, HEIMBECK M. S., EVERITT H. O. and WEGSCHEIDER W., *Phys. Rev. B*, **87** (2013) 245308.

[6] ZUDOV M. A., LU R. R., PFEIFFER N. and WEST K. W., *Phys. Rev. Lett.*, **90** (2003) 046807.

[7] IÑARREA J. and PLATERO G., *Phys. Rev. Lett.*, **94** (2005) 016806; *Phys. Rev. B*, **72** (2005) 193414; *Appl. Phys. Lett.*, **89** (2006) 052109; IÑARREA JESUS, MANI R. G. and WEGSCHEIDER W., *Phys. Rev. B*, **82** (2010) 205321.

[8] IÑARREA J. and PLATERO G., *Appl. Phys. Lett.*, **93** (2008) 062104; IÑARREA J., *Appl. Phys. Lett.*, **90** (2007) 172118; 262101; RAMANAYAKA A. N., MANI R. G., IÑARREA J. and WEGSCHEIDER W., *Phys. Rev. B*, **85** (2012) 205315.

[9] PISTOLESI F. and FAZIO F., *Phys. Rev. Lett.*, **94** (2005) 036806.

[10] ROBINSON JOHN P., KENNETT MALCOM P., COOPER NIGEL R. and FAL'KO VLADIMIR I., *Phys. Rev. Lett.*, **93** (2004) 036804.

[11] VOGL ANTON, PhD Thesis Dissertation, Regensburg University (2011).

[12] RIDLEY B. K., *Quantum Processes in Semiconductors*, 4th edition (Oxford University Press) 1993.

[13] IÑARREA J. and PLATERO G., *Appl. Phys. Lett.*, **89** (2006) 172114; IÑARREA J., *Appl. Phys. Lett.*, **100** (2012) 242103.

[14] IÑARREA J., *EPL*, **106** (2014) 47005; IÑARREA J. and PLATERO G., *Phys. Rev. Lett.*, **111** (2013) 220903.