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LETTER

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Diffraction as a reason for slowing down light pulses in vacuum

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Abstract – The mean velocity of a finite-size short light pulse in a far zone is defined as the vectorial sum of velocities of all rays forming the pulse. Because of diffraction, the mean pulse velocity defined in this way is always somewhat smaller than the speed of light. The conditions are found when this slowing-down effect is sufficiently pronounced to be experimentally measurable. Under these conditions the original Gaussian shape of a pulse is found to be strongly modified with significant lengthening of the rear wing of the field envelope. Schemes for measuring these effects are suggested and discussed.

Introduction. – Propagation of light pulses in vacuum is a rather widely investigated process [1–9], the main features of which are well established and known. But, to the best of our knowledge, there is at least one question which is not sufficiently analyzed in the literature and the importance of which is not fully recognized. This is the question about propagation velocities of light pulses in vacuum: whether the pulse-propagation velocity in vacuum is identical to or different from the speed of light $c = 3 \times 10^{10}\,\mathrm{cm/s}$. For answering this question one needs a clear definition of the pulse-propagation velocity. Such definition used below can be explained qualitatively in terms of geometrical optics with a finite-size light pulse considered as consisting of a very large number of rays, such that in each ray light propagates with the same speed c but with different directions of the individual ray velocities. The difference of ray-propagation directions is assumed to be determined by the diffraction of pulses having finite transverse dimensions. Then, the vectorial sum of appropriately weighted individual velocities of light in rays can be considered as the mean velocity of a pulse $\langle \vec{v}\,\rangle$ or as the velocity of pulse as a whole. Clearly enough, in a general case, the defined in this way pulse-propagation velocity $|\langle \vec{v}\,\rangle|$ is somewhat smaller than the speed of light c. In many cases the difference $c - |\langle \vec{v}\,\rangle|$ is rather small. But, as shown below, in the case of very short and narrow pulses, in their far zone, the difference between $|\langle \vec{v}\,\rangle|$ and c can be sufficiently well pronounced and measurable. Below we find explicit expressions for the pulse-propagation velocity by using a solution of the boundary problem with Gaussian boundary conditions and with the field obeying Maxwell equations. We show that the slowing-down of the pulse propagation is related closely to the rather well pronounced and caused by diffraction lengthening of the pulse at its rear front. Some experimental schemes for observing this effect will be discussed.

Note that, though not used earlier for classical free-space light pulses, the described-above definition of the mean pulse-propagation velocity is not absolutely new. Historically such approach originates from features of the particle propagation in the special-relativity theory applied to pairs of noncollinear photons [10,11]. Evidently, the vectorial sum of momenta of two noncollinear photons is a vector, the absolute value of which is less than the sum of photon energies divided by c. Owing to this the system of two noncollinear photons as a whole can be characterized by its nonzero Lorentz-invariant mass. The appearance of a nonzero mass means immediately that the system of two noncollinear photons as a whole moves with a velocity smaller than the speed of light, though each individual photon of the system moves with its own velocity equal to the speed of light. Moreover, as shown in [10,11], for any pair of noncollinear photons one can identify its rest frame where absolute values of photon's momenta have equal absolute values but opposite directions. In this frame the system of two photons as a whole does not move at all. This is an extreme manifestation of the slowing-down effect owing to the noncollinearity of the wave vectors of photons.

Much later the slowing-down effect arising owing to noncolliearity was seen explicitly in the experiment [12] with pairs of photons obtained in the process of noncollinear Spontaneous Parametric Down-Conversion. In this process the initial beam is a sequences of well-separated from each other pairs of simultaneously produced photons. Owing to different conditions of propagation, photons in one of the two channels were slightly delayed compared to photons in the other channel, and this delay was registered in a very nice way with the help of coincidence measurements in the well-known Hong-Ou-Mandel effect [13]. Two important points to be emphasized concern the reasons of noncollinearity and temporal delay in the experiment [12]. First, the original noncollinearity of photon propagation in the SPDC process arises owing to the appropriately chosen orientation of a crystal which is not related to diffraction. And second, for making the effect more visible and more easily observable, it was strengthened by means of focusing and than defocusing the photon beam in one of the channels in a set of two confocal lenses, which produced a temporal effective additional noncollinearity also not related to diffraction.

At last, in our recent work [14] the ideas of two noncollinear photons and their Lorentz-invariant mass [10,11] were generalized for the case of an extremely high number of photons forming a light pulse. The latter was characterized quantum mechanically as a superposition of infinitely many multiphoton states in the form of multimode coherent states [15]. A light pulse was considered as a relativistic beam of photons and its Lorentz-invariant mass and propagation velocity were found from general relativistic formulas relating total energy, total momentum and mass. In this approach diffraction is immanently present. But it was supposed to be very weakly pronounced and, in analogy with [12], the effect was suggested to be strengthened by focusing and defocusing beam in the confocal lenses. In contrast to this, in the present work we consider a purely classical light beam, and we rely only on diffraction. By comparing the results to be described below with those of ref. [14] we find that the approaches of these two works are absolutely compatible and supporting each other. We believe that both the given above definition of the mean pulse-propagation velocity and its explicit derivation given below are fundamentally important. In principle, these results can be important also in practice, *e.g.*, for correct high-precision measurements of distances with femtosecond laser pulses.

The far-zone structure of short Gaussian pulses. – Let us reproduce briefly the well-known procedure of solving a boundary problem for the electric field of a Gaussian pulse [14]. Let the pulse be propagating along the z-axis, and at the boundary $z=0$ its electric field E be modeled by the Gaussian form both in transverse coordinate $\vec{r}_\perp$ and time t

$$E(\vec{r};t)\big|_{z-=0} = E_0 \exp\left(-\frac{r_\perp^2}{2w^2} - \frac{t^2}{2\tau^2}\right)\cos(\omega_0 t)\,, \quad (1)$$

where w, τ, and ω_0 are the waist of the pulse at $z=0$, pulse duration and central frequency. We do not specify here the polarization of the field and do not consider its magnetic component as this detailing is unnecessary for the problem under consideration.

The Gaussian exponents in eq. (1) can be Fourier-transformed to reduce this equation to the form

$$E(\vec{r};t)|_{z=0} = E_0 \frac{w^2\tau}{(2\pi)^{3/2}} \mathrm{Re}\int d\vec{k}_\perp d\omega\, e^{i\vec{k}_\perp \vec{r}_\perp} e^{-i\omega t} \times \exp\left(-\frac{k_\perp^2 w^2}{2} - \frac{(\omega-\omega_0)\tau^2}{2}\right). \quad (2)$$

With the integrand on the right-hand side of this equation multiplied by $e^{ik_z z}$ with $k_z = (\frac{\omega^2}{c^2} - \vec{k}_\perp^2)^{1/2}$, the same expression determines the field in all the half-plane $z \geq 0$. Let us write down this result in the paraxial approximation, in which $|\vec{k}_\perp| \ll \omega/c$ and, hence, $k_z \approx \frac{\omega}{c} - \frac{c\vec{k}_\perp^2}{2\omega}$. Under this condition

$$E(\vec{r};t) = E_0 \frac{w^2\tau}{(2\pi)^{3/2}} \mathrm{Re}\int d\vec{k}_\perp d\omega\, \exp\left[i\omega\left(\frac{z}{c} - t\right)\right] \times \exp\left[i\vec{k}_\perp \vec{r}_\perp - \frac{k_\perp^2}{2}\left(w^2 + i\frac{cz}{\omega}\right) - \frac{(\omega-\omega_0)\tau^2}{2}\right]. \quad (3)$$

The integral over $d\vec{k}_\perp$ is easily taken to give

$$E(\vec{r},t) = \frac{E_0\tau}{\sqrt{2\pi}} \mathrm{Re}\left\{\int d\omega\, \frac{L_D(\omega)}{iz + L_D(\omega)} \exp\left[i\omega\left(\frac{z}{c} - t\right)\right] \times \exp\left[-\frac{(\omega-\omega_0)^2\tau^2}{2} - \frac{\vec{r}_\perp^2}{2c}\frac{\omega}{iz + L_D(\omega)}\right]\right\}, \quad (4)$$

where $L_D(\omega) = w^2\omega/c \equiv L_D \times [1 + (\omega-\omega_0)/\omega_0]$ is the diffraction length for a light wave with frequency ω and initial (at $z=0$) transverse size w; $L_D \equiv L_D(\omega_0)$. Let us assume now that light pulses under consideration consist of many periods,

$$c\tau \gg \lambda, \text{ or } |\omega - \omega_0| \sim 1/\tau \ll \omega_0, \quad (5)$$

and the function $\omega/(L_D(\omega) + iz)$ in the second line of eq. (4) can be expanded in powers of the parameter $(\omega-\omega_0)/\omega_0 \ll 1$ up to the first order to give

$$\frac{\omega}{iz + L_D(\omega)} \approx \frac{\omega_0}{L_D + iz} + \frac{iz}{(iz + L_D)^2}(\omega - \omega_0).$$

With this expansion substituted into eq. (4) and with $L_D(\omega)$ approximated by L_D in the pre-exponential factor, we get again the Gaussian integral in the frequency

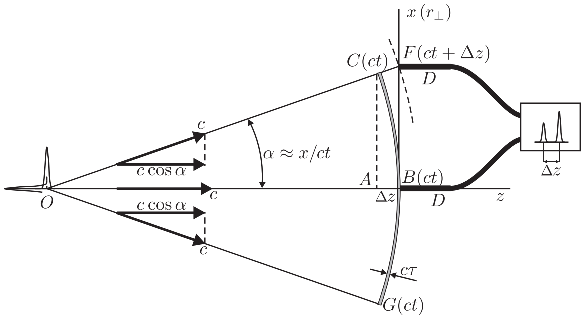

Fig. 1: The field distribution in a far zone in the (xz) plane (the gray area) and the suggested scheme of measurement, indicating the effect of slowing-down propagation along the z-axis of peripheral rays of the pulse; the fat lines with arrows are the propagation velocities of light in rays and their projections on the the z-axis; D denotes detectors (*e.g.*, open-end fibers).

variable ω which is easily taken to reduce eq. (4) to the form

$$E(\vec{r},t) = E_0 \mathrm{Re}\left\{\frac{L_D}{iz+L_D}\exp\left[i\omega_0\left(\frac{z}{c}-t\right)\right]\right. \times \exp\left[-\frac{\omega_0\vec{r}_\perp^{\,2}}{2c\,(iz+L_D)}\right] \left.\times \exp\left[-\frac{1}{2(c\tau)^2}\left(z-ct-\frac{z\vec{r}_\perp^{\,2}/2}{(iz+L_D)^2}\right)^2\right]\right\}. \tag{6}$$

In a general case this expression for the field strength can be rewritten as $E(\vec{r},t) = A(\vec{r},t)\cos[\omega_0(z/c-t)+\Phi(\vec{r},t)]$ with real amplitude $A(\vec{r},t)$ and phase $\Phi(\vec{r},t)$. The phase $\Phi(\vec{r},t)$ determines internal structure of oscillations in the pulse, the analysis of which is beyond the scope of this work. As for the field amplitude, it takes a rather simple form in the far-zone limit, when $z \gg L_D$:

$$A(\vec{r},t) = E_0\frac{L_D}{z}\exp\left[-\frac{\omega_0\vec{r}_\perp^{\,2}L_D}{2cz^2}\right] \times \exp\left[-\frac{1}{2(c\tau)^2}\left(z-ct+\frac{\vec{r}_\perp^{\,2}}{2z}\right)^2\right]. \tag{7}$$

The first of the two exponents in this equation characterizes diffraction. It determines the light diffraction angle: $\alpha = r_\perp/z \le \sqrt{c/\omega_0 L_D} = c/\omega_0 w = \lambda_0/2\pi w$, where λ_0 is the central wavelength of the pulse. The second exponent in eq. (7) characterizes the pulse structure. The field structure is axially symmetric, and its section by the plane (xz) is shown schematically in fig. 1. This picture, as well as all further consideration correspond to the case when the pulse length $c\tau$ is much shorter than the diffraction length L_D and, hence, much shorter than the distance between the initial plane $z=0$ and the observation region around $z \approx ct$. Under these conditions and at any given value of $r_\perp$ the peak of the pulse envelope $A(\vec{r},t)$ (7) is achieved at z and t obeying the equation

$$z - ct + \frac{r_\perp^2}{2z} = 0, \tag{8}$$

the solution of which is given by

$$z_{\text{peak}} = \frac{ct}{2} + \sqrt{\frac{(ct)^2}{4} - \frac{r_\perp^2}{2}} \approx ct - \frac{r_\perp^2}{2(ct)}. \tag{9}$$

In the last approximate expression of this equation it is taken that $r_\perp \ll z \approx ct$. Equation (8) determines the location of a spreading short pulse in the far zone in a region shown as a gray arc in fig. 1. Such distributions have been seen earlier, *e.g.*, in the theoretical researches of refs. [4,9]. Below, this geometry of the field distribution is used for the analysis of the mean propagation velocity of a pulse.

The mean propagation velocity of a diverging light pulse. – The lines OC, OB, OG in fig. 1 describe three examples of rays in a pulse. The light in these rays propagates with the speed of light c for equal distances $OC = OB = OG = ct \gg L_D$. The distance OA is the projection of the upper-ray trajectory OC on the z-axis. As seen clearly from fig. 1, $OA < OB$, *i.e.*, the projections on the z-axis of paths of rays non-parallel to this axis are shorter than their own total path-length ct and than the path-length ct of the ray propagating along the z-axis. At small values of the angle α between the propagation direction of rays and the z-axis the difference of path-lengths is easily estimated as

$$\Delta z = OB - OA = ct - ct\cos\alpha \approx \frac{r_\perp^2}{2ct} \ll ct. \tag{10}$$

As during a given propagation time t the rays non-parallel to the z-axis cross a smaller distance along this axis, the velocities of their propagation in this direction are smaller than that of the ray $\|Oz$, and they are detrmined by the projections of total velocities $\vec{v}$ on the z-axis as shown in the inset of fig. 1:

$$v_z(r_\perp) = c\cos\alpha = c - \frac{\Delta z}{t} = c\left(1 - \frac{r_\perp^2}{2z^2}\right). \tag{11}$$

As mentioned in the introduction we are interested here primarily in the propagation velocity of a pulse as a whole, *i.e.*, in the mean velocity $\langle\vec{v}\rangle$ defined as the vectorial sum of velocities of all rays of a pulse coming into the sum with the weighting function given by the squared diffraction part of the pulse amplitude (the first exponent in eq. (7)). Because of the axial symmetry, transverse components of the mean velocity are equal to zero, $\langle v_x\rangle = \langle v_y\rangle = 0$, whereas its z-component is given by

$$\langle v_z\rangle = c\left\{1 - \frac{1}{2z^2}\frac{\int \mathrm{d}\vec{r}_\perp r_\perp^2 \exp\left[-\frac{\omega_0\vec{r}_\perp^{\,2}L_D}{cz^2}\right]}{\int \mathrm{d}\vec{r}_\perp \exp\left[-\frac{\omega_0\vec{r}_\perp^{\,2}L_D}{cz^2}\right]}\right\}. \tag{12}$$

With integrals easily calculated, we get immediately the final expression for the mean propagation speed of a diverging laser pulse in a far zone:

$$\langle v_z \rangle \equiv |\langle \vec{v} \rangle| = c\left(1 - \frac{c}{2\omega_0 L_D}\right) = c\left(1 - \frac{\lambda_0^2}{8\pi^2 w^2}\right). \quad (13)$$

Interesting enough, this result coincides exactly with that derived in our previous work [14] in the frame of an absolutely different approach. We have considered there diverging pulses as relativistic objects characterized by their nonzero Lorentz-invariant mass, which was calculated. As usual in relativistic physics, objects with nonzero mass move with a velocity smaller than the light speed c. Based on this idea, the mean propagation velocity of a light pulse was found from relativistic equations connecting the total energy, momentum, invariant mass and propagation velocity of an object. In this way the mean pulse-propagation velocity was found to be given by the same equation as derived here, eq. (13). We believe that the present derivation is also very important because it clarifies the physics of the phenomenon in terms of geometrical optics and diffraction of finite-size light pulses.

The mean pulse-propagation velocity (13) is related directly to the mean shortening of the pathways along the z-axis, summed over all rays of a pulse:

$$\langle \Delta z \rangle = (c - \langle v_z \rangle)t = z\left(1 - \frac{\langle v_z \rangle}{c}\right) = z\frac{\lambda_0^2}{8\pi^2 w^2}. \quad (14)$$

Conventionally, the quantity $z - \langle \Delta z \rangle$ can be referred to as the z-coordinate of the pulse's "center of gravity" though, of course, it is not related at all to the true gravity features of light.

Observation in experiments. – Concerning seeing in experiments the described slowing-down effect, one of the possible schemes is shown in fig. 1. Measurements are assumed to be done by two detectors located at different distances from the z-axis but in the same observation plane perpendicular to the z-axis. Detectors can be represented simply by open-end fibers. With parameters given by eq. (16) detectors can be installed at a distance of $z = 1\,\mathrm{m}$ from the light source, for example, in the horizontal plane, at transverse coordinates $x_1 = 0$ and $x_2 = 1\,\mathrm{cm}$. With fibers of equal length signals from detectors can be sent to the knife-edge prism to merge into a single beam which has to be sent to a device like FROG (Frequency-Resolved Optical Gating) pulse analyzer. The expected result to be seen at the monitor has to represent a double-pulse structure qualitatively shown in fig. 1 with the spacing between pulses to be given by Δz of eq. (10) numerically equal to $\sim 0.128\,\mathrm{mm}$ at values of parameters of eq. (16).

Note that splitting of two pulses in this scheme is well pronounced if $\langle \Delta z \rangle \gg c\tau$. Combined with the assumption (5), this gives the following conditions when the slowing-down effect is well pronounced and can be observed:

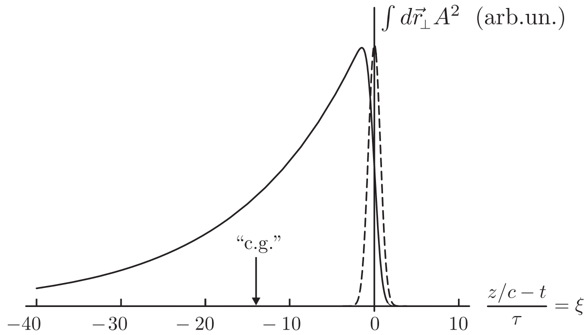

Fig. 2: The pulse shape in a far zone (solid curve) and the original pulse (dashed curve); "c.g." is the "center of gravity" of the pulse in units of $c\tau$, $(\langle \Delta z \rangle / c\tau)$.

$$\langle \Delta z \rangle = z\frac{\lambda_0^2}{8\pi^2 w^2} = \lambda_0 \frac{z}{2L_d} > c\tau \gg \lambda_0, \quad (15)$$

which assumes in particular that $z \gg L_D$. Below is an example of parameters, which are realistic and at which the conditions (15) are satisfied:

$$\tau = 30\,\mathrm{fs},\ \lambda_0 = 1\,\mu\mathrm{m},\ w = 10\lambda_0 = 10\,\mu\mathrm{m},\ z = 1\,\mathrm{m}. \quad (16)$$

These parameters correspond to the pulse length $c\tau = 9\,\mu\mathrm{m}$, diffraction length $L_D = 628\,\mu\mathrm{m}$, diffraction angle $\alpha \sim \lambda_0/2\pi w = 0.016 = 9°$, transverse size of the light spot at the measurement plane $x_D \sim \alpha z = 1.6\,\mathrm{cm}$, peripheral-ray path lengthening distance $\Delta z \sim x_D^2/2z = 128\,\mu\mathrm{m} = 14c\tau$, and parameter $z/2L_D = 796 \gg 1$.

In addition to the mean, averaged, parameters of a light pulse, presentation of its amplitude $A(r_\perp, z, t)$ in the form of eq. (7) can be used to find the total pulse shape by means of integration of A^2 over transverse coordinates at given z and t. The result of integration is given by

$$F(\xi) = \int \mathrm{d}\vec{r}_\perp A^2(r_\perp, z, t) = \frac{E_0^2 L_D^2 \pi^{3/2}}{2z^2} \times \exp\left(\frac{a^2}{4} + a\xi\right)\mathrm{erfc}\left(\frac{a}{2} + \xi\right), \quad (17)$$

where $\xi = \frac{z/c-t}{\tau}$, "erfc" is the complementary error function, and $a = (\frac{2\pi w}{\lambda_0})^2 \frac{2c\tau}{z}$. At the above given values of the pulse parameters (16) $a = 0.07$, and the function $F(\xi)$ determining the pulse shape is shown in fig. 2. The dashed curve is the appropriately normalized original pulse. The solid curve in fig. 2 indicates a rather significant lengthening and slowing-down of the pulse as a whole owing to diffraction. The arrow indicates a position of the above-defined "center of gravity" $\langle \Delta z \rangle$. Figure 2 indicates the existence of not only a rather well-pronounced effect of slowing-down of the pulse as a whole, but even a small delay of its peak value compared to the diffraction-free position at $\xi = 0$. Different parts of the curve arise from different rays forming the light pulse: the part close to $\xi = 0$ arises from the central rays, whereas the long wing at $\xi < 0$ corresponds to delayed peripheral rays of the pulse. In principle, lengthening of the pulse rear wing can be observed experimentally in the same way as shown in fig. 1 but with a series of additional fiber detectors installed along the z-axis between the upper and lower ones, with equalized lengths of fibers from detectors at $z = \mathrm{const}$ to the arrival-time resolving device. In a more sophisticated way, for reconstructing all the curve in fig. 2, detectors can be installed along rings around the z-axis with equal spacings between detectors in all rings and with signals from detectors in each given ring summed together.

Conclusion. – In conclusion, the conditions (15) are identified under which the effect of slowing-down propagation of short light pulses is well pronounced and can be observed experimentally. Under these conditions the pulse shape in the far zone changes significantly compared to the original one: its rear wing significantly lengthens as shown in fig. 2. Owing to this, the "center of gravity" of a pulse appears to be pronouncedly delayed compared to its front wing, and even the location of the peak of the pulse envelope appears to be slightly delayed too. All this indicates unambiguously that owing to diffraction a light pulse obeying the conditions (15) propagates as a whole with a mean velocity $\langle \vec{v} \rangle$ smaller than the speed of light c. Schemes of experiments for observing these effects directly are discussed. Once again, we believe that it is fundamentally important having a) a clear definition of the mean (or "gravity-center") velocity of light pulses, b) its explicit expressions for originally Gaussian pulses, c) identification of diffraction as the reason for slowing down diverging pulses, and d) identification of conditions when this effect can occur. Concerning measurements with very short pulses, the carried out consideration shows that it is important to know what is felt by a target, arrival of the front wing of a pulse or of its "center of gravity" which propagates with a velocity $\langle v_z \rangle$ (13) smaller than the light speed c.

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