LETTER

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The nuclear configurational entropy impact parameter dependence in the Color-Glass Condensate

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Abstract – The impact parameter (b) dependence on the saturation scale, in the framework of the Color-Glass Condensate (b-CGC) dipole model, is investigated from the configurational point of view. During the calculations and analysis of the quantum nuclear states, the critical points of stability in the configurational entropy setup are computed, matching the experimental parameters that define the onset of the quantum regime in the b-CGC in the literature with very good accuracy. This new approach is crucial and important for understanding the stability of quantum systems in the study of deep inelastic scattering processes.

Introduction. – The nuclear configurational entropy [1] takes into the cross-section as a natural localized, square-integrable function, driving the configurational entropy setup in nuclear physics. This concept has been already applied to gauge dualities involving quantum chromodynamics (QCD) models in refs. [2,3] for studying the stability of mesons and scalar glueballs. The nuclear configurational entropy is based upon the information entropy, implemented by the recently introduced configurational entropy [4–7], for spatially-localized physical systems. As the cross-section of any nuclear reaction is spatially-localized and employed to characterize the probability that any reaction occurs, the analysis of the shape complexity of classical field configurations can be implemented in the context of cross-sections. In nuclear physics one measures the cross-section values for the systems with a finite spatial extent. In order to describe the system, thus, one needs as much information, as the higher the informational entropy is. A brief analysis between the configurational entropy lattice approach and statistical mechanics has been paved in ref. [2]. In a similar context, the Hawking-Page transition was analysed [8] and Bose-Einstein condensates have been studied [9] with interesting results.

The strong interaction at high energy regimes has been studied in both experiment and theory. QCD is a theory of strong interactions that is able to explain most of their underlying experimental results to remarkable accuracy. Another important source of information is the numerical lattice calculation, which produces a correct solution to the QCD field equations. However, a lot of questions are still open and the lattice data still need to be interpreted in order to find the mechanisms and principles that lead to these numerical results. In this case, the Shannon-based informational entropy can shed some light onto the principal results based on the lattice QCD setup and introduces a quantitative theoretical apparatus for studying the instability of a highly-excited nuclear system. In this work, the cross-sections will be analysed with regard to some principal points of the associated informational entropy. Our focus in this letter is to apply the Color-Glass Condensate to nuclear collisions.

Colliding heavy nuclei with each other makes QCD predict that a new state of matter, called the quark-qluon plasma (QGP), is then formed [10]. The most precise measurements of the quark and gluon structure of the proton come from the HERA particle accelerator, which collided electrons and positrons with protons. In these experiments, the proton can look very differently, depending at which scale it is measured. When the proton structure is probed with a photon that has a long wavelength compared with the proton size, a charged particle with electric charge $+e$ is seen. The inner structure of the proton becomes visible when the photon wavelength is decreased to the order of the proton radius. First, one observes three valence quarks having a fractional electric

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charge and carrying a fraction $\sim 1/3$ of the proton longitudinal momentum, viewed in the frame where the proton energy is very large. When the wavelength of the photon decreases more, a richer structure becomes visible. The photon starts to see a large number of sea quarks and antiquarks that carry a small fraction of the proton longitudinal momentum, denoted by Bjorken-x [11–14]. These quarks originate from gluon splittings to quark-antiquark pairs. When the proton structure is measured at smaller x, more sea quarks and gluons are seen. The mechanism of the reaction based on QCD can be investigated by the dependence of the total hadronic cross-section on the energy, as well as the study of the appropriate role of partons during the interaction with high energy hadrons. Such important questions which have arose since the early days of the strong interactions, as, for instance, the behavior of cross-sections at high energies, or the universality of hadronic interactions, as well as the nature of multi-particle production, have acquired a new vision. In the KKT model [11], it was assumed that the gluons inside particles are seen by other particles as a gluon wall, that describes the Color-Glass Condensate itself. In other words, gluons from the inside have a high density distribution. Increasing the energy increases the momentum states that are occupied by the gluons, forcing a weaker coupling among the gluons [11–13]. This leads to the gluon saturation effect, which corresponds to a multiparticle Bose condensate state.

QCD at high energies can be described as a many-body theory of partons which are weakly coupled albeit non-perturbatively due to the large number of partons. Such a system is called a Color-Glass Condensate (CGC) and describes an effective perturbative weak-coupling field theory approach in the small-x regime of QCD. The initial conditions for high energy collisions are determined by the free partons in the wave functions of the colliding nuclei.

Partons represent the localized-energy systems, they can be considered as the spatially-coherent field configurations in an informational entropic context, together with the Shannon entropy of information theory [6]. The critical points of the configurational entropy correspond to the onset of instability of a spatially-bound configuration. In a considerable amount of studies, the configurational entropy has been applied to analyze aspects in a variety of models, which comprise the higher spin mesons and glueballs stability [2,3] as well as Bose-Einstein condensates of long-wavelength gravitons, which describe black holes [8]. The physical systems that were studied in the above-mentioned works had configurations of classical fields and have been successfully analyzed from the view of critical points, including the derivation of the Higgs mass [15,16].

It is interesting to analyze the configurational entropy from the point of view of spatially-localized reaction cross-sections as [1]

$$\sigma(\vec{k}\,) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} \sigma(\vec{r}\,)e^{i\vec{k}\cdot\vec{r}}\mathrm{d}\vec{r}. \qquad (1)$$

Then the modal fraction can be expressed as [4–7]:

$$f_\sigma(\vec{k}\,) = \frac{|\sigma(\vec{k}\,)|^2}{\iint_{-\infty}^{\infty}|\sigma(\vec{k}\,)|^2\mathrm{d}\vec{k}}. \qquad (2)$$

Finally, one can define the configurational entropy analogously to what has been defined for the energy density in ref. [6], but this time for the cross-section, as

$$S_c[f] = -\iint_{-\infty}^{\infty} \dot{f}_\sigma(\vec{k})\log\dot{f}_\sigma(\vec{k})\mathrm{d}\vec{k}, \qquad (3)$$

where $\dot{f}(\vec{k}) = f(\vec{k})/f_{\max}(\vec{k})$. Critical points of the configurational entropy imply that the system has informational entropy that is critical with respect to the maximal entropy $f_{\max}(\vec{k})$, corresponding to more dominant states [2,8,9].

The configurational entropy and KKT model. – The study of deep inelastic scattering (DIS) reactions and exclusive diffractive processes of leptons on protons and/or nuclei, such as, for instance, exclusive vector mesons production and virtual Compton scattering at small-x, has turned toward understanding the mechanism of QCD. Many interesting questions which have arisen since the beginning of the investigation of strong interactions, such as the behavior of cross-sections at high energies, the universality of hadronic interactions at high energies, have acquired fresh vigor. In the CGC approximation, one of the important ingredients for particle production is the universal dipole amplitude, represented by the imaginary part of the quark-antiquark scattering amplitude on a target.

The impact parameter (b) dependence of the dipole amplitude is essential for understanding exclusive diffractive processes in the CGC or the color dipole approach. Therefore, in this work we will briefly represent a simple dipole model, b-CGC, that incorporates all known properties of the gluon saturation, and the impact parameter dependence of the dipole amplitude [17]. The b-CGC model has been applied to various reactions, as deep inelastic scattering and diffractive processes [17] and proton-nucleus [18] collisions at RHIC and the LHC.

At small x, the deep inelastic scattering is characterized by the fluctuation of the virtual photon $\gamma^\star$ into a quark-antiquark pair $q\bar{q}$ with size r, which then scatters off the hadronic or nuclear target via gluon exchanges. Then, the total deep inelastic cross-section, $\gamma^\star p$, for a given Bjorken x and virtuality Q^2 will be expressed as [19]:

$$\sigma_{L,T}^{\gamma^* p}(Q^2,x) = 2\sum_f\int\mathrm{d}^2\vec{r}\int\mathrm{d}^2\vec{b}\int_0^1\mathrm{d}z|\Psi_{L,T}^{(f)}(r,z,m_f;Q^2)|^2 \times\mathcal{N}(x,r,b), \qquad (4)$$

where z is the fraction of the light-front momentum of the virtual photon carried by the quark, m_f is the quark mass, and $\mathcal{N}(x,r,b)$ is the imaginary part of the forward $q\bar{q}$ dipole-proton scattering amplitude with dipole size r

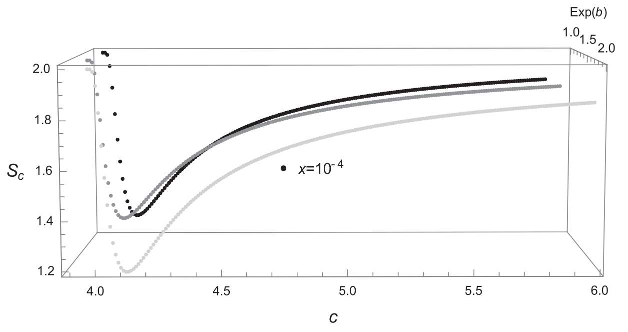

Fig. 1: Configurational entropy as a function of the onset of the gluon anomalous dimension, in the Color-Glass Condensate regime, for distinct values of the Bjorken variable $x = 10^{-4}$. Each curve was generated for intervals of $c = 0.01$. The black curve is for $b = 0\,\mathrm{GeV}^{-1}$, the light grey curve is for $b = 0.7\,\mathrm{GeV}^{-1}$ and the grey curve represents $b = 0.4\,\mathrm{GeV}^{-1}$.

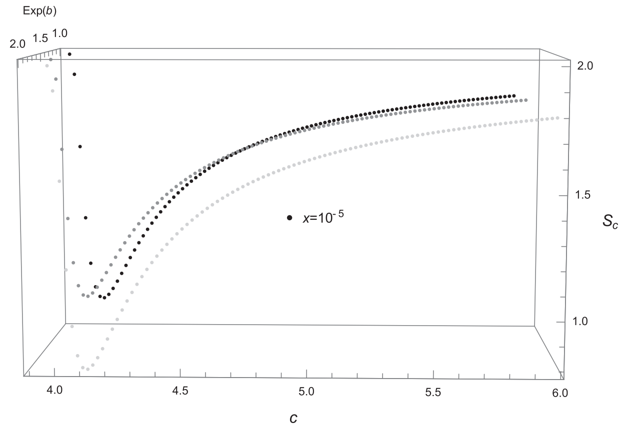

Fig. 2: Configurational entropy as a function of the onset of the gluon anomalous dimension, in the Color-Glass Condensate regime, for distinct values of the Bjorken variable $x = 10^{-5}$. Each curve was generated for intervals of $c = 0.01$. The black curve is for $b = 0\,\mathrm{GeV}^{-1}$, the light grey curve is for $b = 0.7\,\mathrm{GeV}^{-1}$ and the grey curve represents $b = 0.4\,\mathrm{GeV}^{-1}$.

and impact parameter b. The form of the light-front wave function $\Psi^{(f)}_{L,T}$, for $\gamma^\star$ fluctuations into $q\bar{q}$, was taken from ref. [20]. The subscripts L, T in eq. (4) denote the longitudinal and transverse polarizations of the virtual photon. In ref. [21] it was suggested a simple dipole model, which links the two limiting behaviors of eq. (4), namely the case in the vicinity of the saturation line for small dipole sizes, $r \ll 1/Q_s$, and the case of deep inelastic scattering inside the saturation region for larger dipoles, $r \gg 1/Q_s$. Such a model is historically called CGC dipole model, in which the color dipole-proton amplitude is given by

$$N(x,r,b) = \begin{cases} N_0 \left(\dfrac{rQ_s}{2}\right)^{2\gamma_{\mathit{eff}}}, & rQ_s \le 2, \\ 1 - \exp(-\mathcal{A}\log^2(\mathcal{B} rQ_s)), & rQ_s > 2, \end{cases} \tag{5}$$

where the effective anomalous dimension is expressed by the formula

$$\gamma_{\mathit{eff}} = \gamma_s + \frac{1}{\kappa\lambda Y}\log\left(\frac{2}{rQ_s}\right), \tag{6}$$

where $Y = \log(1/x)$ and $\kappa = \chi''(\gamma_s)/\chi'(\gamma_s)$, with χ being the characteristic function. The scale Q_s in eqs. (5), (6), is generally called the saturation scale. In the CGC dipole model, the scale Q_s is given by following expression:

$$Q_s \mapsto Q_s(x) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}}\,\mathrm{GeV}. \tag{7}$$

The parameters $\mathcal{A}$ and $\mathcal{B}$ in eq. (5) are determined from the matching of the dipole amplitude and its logarithmic derivatives at $rQ_s = 2$:

$$\mathcal{A} = -\frac{N_0^2\gamma_s^2}{(1-N_0)^2\log(1-N_0)}, \quad \mathcal{B} = \frac{1}{2}(1-N_0)^{-\frac{1-N_0}{N_0\gamma_s}}. \tag{8}$$

The amplitude is considered to be independent of the impact parameter in the Color-Glass Condensate model. Thus, the integral over the impact parameter in eq. (4) can be considered as a normalization factor $\sigma_0 = 2\int \mathrm{d}^2 b$, and is determined by a fit to data. Therefore, the total dipole cross-section will be estimated as $\sigma_{q\bar{q}} = \sigma_0\mathcal{N}(x,r)$. The parameters $\kappa = 9.9$ and $N_0 = 0.7$ are fixed [17,21], and the other four parameters, namely $\gamma_s, x_0, \lambda, \sigma_0$, are obtained by a fit to the HERA data via a χ^2 minimization procedure.

In the so-called b-CGC model [17] the authors have extended the CGC dipole model by involving the dependence of an amplitude of an impact parameter. Therefore, the dependence of the saturation scale on the impact parameter in expression (8) can be changed by

$$Q_s \mapsto Q_s(x,b) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}}\exp\left(-\frac{b^2}{4\gamma_s B_{CGC}}\right)\mathrm{GeV}. \tag{9}$$

In eq. (9), instead of σ_0 in the CGC dipole model, the free parameter is B_{CGC}. It was determined by other reactions, for instance, the t-distribution of the exclusive diffractive processes at HERA.

In order to compute the informational entropy associated to the Color-Glass Condensate, let us start by calculating the spatial Fourier transform of the total deep inelastic cross-section, and thus the color dipole-proton amplitude is defined in the b-CGC model [17]. For such a purpose, eqs. (1)–(3) are used in the 2-dimensional case. First of all, the Fourier transform of the total deep inelastic cross-section (4) together with eq. (5) can be computed using eq. (1). After that, the result of the calculations is used to estimate the modal fraction, eq. (2). At last, using eq. (3) one can calculate the informational entropy utilizing the modal fraction.

The numerical results follow after awkward algebraic manipulations. The results were obtained for three given values of the impact parameter, *i.e.* $b = 0$; $b = 0.404$, and $b = 0.693$, for $x = 10^{-4}$ (fig. 1) and $x = 10^{-5}$ (fig. 2), respectively. As one can see from fig. 1, the black dotted curve that depicts the minimum of the nuclear configurational entropy for $b = 0$ corresponds to the value of the parameter $c = 4.100$. In the case of the impact parameter $b = 0.404$ (grey dotted curve), the nuclear configurational entropy minimum occurs at $c = 4.050$ and in the more peripheral collision, when $b = 0.693$, which is shown by the light grey curve in fig. 1, the minimum of the informational entropy falls at $c = 4.120$, establishing the onset of the quantum regime in the CGC.

In the case of the Bjorken variable given by $x = 10^{-5}$, which is shown by fig. 2, the minimum in the black dotted curve for the impact parameter value $b = 0$ corresponds to $c = 4.080$. When the value of the impact parameter turns to $b = 0.404$ (grey curve), the minimum of the nuclear configurational entropy corresponds to $c = 4.040$ and in the more peripheral collision when $b = 0.693$, which is shown by the light grey curve in fig. 2, the minimum of the nuclear configurational entropy falls at $c = 4.120$. Therefore, one can conclude that the best result for the impact parameter is $b = 0.404$, that corresponds to the fitted parameter c, and was assumed to be $c = 4.01$, in ref. [1], to derive the results therein presented. These results match those in ref. [12].

The calculation shows that such is a natural choice provided by the analysis of the minima of the nuclear configurational entropy, which derives the universal dipole amplitude with impact parameter of the collision in the Color-Glass Condensate model setup. The uncertainties of the calculation are upper bound by $\sim 1.23\%$.

Outlook. – The nuclear configurational entropy was used to derive the onset of the CGC, with impact parameter dependence, matching the results in the literature with a very good accuracy [1,12], of around ∼1%. Figures 1 and 2, and the analysis that respectively follows, illustrate the critical points of the nuclear configurational entropy as a way to derive the onset of the quantum regime in the CGC, for different values of the impact parameter b. A direction to be further studied encompasses quantum mechanics fluctuations. The wave function used in those equations can be explored in the context of the topological defects proposed, *e.g.*, in refs. [22–28], in the configurational entropy setup. In order to improve the understanding of the nature of deep inelastic scattering, as well as the diffractive processes, further systematical investigation is needed concerning the gluon saturation in the proton wave function.

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REFERENCES

[1] Karapetyan G., *EPL*, **117** (2017) 118001.
[2] Bernardini A. E. and da Rocha R., *Phys. Lett. B*, **762** (2016) 107.
[3] Bernardini A. E., Braga N. R. F. and da Rocha R., *Phys. Lett. B*, **765** (2017) 81.
[4] Gleiser M. and Stamatopoulos N., *Phys. Lett. B*, **713** (2012) 304.
[5] Gleiser M. and Jiang N., *Phys. Rev. D*, **92** (2015) 044046.
[6] Gleiser M. and Stamatopoulos N., *Phys. Rev. D*, **86** (2012) 045004.
[7] Gleiser M. and Sowinski D., *Phys. Lett. B*, **747** (2015) 125.
[8] Casadio R. and da Rocha R., *Phys. Lett. B*, **763** (2016) 434.
[9] Braga N. R. F. and da Rocha R., *Phys. Lett. B*, **767** (2017) 386.
[10] Song H. and Heinz U. W., *J. Phys. G*, **36** (2009) 064033.
[11] Kharzeev D., Kovchegov Y. V. and Tuchin K., *Phys. Lett. B*, **599** (2004) 23.
[12] Carvalho F., Durães F. O., Gonçalves V. P. and Navarra F. S., *Mod. Phys. Lett. A*, **23** (2008) 2847.
[13] McLerran L. and Venugopalan R., *Phys. Rev. D*, **49** (1994) 2233.
[14] Carvalho F., Durães F. O., Navarra F. S. and Gonçalves V. P., *Acta Phys. Pol. B*, **39** (2008) 2511.
[15] Alves A., Dias A. G. and da Silva R., arXiv:1703.02061 [hep-ph] (2017).
[16] Alves A., Dias A. G. and da Silva R., *Physica*, **420** (2015) 1.
[17] Watt G. and Kowalski H., *Phys. Rev. D*, **78** (2008) 014016.
[18] Rezaeian A. H., *Phys. Rev. D*, **85** (2012) 014028.
[19] Nikolaev N. N. and Zakharov B. G., *Z. Phys. C*, **49** (1991) 607.
[20] Golec-Biernat K. J. and Wusthoff M., *Phys. Rev. D*, **59** (1998) 014017.
[21] Iancu E., Itakura K. and Munier S., *Phys. Lett. B*, **590** (2004) 199.
[22] Correa R. A. C., Dantas D. M., Almeida C. A. S. and da Rocha R., *Phys. Lett. B*, **755** (2016) 358.
[23] Correa R. A. C. and da Rocha R., *Eur. Phys. J. C*, **75** (2015) 522; *Ann. Phys.*, **359** (2015) 198.
[24] Bernardini A. E. and da Rocha R., *Phys. Lett. A*, **380** (2016) 2279.
[25] Bernardini A. E. and da Rocha R., *Adv. High Energy Phys.*, **2016** (2016) 3650632.
[26] Bazeia D., Menezes R. and da Rocha R., *Adv. High Energy Phys.*, **2014** (2014) 276729.
[27] Bazeia D., Losano L., Menezes R. and da Rocha R., *Eur. Phys. J. C*, **73** (2013) 2499.
[28] Bernardini A. E. and da Rocha R., *Adv. High Energy Phys.*, **2013** (2013) 304980.