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LETTER

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MOND and cosmological issues from entropic gravity and nonextensive thermostatistics correspondence

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Abstract – It is an old idea to realize Einstein's equations as a thermodynamical equation of state. Hence, to understand the actual role of the holographic screen is a very relevant issue. In this letter we have analyzed the entropy as a function of the holographic screen in some different scenarios and calculated a modified Newton's gravitational law for each one of them. We have also disclosed the modified Newtonian dynamics (MOND) from Verlinde's ideas. Besides, we have calculated some cosmological elements using the same concepts.

One of the main challenges of modern theoretical physics is to unify the concepts of quantum mechanics and gravitation. Although it is an old issue in the literature, it was questioned if this difficulty is caused by the fact that we are really trying to quantize an effective theory. Another question would be whether gravity is not an underlying force. In its defense we can say that general relativity is an exact theory that describes the dynamics of the objects that comprise our Universe. Recently, the detection of gravitational waves by LIGO Collaboration [1] corroborates the predictions of general relativity. But, in spite of its success, the question about its fundamentality is still on.

There are some theoretical evidences that show the thermodynamical feature of gravity. For instance, the works of Bekenstein and Hawking (BH) [2–4] in which the authors connect the laws of black holes to the ones about thermodynamics concerning the creation of particles by a gravitational field. Or the obtention of Einstein's equations from the entropy proposed by Jacobson [5–7], where he proposed that gravitation must be an effective theory. Besides, we can mention the works of van Raamsdonk *et al.* [8–10] where Einstein's equations can be derived from the entanglement laws. Jacobson established the connection to entropy in [5–7], where he derived Einstein's equations. He used both the Clausius relation $\delta E = T\delta S$ and the concept that the matter present can be considered as part of the energy.

At the quantum level, we can understand spacetime geometry as the entanglement structure of the microscopic quantum state, from which gravity emerges depicting the change in entanglement that originates from matter. Namely, gravity emerges from the viewpoint of quantum information. The best way to understand these new ideas is to consider an anti-de Sitter space, where the description of a dual conformal field theory allows one to obtain the microscopic entanglement in a well-constructed setting [11].

Recently, Verlinde [12] has derived Newton's law of gravity by using holographic arguments. He has used the BH entropy-area relation for black holes. Verlinde also suggested that gravity is an entropy manifestation. The gravitational force results from information entropy modifications, which would be stored in a holographic sphere. Verlinde considered entropy as the information relative to the positions of material bodies around a point mass M at a distance R. Besides, all points at this distance are useful to define a sphere S embedded into a $D = 3$ space. To sum up, when we change the bits of information that are localized on the screen, a force appears as a reaction to

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that modification [13–17]. This will be shown in eq. (13) below.

The formalism known as Modified Newtonian Dynamics (MOND) was constructed by Milgrom [18–20], to explain the observed general properties of galaxies such as the stars velocities inside the galaxies. The observed values are larger than the expected ones calculated by using Newtonian mechanics. In MOND, for extremely small accelerations, we would have a violation of Newton's laws. However, a cosmological model based on MOND's concepts has not been constructed yet. In this work we have used Verlinde's ideas in a Tsallis nonextensive statistics background to derive MOND's concepts among other results.

Tsallis' statistics [21–23], which is an extension of Boltzman-Gibbs's (BG) statistical theory, defines a nonadditive entropy given by

$$S_q = k_B \frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \qquad \left(\sum_{i=1}^{W} p_i = 1\right), \tag{1}$$

where p_i is the probability of a system to exist within a microstate, W is the total number of configurations and q, known in the current literature as being the Tsallis parameter or NE parameter, is a real parameter which measures the degree of nonextensivity. It has different values for each system.

The definition of entropy in Tsallis statistics has the standard properties of positivity, equiprobability, concavity and irreversibility. This approach has been successfully used in many different physical systems. For instance, we can mention the Levy-type anomalous diffusion [24], turbulence in a pure-electron plasma [25], gravitational systems [26,27] and in the scenario of entropic gravity [28–32]. It is noteworthy to mention that Tsallis thermostatistics formalism has the BG statistics as a particular case in the limit $q \to 1$, where the entropy standard additivity property can be recovered.

In the microcanonical ensemble, where all the states have the same probability, Tsallis entropy reduces to [33]

$$S_q = k_B \frac{W^{1-q}-1}{1-q}, \tag{2}$$

where, at the limit $q \to 1$, we can recover the usual Boltzmann entropy formula, $S = k_B \ln W$.

Let us consider a number of microstates W, in a general scaling of Tsallis' entropy scenario, so that W can be written like the one in [34] such that

$$W = b\left(\frac{A}{4l_p^2}\right)^{\alpha}, \tag{3}$$

where b is a dimensionless constant, A is the area of Σ, l_p is the Planck's length in nonextensive entropy, eq. (2), and α is an undetermined parameter that shows the general scaling purpose here. For example if $\alpha = 3/2$ in (3), we would have a volume scaling. Our objective here is to analyze what happens if the exponent in (3) is chosen to be a general one.

The term in eq. (3) was calculated through loop quantum gravity (LQG) considerations in [35] as a correction term for the entropy. In [35], eq. (3) is the volume correction relative to the area law, which is also motivated by a model for the microscopic degrees encompassing the black hole entropy in LQG. With reference to our work specifically, the important feature is that this term results in the $1/R$ correction term in Newton's laws in MOND as an explanation for the registered anomalous galactic rotation curves. This idea is the path that allows us to use Tsallis nonextensive statistics. It will be clearer soon.

The objectives here are to use eq. (3) in the definition of entropy in eq. (2) and to analyze the q-parameter effect together with the α parameter to see their cosmological effects. Notice that for $q = 0$ we recover the volume correction term to Newton's laws.

To begin with, let us review that Verlinde [12] has stated that the entropy ΔS of Σ connected to a test particle of mass m moving by a distance Δx orthogonal to the screen can be written as

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x, \tag{4}$$

which shows that the entropy obtained is proportional to the information loss of the test particle, where $\lambda_m = \hbar/mc$ is the Compton wavelength and we can write $\Delta S = 2\pi k_B \Delta x/\lambda_m$. The general expression of the force which is governed by the usual thermodynamic equation is

$$F = T\frac{\Delta S}{\Delta x} = T\frac{\mathrm{d}S}{\mathrm{d}A}\frac{\Delta A}{\Delta x}, \tag{5}$$

where $A = 4\pi R^2$ is the area of the holographic screen. Suppose we have two masses, one is the test mass m and the other, M, is considered as the source. The holographic screen will be centered around the source mass M. The energy of the holographic screen is given by

$$E = Mc^2. \tag{6}$$

We will use that the bits of information are proportional to the area of the holographic screen as [34]

$$A = QN, \tag{7}$$

where N is the number of bits and the constant Q the fundamental charge [34] or area gap ($\Delta A|_{N=1} = Q$) determined by the microscopic theory. It is important to explain here that we have a relation between entropy and the distance from the horizon, *i.e.*, $\Delta S \propto \Delta x$. However, we also have a relation between entropy and the area of Σ, *i.e.*, $\Delta S \propto \Delta A$. So, we can write that $\Delta A \propto \Delta x$ and consequently that $Q \propto \Delta x$, which shows clearly that we can expect a relation between Q and η, this issue will be clear in a moment. The total bits energy on the screen is given by the energy equipartition law

$$E = \frac{1}{2}Nk_BT. \tag{8}$$

When the test mass m is at a distance

$$\Delta x = \eta \lambda_m, \tag{9}$$

away from Σ, the entropy of the screen modifies when the test mass moves through a certain distance where $\lambda_m = \hbar/mc$ is the Compton wavelength and η is a scale factor. The entropy gradient points radially from the outside of the screen to the inside, as can be seen from

$$\Delta S = \frac{\partial S}{\partial A}\Delta A. \tag{10}$$

Then, from eq. (7) we have that

$$\Delta A = Q, \tag{11}$$

where it was assumed that $\Delta N = 1$. Combining eqs. (5)–(9), (11) and using the fundamental relation $l_P^2 = \hbar G/c^3$, we have that

$$F = \frac{GMm}{R^2}\frac{Q^2}{2\pi\eta l_p^2 k_B}\frac{\mathrm{d}S}{\mathrm{d}A}. \tag{12}$$

Defining conveniently that $Q^2 = 8\pi\eta l_p^4$, which provides Newton's laws to first order such as in [34] we can write that

$$F = \frac{GMm}{R^2}4\frac{l_p^2}{k_B}\frac{\mathrm{d}S}{\mathrm{d}A}. \tag{13}$$

Notice that this expression is sufficiently general in order to permit the consideration of any kind of entropy, which is our purpose at this point. We will use the well-known particular relation from black hole entropy $S = k_B A/4l_p^2$, the BH formula, where in this case A is the area of the event horizon as a constraint, *i.e.*, a screen that sets the point of no return. Quantically speaking, a black hole creates and emits particles as if it was a black body with temperature T [36].

It can be shown that we can obtain the usual Newton's law of gravitation, $F = GMm/R^2$ from eq. (13). Newton's gravitational theory just states how the law works but, however, it does not tell us why they work.

Using eqs. (2) and (3) into eq. (13) we obtain a modified Newton's law of gravitation written as

$$F = \frac{GMm}{R^2}\alpha b^{1-q}\left(\frac{A}{4l_p^2}\right)^{\alpha(1-q)-1}. \tag{14}$$

We can observe that when we make $\alpha = 1/(1-q)$ in this last equation we can recover the usual Newton's law of gravitation if $b = (1-q)^{\frac{1}{1-q}}$. In fig. 1, we have plotted eq. (14) as a function of the parameter q. Substituting eq. (3) into (2) it is straightforward to see that

$$S = k_B\left(\frac{A}{4l_p^2}\right), \tag{15}$$

where we have neglected a constant term, $-k_B/(1-q)$, in comparison with the A term.

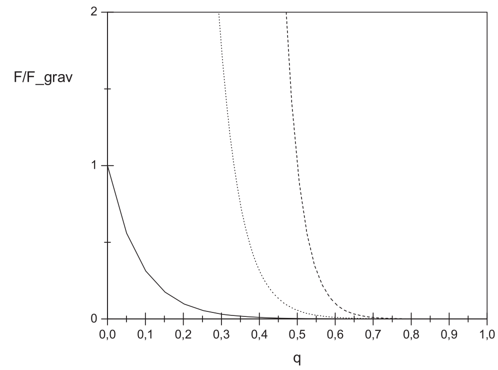

Fig. 1: The generalized gravitational force, normalized by the usual Newton gravitational law, as a function of the parameter q, eq. (14). We consider the product $\alpha b^{(1-q)} = 1$ and $\frac{A}{4l_p^2} = 100000$. The solid line represents $\alpha = 1$, the dotted line, $\alpha = 1.5$ and the dashed line, $\alpha = 2$.

This last equation is the BH formula. Namely, in Newton's gravitational scenario we can have the black hole entropy. One can think that this result connects the thermodynamical BH formula for black holes to the classical Newton's expression for gravity, which could suggest a thermodynamical emergent gravitation. On the other hand, Botta Cantcheff and Nogales [37] have shown that we can derive the usual entropy of black holes by using a volume microstates scaling law (3) and Tsallis' nonextensive entropy (2).

For $q = 1$ (BG) scenario and any b we have that

$$F_{q=1} = \frac{GMm}{R^2}\alpha\frac{4\,l_p^2}{A}, \tag{16}$$

which of course is not Newton's second law.

From eq. (14) if we have that $\alpha = 1.5/(1-q)$ we can write

$$\begin{aligned} F &= \frac{GMm}{R^2}\frac{3}{2(1-q)}b^{1-q}\left(\frac{A}{4l_P^2}\right)^{1/2} \\ &= \frac{GMm}{R}\frac{3\sqrt{\pi}}{1-q}\frac{b^{1-q}}{2l_p}, \end{aligned} \tag{17}$$

where we have used that $A = 4\pi R^2$ and which is just the Newtonian force established by MOND's approach [18].

MOND's success is due mainly to its capacity to explain the majority of the galaxies rotations. It reproduces the well-known Tully-Fisher relation [38]. In this way, it can be an alternative to the dark matter model. However, it has problems to explain both the temperature profile of galaxy clusters and the confrontation with cosmology as well. In few words it is basically a modification of Newton's second law where the force can be written as

$$F = m\mu\left(\frac{a}{a_0}\right)a, \tag{18}$$

where a_0 is a constant that will be defined below and $\mu(x) \approx 1$ for $x \gg 1$ and $\mu(x) \approx x$ for $x \ll 1$. For simplicity, it is usual to suppose that the variation of $\mu(x)$

between the asymptotic limits happens abruptly at $x = 1$ or $a = a_0$.

From the rotational movement of the galaxies we have that the Tully-Fisher relation is given by $v^2 = \sqrt{GMa_0}$, where a_0, the MOND's constant (an acceleration scale) value is $a_0 \approx 1.2 \cdot 10^{-8}\,\mathrm{cm/s^2}$ and, substituting this velocity into eq. (17) we have that

$$\frac{v^2}{R} = \frac{GM}{R}\frac{3}{2}\frac{\sqrt{\pi}}{1-q}\,\frac{b^{1-q}}{l_p}, \tag{19}$$

hence

$$b = \left[\frac{2}{3}(1-q)l_p\sqrt{\frac{a_0}{\pi GM}}\,\right]^{\frac{1}{1-q}}, \tag{20}$$

which is a viable result for b. Let us analyze other consequences of this b-value, which reproduces MOND, as we saw above.

Substituting this last value for b into eq. (3), we have that

$$W = \left[\frac{2}{3}(1-q)l_P\right]^{\frac{1}{1-q}}\left(\frac{a_0}{\pi GM}\right)^{\frac{1}{2(1-q)}}\left(\frac{A}{4l_P^2}\right)^{\alpha}, \tag{21}$$

which clearly shows no divergence in the $M \to 0$ case when $q > 1$, which constrains q to be a very nonextensive system feature.

From eq. (13) we have the thermodynamical expression for the Newtonian force. So,

$$m\ddot{R} = m\ddot{a}r = \frac{GMm}{a^2r^2}\frac{4l_p^2}{k_B}\frac{\mathrm{d}S}{\mathrm{d}A}. \tag{22}$$

where R, the radius of the holographic screen, is the apparent horizon, *i.e.*, $R(t,r) = a(t)r$, and r is the radial comoving coordinate,

$$\Longrightarrow \ddot{a} = \frac{GM}{a^2r^3}\frac{4l_p^2}{k_B}\frac{\mathrm{d}S}{\mathrm{d}A}. \tag{23}$$

Based on [39], the acceleration in eq. (23) results from the active gravitational mass, which is the well-known Tolman-Komar (TK) mass [40,41] given by

$$M = (\rho + 3p)\frac{4\pi}{3}a^3r^3, \tag{24}$$

which is proportional to the scale function. Substituting eq. (24) into eq. (23) we have that

$$\frac{\ddot{a}}{a} = \frac{16\pi}{3}G(\rho + 3p)\frac{l_p^2}{k_B}\frac{\mathrm{d}S}{\mathrm{d}A}, \tag{25}$$

and by multiplying both sides of this last equation by $\dot{a}a$, we have that

$$\dot{a}\ddot{a} = \frac{16\pi}{3}\frac{\mathrm{d}}{\mathrm{d}t}G(\rho a)\frac{l_p^2}{k_B}\frac{\mathrm{d}S}{\mathrm{d}A}. \tag{26}$$

Hence, the integration of both sides is

$$H^2 + \frac{k}{a^2} = \frac{32\pi G}{3k_B}\frac{l_p^2}{a^2}\int \mathrm{d}(\rho a^2)\frac{\mathrm{d}S}{\mathrm{d}A}, \tag{27}$$

which is an entropic version of the Friedmann equation and where we have used the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{28}$$

So, to calculate the active mass, the TK mass, we have to solve the differential equation in (23), which can be written as

$$\begin{aligned}\ddot{a} &= \frac{GM}{a^2r^3}\frac{4l_p^2}{k_B}\frac{\mathrm{d}S}{\mathrm{d}A}\\ &= \frac{GM}{a^2r^3}\alpha b^{1-q}\left(\frac{A}{4l_p^2}\right)^{\alpha(1-q)-1},\end{aligned} \tag{29}$$

where we have used eq. (21). Hence, for $A = 4\pi R^2 = 4\pi a^2r^2$, we have that

$$\ddot{a} = GM\alpha b^{1-q}\left(\frac{\pi}{l_p^2}\right)^{\alpha(1-q)-1} a^{2[\alpha(1-q)-2]}r^{2\alpha(1-q)-5}, \tag{30}$$

and for $q = 1$ (the BG limit) we have that

$$\ddot{a} = \frac{GM\alpha l_p^2}{\pi a^4r^5}, \tag{31}$$

which is also a nonlinear differential equation that has a numerical solution for the scale factor. Numerical computation is out of the scope of this letter.

In this letter we have explored the role of the holographic screen under the point of view of Tsallis thermostatistics. We have seen through a generalized correction term of the entropy [34], which is connected to MOND's ideas, that the q-parameter can be fixed since viable cosmological issues were considered. We have found that the b-parameter used in [34] can be also fixed according to the physical scenario.

With these points in mind, we have shown that we can obtain the BH formula for black holes from entropic considerations of Newton's gravitational law. Besides, through a convenient choice for the α-parameter we have derived MOND's equation for the Newtonian force.

From eq. (13) we have derived a generalized Newton's law of gravitation in the context of the nonextensive statistical mechanics. From eq. (14) we can see that the BH's expression for the black hole entropy can be found also in Newton's gravitation law.

We have computed the galaxies rotation velocity. The result has showed us a possible value for the b-parameter and consequently the number of microstates, constraining q to be greater than 1, the very nonextensive case. Finally, we have demonstrated the entropic version of the

Friedmann equation. We have shown that the scale factor obeys a differential nonlinear equation for the scale factor with numerical solution, which can be a target for future investigations.

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