LETTER

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Generation of platicons and frequency combs in optical microresonators with normal GVD by modulated pump

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Abstract – We demonstrate that flat-topped dissipative solitonic pulses, "platicons", and corresponding frequency combs can be excited in optical microresonators with normal group velocity dispersion using either the amplitude modulation of the pump or the bichromatic pump. Soft excitation may occur in a particular frequency range if the modulation depth is large enough and the modulation frequency is close to the free spectral range of the microresonator.

Optical frequency combs generated in mode-locked lasers have revolutionized optical measurements. Last years optical microresonators have been attracting growing attention as a promising platform for microresonator-based Kerr frequency combs [1–4]. It was demonstrated both theoretically and experimentally that frequency comb consisting of equidistant optical lines in spectral domain may appear in a nonlinear microresonator pumped by a continuous wave (c.w.) laser due to the cascaded four-wave mixing processes. The frequency spacing between comb lines corresponds to the free spectral range (FSR, typically tens of GHz up to THz) of the microresonator which is the inverted roundtrip time of light in the cavity. Microresonator-based combs were shown to perform at the level required for optical-frequency metrology applications [5] and for optical communications [6]. However, such systems often suffer from significant frequency and amplitude noise [7,8] due to the formation of sub-combs [9]. In contrast to conventional mode-locked laser-based frequency combs microresonator combs do not correspond, generally, to stable ultrashort pulses in the time domain because of arbitrary phase relations between the comb lines obtained in the process of formation. Kerr solitons solve this problem with a c.w. laser beam converted into a train of pulses, corresponding to a low-noise frequency comb having smooth spectral envelope in the spectral domain. This regime was demonstrated experimentally in optical crystalline and integrated ring microresonators with anomalous group velocity dispersion (GVD) [10,11]. The limitation of this approach is the difficulty to obtain anomalous GVD in broad band for arbitrary centered wavelength in microresonators since material GVD in the visible and near IR is mostly normal. In this way, the development of new methods that enable the implementation of materials with normal GVD for frequency comb generation is interesting.

Frequency combs in normal GVD microresonators are theoretically possible [12–14]. Besides, narrow comb-like spectra from normal GVD crystalline resonators [15–17] and integrated microrings [18] have been demonstrated experimentally. Microresonators with normal GVD can also support dark optical solitons [14,19].

A novel type of solitonic pulses, "platicons", was predicted in microresonators with normal dispersion under the condition of the shifted pump mode frequency resonance [20]. In real microresonators such frequency shift may occur either due to the normal mode coupling between different mode families [19,21,22], or as a result of self-injection locking [16], both providing equivalent local anomalous dispersion [21–24]. It was demonstrated that one may continuously change in wide range the duration of generated platicons varying the pump detuning and that the generation of platicons is significantly more efficient than the generation of bright soliton trains in microresonators for the same absolute value of anomalous GVD in terms of conversion of the c.w. power into the power of the

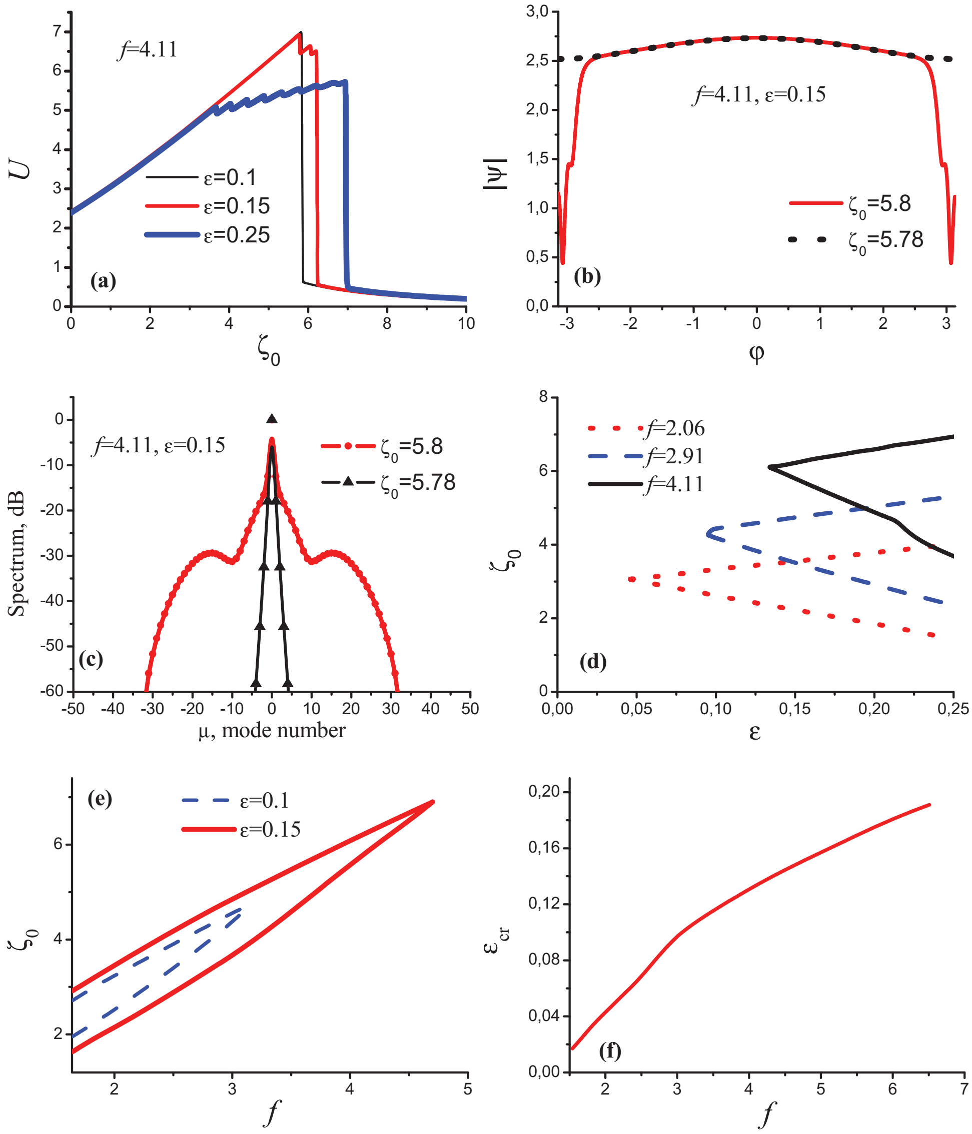

Fig. 1: (Color online) (a) Averaged intracavity intensity U *vs.* normalized detuning ζ_0 at $f = 4.11$ for different relative sideband pump amplitudes ε. (b) Profiles and (c) spectra of low-contrast wave (black line) and platicon (red line) at $\varepsilon = 0.15$, $f = 4.11$. Excitation domains (d) for different primary pump amplitudes f and (e) for different relative sideband pump amplitudes ε. (f) Platicon generation critical value of relative sideband pump amplitude *vs.* primary pump amplitude. In all cases $D_2/\kappa \approx -1.036 \times 10^{-2}$, $\Delta = 0$.

comb [20]. The problem with the proposed method is that it requires an artificial perturbation of the local dispersion. In this paper we show that platicon generation is possible even in the absence of the pump mode shift when a bichromatic or amplitude-modulated pump is used. This method is efficient if the pump modulation frequency or the frequency difference between two pump waves is close to one or several FSRs. The proposed approach is experimentally feasible since comb generation from a bichromatic pump has been already studied experimentally [25] and theoretically [26] in the case of anomalous dispersion. Recently, frequency comb generation by dual-pump input signal was also discussed in normally dispersive optical fibers [27].

We consider first the case of the amplitude-modulated pump $|P| = P_0 \{1 + 2\varepsilon \cos(\Omega t)\}^2$ with modulation frequency Ω and relative amplitude of the sideband pump ε. Our numerical model is based on the system of dimensionless coupled nonlinear mode equations [9,28] modified to take into account the complexity of the pump field. Assuming that the pump modulation frequency is close to one FSR, we have

$$\frac{\partial a_\mu}{\partial \tau} = -(1 + i\zeta_\mu) a_\mu + i \sum_{\mu' \le \mu''} (2-\delta_{\mu'\mu''}) a_{\mu'} a_{\mu''} a^*_{\mu'+\mu''-\mu}$$
$$+ \delta_{0\mu} f + \varepsilon \{\delta_{1\mu} f \exp(i\Delta\tau) + \delta_{-1\mu} f \exp(-i\Delta\tau)\}. \qquad (1)$$

Here $\delta_{\mu'\mu''}$ is the Kronecker delta, a_μ is the slowly varying amplitude of the comb modes for the mode frequency ω_μ, $\tau = \kappa t/2$ denotes the normalized time, $\kappa = \omega_p/Q$ is the cavity decay rate, Q is the loaded quality factor, ω_p is

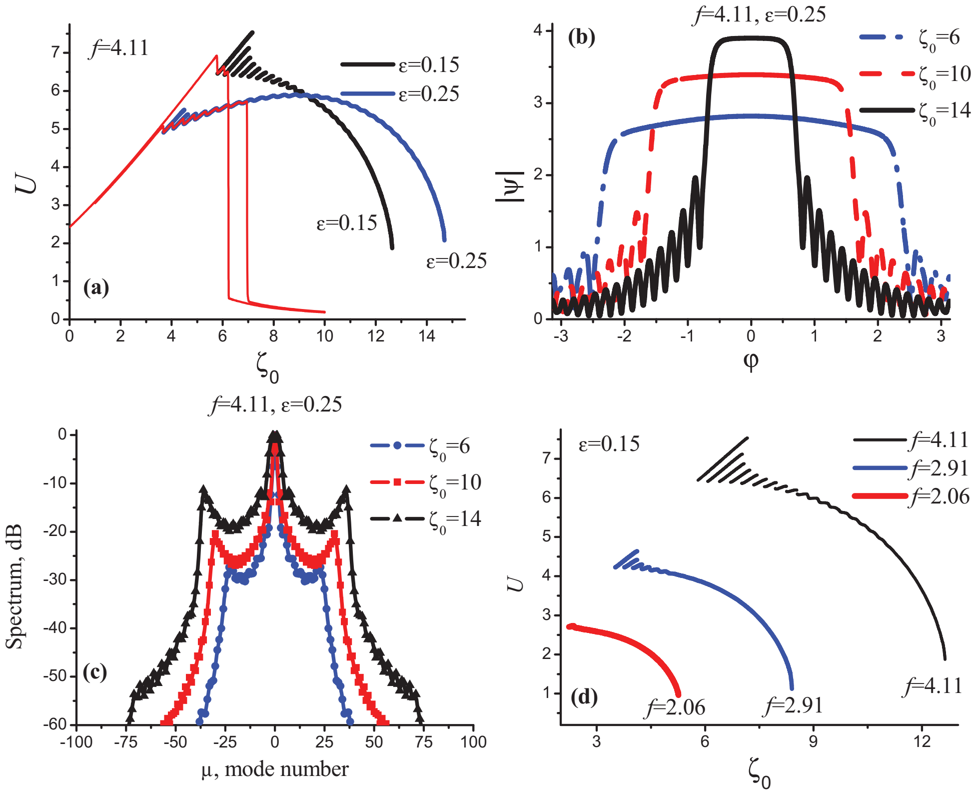

Fig. 2: (Color online) (a) Averaged intracavity intensity U of platicons *vs.* normalized detuning ζ_0 at $f = 4.11$ for $\varepsilon = 0.15$ (black line) and $\varepsilon = 0.25$ (blue line). Thin red lines indicate the results of soft excitation. (b) Profiles and (c) spectra of platicons for different values of ζ_0 at $\varepsilon = 0.25$, $f = 4.11$. (d) Averaged intracavity intensity U of platicons *vs.* normalized detuning ζ_0 for different values of f at $\varepsilon = 0.25$. In all cases $D_2/\kappa \approx -1.036 \times 10^{-2}$, $\Delta = 0$.

the pump frequency; f stands for the dimensionless pump amplitude, $\Delta = 2(D_1 - \Omega)/\kappa$ is the normalized modulation frequency mismatch, D_1 is FSR. All mode numbers μ are defined relative to the pumped mode. We consider the Taylor expansion of the dispersion law $\omega_\mu = \omega_0 + D_1\mu + \frac{1}{2}D_2\mu^2 + \ldots$ and neglecting third-order dispersion we get the following expressions for the normalized detunings: $\zeta_0 = 2(\omega_0 - \omega_p)/\kappa$, $\zeta_\mu = \zeta_0 + (D_2/\kappa)\mu^2$. Note that $D_2 < 0$ for normal GVD. For the numerical analysis we consider the range of parameters corresponding to experimentally feasible (realistic) microresonators: $f \in [1.5; 6.5]$ (the value $f = 1$ corresponds to bistability and comb threshold [10,14]), $D_2/\kappa \in \left[-5 \times 10^{-2}; -10^{-3}\right]$, $\zeta_0 \in [0; 15]$. For example, for MgF_2 microresonator [16] ($\lambda = 780\,\mathrm{nm}$, $Q = 2.5 \times 10^9$, $D_2/2\pi = -2.96\,\mathrm{kHz}$) $D_2/\kappa \approx -0.019$ and the pump power of 1 mW corresponds to $f \approx 3.83$; for Si_3N_4 microrings [18] ($\lambda = 1550\,\mathrm{nm}$, $Q = 1.1 \times 10^6$, $D_2/2\pi = -300\,\mathrm{kHz}$) $D_2/\kappa \approx -0.0017$ and the pump of 1 W corresponds to $f \approx 2.76$.

In our simulations we used initial weak noise-like inputs for seeding. Coupled-mode equations for 501 modes were numerically propagated in time using the Runge-Kutta integrator. Nonlinear terms were calculated using a fast method proposed in [29]. We also checked that results do not change with the increase of the number of modes. For the analysis we calculated the average intracavity intensity $U = \sum_\mu |a_\mu|^2$ for different values of the normalized detuning ζ_0 (see fig. 1(a)) and corresponding waveforms $\psi(\varphi) = \sum_\mu a_\mu \exp(i\mu\varphi)$.

For the resonant modulation ($\Delta = 0$) it was found that at $\varepsilon \neq 0$ due to the presence of the sideband pump conventional c.w. solutions turn into low-contrast waves with narrow spectrum (see figs. 1(b) and (c)). For small sideband pump amplitudes ε internal intensity *vs.* detuning dependence has a conventional triangular shape and only low-amplitude sideband modes may be observed. However, if the sideband pump amplitude is large enough, this dependence deviates from the expected triangular shape, the characteristic step-like dependence of intensity *vs.* detuning appears and at a particular frequency range, in the vicinity of this step, one may observe the generation of high-contrast optical pulses with a pronounced dip located at the minima of the low-contrast wave (see fig. 1(b)) analogously to platicons [20]. Platicon generation is accompanied by a dramatic spectrum widening (fig. 1(c)). Similarly to experimental spectra observed in [16] these are characterized by two pronounced wings (figs. 1(c) and 2(c)). Thus, the amplitude modulation of the pump power may provide soft excitation (from noise-like inputs) of platicons without a tricky laser scanning necessary for bright solitons in case of anomalous GVD [10]. Such dramatic profile and spectrum transformation can be explained by the phenomenon of optical wave breaking [30] due to the double shock formation described in ref. [27]. Shock formation is accompanied by the emergence of multiple frequency components that mix nonlinearly to produce new frequency components by four-wave mixing resulting in a significant spectrum broadening.

Generated platicons were found to be stable against perturbations. To elucidate their stability we propagated them up to large periods of time (up to $\tau \sim 500$) in the presence of added white-noise–like perturbations ($U_{noise} \sim 0.1$) by solving the governing equations (eq. (1)). Platicons rapidly (at time periods ~ 10) clean up the noise and propagate in a stable fashion over indefinitely large periods of time.

Platicons generated in optical microresonators look similar to "flaticons" demonstrated in optical fibers with normal GVD [31] or dispersive shock waves in an externally driven passive Kerr resonator with weak normal dispersion [32]. However, in contrast to quickly evolving flaticons and shock waves platicons are stationary soliton-like waveforms in time. It should be noted that in addition to the coupled-mode approach platicons may be also described by the Lugiato-Lefever equation (driven and damped nonlinear Schrödinger equation) [14,20] as it was done for dispersive shock waves [32]. However, the considered microresonators are comparatively small and pulse duration is comparable with the roundtrip time. Thus, periodic boundary conditions play an important if not determining role providing stationary solutions.

We found also that the growth of the sideband pump amplitude results in a widening of the excitation domain (fig. 1(d)). The position of the excitation domain depends on the primary pump amplitude f: for larger amplitude values the excitation domain shifts to larger values of ζ_0 and becomes narrower (fig. 1(e)). It is interesting to note, that for a larger primary pump amplitude platicon generation requires larger values of the relative sideband pump amplitude ε (fig. 1(f)).

In order to study the possibility of soft excitation by a phase-modulated pump we modified eq. (1) as

$$\frac{\partial a_\mu}{\partial \tau} = -(1+i\zeta_\mu)\,a_\mu + i\sum_{\mu'\le\mu''}(2-\delta_{\mu'\mu''})\,a_{\mu'}a_{\mu''}a^*_{\mu'+\mu''-\mu} + \delta_{0\mu}f + \varepsilon\left\{\delta_{1\mu}f\exp(i\Delta\tau) - \delta_{-1\mu}f\exp(-i\Delta\tau)\right\}. \qquad (2)$$

and found that phase modulation does not lead to the generation of platicons. Contrastingly, temporal cavity bright soliton excitation via direct phase modulation of the cavity driving field was demonstrated numerically [33] and experimentally [34].

Using an initial platicon as an input for our simulations we found all possible platicon solutions (fig. 2(a)). Interestingly, the excitation domain, defined as detuning range where platicon excitation from noise-like inputs is possible, is significantly narrower than the full existence domain and lies at its lower bound (in terms of ζ_0) (compare widths of characteristic steps on thin red lines in fig. 2(a) corresponding to excitation domains and widths of existence domains where the thick blue and black lines lie). Moreover, it was revealed that noise-like inputs provide the excitation of wide platicons only. However, having a wide platicon, generated inside the excitation domain, one may further find all other possible platicon solutions varying slowly the pump frequency (or the detuning value ζ_0) and, consequently, may determine full existence domain. It should be noticed that if several platicon solutions may exist at the same pump frequency, soft excitation provides the generation of a low-power platicon.

It was found that while the sideband pump amplitude increases, the existence domain of platicons becomes wider and the energy spectrum contains a smaller number of steps. In this way, in the considered system the parameter ε behaves analogously to the parameter describing the pump mode shift in [20] and similarly the growth of ε results in the transformation of the discrete energy spectrum of dark solitons into the quasi-continuous spectrum of platicons.

We revealed that similarly to platicons described in [20] one can effectively control the duration and, consequently, the power of generated pulses by slow pump frequency tuning. Figure 2(b) shows that while the pump frequency decreases (the corresponding detuning ζ_0 increases), the pulse duration also decreases. Consequently, the width of the corresponding frequency comb may also be tuned (fig. 2(c)).

It was found that while the pump power decreases, the existence domain of platicons becomes narrower and shifts to smaller values of ζ_0 (fig. 2(d)). In this way, one can tune the platicon duration varying the primary pump power: decreasing the pump one may generate shorter pulses.

To prove that the proposed approach is applicable for a wide range of materials used for the fabrication of microresonators with normal GVD we make calculations for a wide range of GVD values from $D_2/\kappa \approx -1.036\times10^{-3}$ up to $D_2/\kappa \approx -4.142\times10^{-2}$. It was found that the growth of the absolute value of D_2 results in a weak increase of the sideband pump power necessary for platicon generation (see fig. 3(a)). The decrease of the absolute value of the dispersion coefficient results in more localized pulses with sharper profiles and wider spectrum (figs. 3(b) and (c)). The positions of spectrum peaks and, consequently, the spectrum width may be estimated rather accurately by the simple phase-matching condition [35]: $\zeta_\mu = \zeta_0 + (D_2/\kappa)\,\mu^2 = 0$.

Also, while the existence domain weakly depends on D_2, the increase of the absolute GVD value leads to a more pronounced discreteness of the platicon energy spectrum (fig. 3(d)).

We also found that the method considered is very sensitive to the pump modulation frequency. While the mismatch between modulation frequency and FSR increases, existence (fig. 4(a)) and excitation (fig. 4(b)) domains

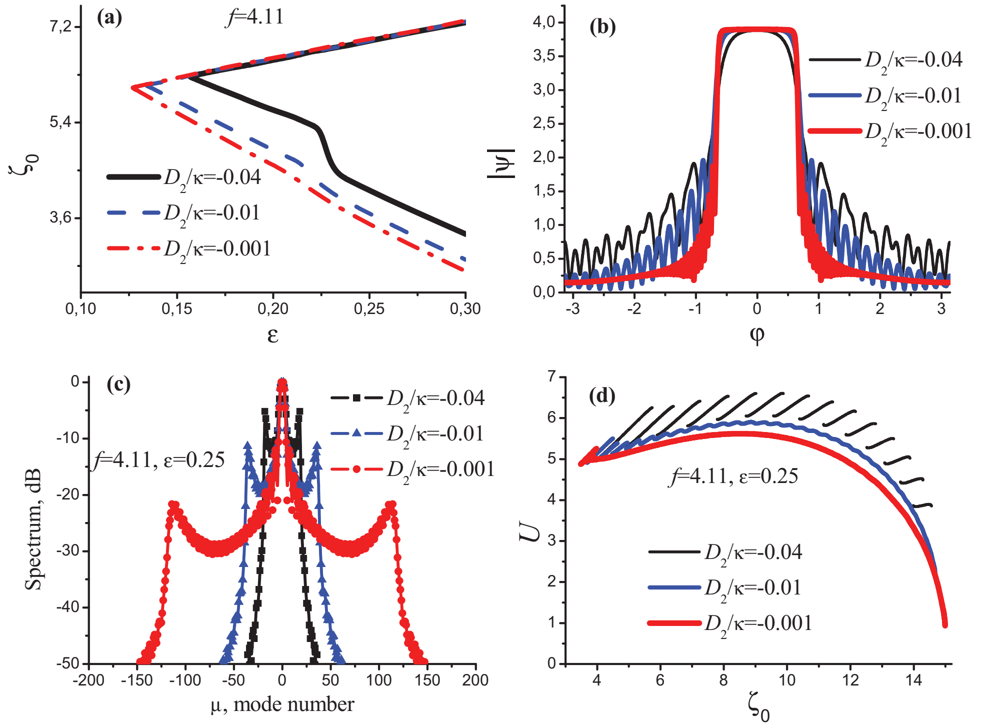

Fig. 3: (Color online) (a) Excitation domains for different values of GVD at $f = 4.11$. (b) Profiles and (c) spectra of platicons for different values of GVD at $\varepsilon = 0.25$, $f = 4.11$, $\zeta_0 = 14$. (d) Averaged intracavity intensity U of platicons *vs.* normalized detuning ζ_0 at $f = 4.11$, $\varepsilon = 0.25$ for different values of GVD. In all cases $\Delta = 0$.

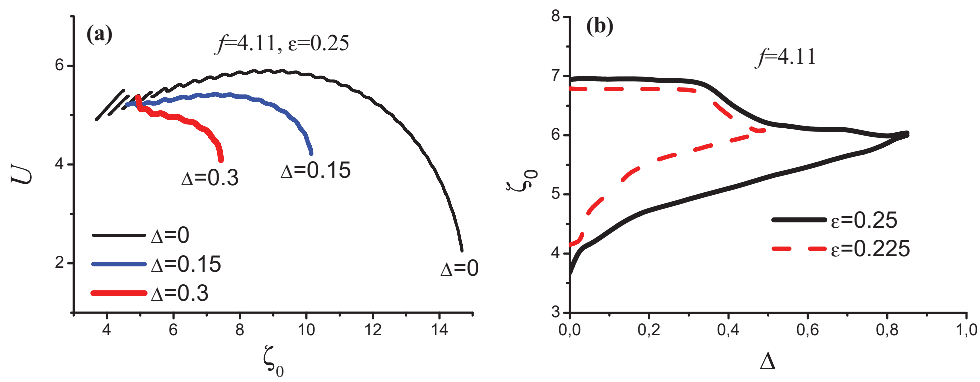

Fig. 4: (Color online) (a) Averaged intracavity intensity U of platicons *vs.* normalized detuning ζ_0 at $f = 4.11$, $\varepsilon = 0.25$ for different values of the modulation frequency mismatch Δ. (b) Excitation domain *vs.* mismatch value at $f = 4.11$, $\varepsilon = 0.25$ (black line) and $\varepsilon = 0.225$ (red line). $\varepsilon = 0.25$. In all cases $D_2/\kappa \approx 1.036 \times 10^{-2}$.

become significantly narrower and disappear at some critical value of mismatch. The critical for platicon excitation mismatch value increases with the sideband pump amplitude ε (see fig. 4(b)). This may be explained by the inefficiency of sideband pumping at large mismatches. In this case pump becomes effectively monochromatic and, thus, it cannot provide soft excitation of platicons or dark solitons at the considered dispersion law as it was shown earlier [14,20].

It was also revealed that for frequency combs generated by the amplitude-modulated pump the spacing between the comb lines is equal to the modulation frequency but not to the FSR of a microresonator. Interestingly, but not surprisingly, if the modulation frequency is equal to an integer number of FSR, the corresponding number of platicons per roundtrip can be generated.

Rewriting eq. (1) in the following form:

$$\frac{\partial a_\mu}{\partial \tau} = -(1 + i\zeta_\mu) a_\mu + i \sum_{\mu' \le \mu''} (2 - \delta_{\mu'\mu''}) a_{\mu'} a_{\mu''} a^*_{\mu'+\mu''-\mu} + \delta_{0\mu} f + \tilde{\varepsilon} \delta_{1\mu} f \exp(i\Delta\tau), \tag{3}$$

we also studied the usage of a biharmonic pump for platicon generation. It was found that the results obtained for a biharmonic pump practically coincide with the results for the amplitude-modulated pump if $\tilde{\varepsilon} \approx 2\varepsilon$. The main difference is that in contrast with the amplitude-modulated pump biharmonic pump is sensitive to the sign of modulation frequency mismatch (compare eqs. (1) and (3)). Interestingly, platicon generation is possible even if there is a constant phase shift between two pumping waves.

To summarize, we showed that soft excitation (from noise-like inputs) of platicons is possible for an amplitude-modulated or a biharmonic pump if the sideband pump power is large enough. For the effective platicon excitation the modulation frequency (or the frequency difference between two pump waves) should be close to the FSR of a microresonator. The excitation domain strongly depends on the sideband pump power and platicon duration may be altered significantly by tuning the pump frequency.

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