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LETTER

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Milling-induction and milling-destruction in a Vicsek-like binary-mixture model

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Abstract – Milling is a collective circular motion often observed in nature (*e.g.*, in fish schools) and in many theoretical models of collective motion. In these models particles are considered to be identical. However, this is not the case in nature, where even individuals of the same species differ from each other in one or more traits. In order to get insights into the mechanisms of milling formation in heterogeneous systems (*i.e.*, with non-identical particles), the emergence of milling in a binary mixture of particles that differ in one trait is investigated for the first time. Depending on parameter values, particles that in single-type systems do not mill can either be induced to mill or destroy the milling of other particles. Milling-induction and milling-destruction are studied varying the speed, the field of view, and the relative amount of the two types of particles.

Introduction. – Milling is a very fascinating and eye-catching collective motion pattern that has been often observed in nature, *e.g.*, in fish schools [1–3], and in models of collective motion [4–9]. It is a rotating circular formation, where individuals turn around a common centre. Its biological function is still unclear [10].

One of the most famous models of collective motion is the Vicsek model [11]. It is based on alignment interactions only, and shows a phase transition from disordered to ordered motion, but no collective circular motion. Recently, the Vicsek model has been modified limiting the field of view and the maximal angular velocity of particles, resulting in the first model based on only alignment interactions that shows spontaneous emergence of milling [4]. Other models of collective motion that have reproduced collective circular motion employ different interaction types among particles, like attraction and avoidance [5–9], or consider chiral particles [12,13].

However, all these models consider identical particles, which is not the case in nature, where intra-group differences are always present. Theoretical models of heterogeneous systems have mostly investigated the spontaneous or induced separation of the different types of particles [14–21], while the emergence of milling in binary mixtures has been investigated only for chiral particles [22]. It is an open question if (and under which conditions) particles that in single-type systems do not mill can be induced to mill by the interactions with other particle types. The aim of this letter is to investigate this in a minimal model of collective motion of binary mixtures based on only alignment interactions.

I study the collective motion of particles that differ in speed or in field of view, and find the existence of milling-induction and milling-destruction. The occurrence of these two phenomena is investigated as a function of the difference of speed, difference of field of view, and as a function of the proportion of the two particle types.

Model. – N point-particles move on a two-dimensional quadratic box of size L with periodic boundary conditions. Particles are characterised by their position $\mathbf{x}_i$ and their orientation, described by the angle $\theta_i \in [0°, 360°)$. Random positions and random orientations are used as starting configuration. Particles move at constant speed v in direction of their orientation θ_i and interact with other particles that are in their field of view ϕ (particles have a blind angle behind them, fig. 1) and that are closer than the interaction range r, which is chosen as unit of length ($r = 1$). The interaction consists of (partial) alignment, *i.e.*, a rotation (limited by the maximal angular velocity ω)

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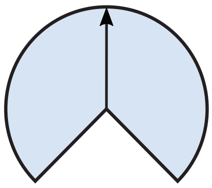

Fig. 1: Sketch of the field of view ϕ. Particles have a blind angle behind them. The black arrow represents the orientation of the particle.

towards the average orientation of neighbours. The time unit is the time interval between two updates of positions and orientations, $\Delta t = 1$. The equation of motion reads

$$\mathbf{x}_i(t+\Delta t) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t, \tag{1}$$

where $\mathbf{v}_i$ is the velocity vector with speed v and orientation θ_i. The orientation updating is

$$\theta_i(t+\Delta t) = \begin{cases} \langle\theta_j(t)\rangle_{r,\phi} + \Delta\theta_i, & \text{for } |\Delta\Theta_i| < \omega\Delta t, \\ \theta_i(t) + \omega\Delta t + \Delta\theta_i, & \text{for } \Delta\Theta_i \geq \omega\Delta t, \\ \theta_i(t) - \omega\Delta t + \Delta\theta_i, & \text{for } \Delta\Theta_i \leq -\omega\Delta t, \end{cases} \tag{2}$$

where $\langle\theta_j(t)\rangle_{r,\phi}$ denotes the average orientation of particles in the interaction range $r = 1$ and in the field of view ϕ (including particle i), ω is the maximal angular velocity, and $\Delta\Theta \in [-180°, 180°)$ is the difference in orientation between the current orientation and the average orientation of the interacting particles. For example, if the current orientation $\theta_i = 10°$ and the average orientation of the interacting particles $\langle\theta_j(t)\rangle_{r,\phi} = 350°$, then $\Delta\Theta = -20°$. $\Delta\theta$ represents the rotational noise, which is a random variable uniformly distributed in the interval $[-\eta/2, \eta/2]$. The motion of particles is updated synchronously, *i.e.*, first all average orientations $\langle\theta_j(t)\rangle_{r,\phi}$ are computed and then particles are moved. The free parameters of the model are the ratio of speed over maximal angular velocity $v/(r\omega)$ (made adimensional dividing by $r = 1$), the field of view ϕ, the particle density $\rho = N/L^2$, and the noise η.

Binary mixtures of particles differing in speed and in field of view are considered separately:

1) Difference in speed: all particles have the same field of view, while n_1 particles have speed v_1 and n_2 particles have speed v_2.

2) Difference in field of view: all particles have the same speed, while n_1 particles have field of view ϕ_1 and n_2 particles have field of view ϕ_2.

Measured quantities. – Milling is identified using two quantities: the absolute value of the average normalized velocity (the polar order parameter)

$$v_a = \frac{1}{N}\left|\sum_{i=1}^{N} \mathbf{u}_i\right| \tag{3}$$

($\mathbf{u}_i = \mathbf{v}_i/v$ is the velocity unit vector) which is one when all the particles move in the same direction and zero when particles move in random directions, and the average absolute value of the normalized angular momentum

$$m_a = \frac{1}{N}\sum_{i=1}^{N} \frac{|\mathbf{r}_{cm,i} \times \mathbf{u}_i|}{|\mathbf{r}_{cm,i}|}, \tag{4}$$

where $|\mathbf{r}_{cm,i}| = |\mathbf{r}_i - \mathbf{r}_{cm}|$ is the distance of particle i to the centre of mass of its cluster. A cluster is defined as a set of particles where the distance between particles is smaller than $d_c = 0.5$. The value of d_c has been arbitrarily chosen in order to optimise milling detection, being the radius of the mills always of the order of magnitude of the interaction range $r = 1$ (see appendix for details on the radius of mills). m_a ranges from zero to one, and is close to one when particles rotate in (multiple and counter-rotating) mills. However, in order to detect milling, m_a alone is not sufficient. For example, when particles form bands, $v_a \simeq 1.0$ and $m_a \simeq 0.9$ (due to the absolute value in eq. (4). The milling state is detected accurately by the condition $v_a < 0.5$ *and* $m_a > 0.7$ (fig. A2 of [4]).

For each parameter setting 100 simulations of 2000 time steps are run, and the described quantities are measured (separately for the two particle types) taking a time average over the last 500 steps, where the system is in the stationary state. Each run can result being either in the milling state or in the non-milling state. The milling proportion p_{mill} is computed as the number of runs where the system is in the milling state divided by the total number of runs. $N = 1000$ particles are used in a quadratic box of size $L = 20$, such that the particle density is $\rho = 2.5$. This box size is sufficiently large to avoid finite-size effects in the parameter regime explored in this work [4]. The maximal angular velocity is kept constant at $\omega = 10°/\Delta t$, and the ratio between speed and maximal angular velocity $v/(r\omega)$ is varied changing the speed in the interval $[0.05, 0.35]$. The ratio between noise and maximal angular velocity is kept constant at $\eta/(\omega\Delta t) = 0.5$.

Results: speed difference. – The behaviour of binary mixtures ($n_1 = n_2$) of particles differing only in their speed is here investigated. The milling proportion p_{mill} of the two types of particles is measured varying the speed of particles of type 1 from $v_1/(r\omega) = 0.11$ to $v_1/(r\omega) = 2$, while particles of type 2 have $v_2/(r\omega) = 0.8$, a value for which particles in single-type systems show a high milling proportion. The field of view of both particle types is kept at $\phi = 180°$.

At $v_1/(r\omega) = 0$, particles of type 1 are not moving and align with the circular motion of particles of type 2. The milling particles are not influenced by the presence of non-moving particles. For $0 < v_1/(r\omega) \leq 0.25$, particles of type 1 (slower particles) cannot follow the circular motion of particles of type 2. They form dense clusters close to mills of particles of type 1, which eventually destroy the milling of particles of type 2 (fig. 2(a) and supplementary videos `Movie_1.avi` and `Movie_2.avi`). Both particle

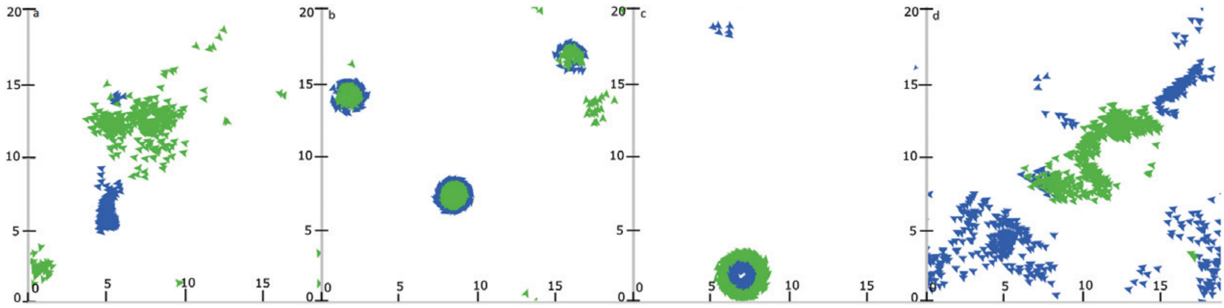

Fig. 2: Snapshots of the entire simulation box ($L = 20$): the speed of particles of type 1 (green) increases from left to right, the speed of particles of type 2 (blue) is constant ($v_2/(r\omega) = 1.0$). For both types of particles $\phi = 180°$, $\eta/(\omega\Delta t) = 0.5$, $\rho = 2.5$. (a) Milling-destruction effect, $v_1/(r\omega) = 0.1$. (b) Milling-induction effect, $v_1/(r\omega) = 0.4$. (c) Milling-induction effect, $v_1/(r\omega) = 1.5$. (d) Milling-destruction effect, $v_1/(r\omega) = 1.7$.

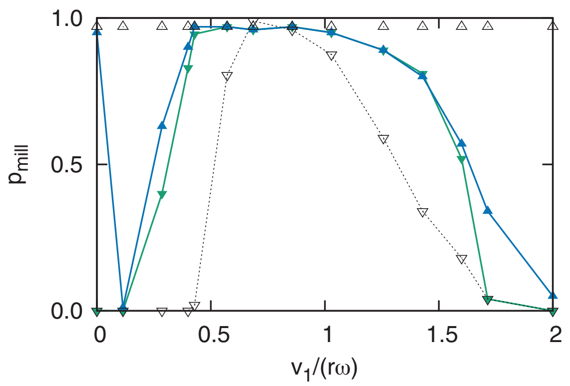

Fig. 3: Milling proportion p_{mill} (computed over 100 runs) as a function of the ratio between speed and maximal angular velocity of particles of type 1, $v_1/(r\omega)$. ($v_2/(r\omega) = 0.8$, $\phi = 180°$, $\eta/(\omega\Delta t) = 0.5$, $\rho = 2.5$). Empty down-triangles refer to a single-type system of particles of type 1: their milling proportion depends on their speed. Empty up-triangles refer to a single-type system of particles of type 2: their milling proportion does not depend on the speed of particles of type 1; the points are repeated at a constant value as a guide for the eye. Filled symbols refer to the binary mixture: filled down-triangles represent particles of type 1, filled up-triangles represent particles of type 2.

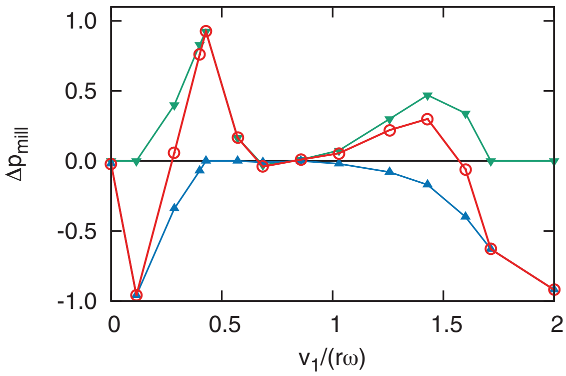

Fig. 4: Difference of milling proportion Δp_{mill} (computed over 100 runs) as a function of the ratio between speed and maximal angular velocity of particles of type 1, $v_1/(r\omega)$. ($v_2/(r\omega) = 0.8$, $\phi = 180°$, $\eta/(\omega\Delta t) = 0.5$, $\rho = 2.5$). Down-triangles: particles of type 1; up-triangles: particles of type 2; circles: sum of the two differences ($\Delta p_{mill,1} + \Delta p_{mill,2}$).

types have the same milling proportion $p_{mill} \simeq 0$, showing that the milling-destruction effect is acting on all particles of type 2 (fig. 3).

For $0.25 < v_1/(r\omega) < 0.4$ and for $1.0 < v_1/(r\omega) \leq 1.6$, the milling proportions of the two particle types differ from values of single-type systems, meaning that a competition between milling-destruction and milling-induction is taking place. When milling-induction prevails on milling-destruction, particles of type 1 follow the circular trajectory of particles of type 2, with faster particles being in the outer part of the mill (fig. 2(b) and (c)).

For $0.25 < v_1/(r\omega) < 0.4$ and for $v_1/(r\omega) > 1.5$, the milling proportions of the two particle types differ from each other, showing that they may perform different patterns of collective motion (fig. 3). On the other hand, for $0.4 \leq v_1/(r\omega) \leq 1.5$, the values of the milling proportion of the two particle types do not differ from each other, indicating that the two types of particles are displaying the same pattern of collective motion (mostly milling) (fig. 3). For $0.6 < v_1/(r\omega) \leq 1.0$ particles of type 1 do already mill in single-type systems and the milling persists also in the binary mixture (fig. 3).

For $1.6 < v_1/(r\omega) < 2.0$ particles of type 1 decrease the milling proportion of particles of type 2 (fig. 3). For $2.0 \leq v_1/(r\omega)$ particles of type 1 destroy completely the milling of particles of type 2, resulting in both particle types moving in aligned flocks (fig. 2(d) and fig. 3).

To measure the relative strength of the two competing effects, the difference of the milling proportion in the binary mixture with the milling proportion in the single-type system $\Delta p_{mill} = p_{mill,b} - p_{mill,s}$ is computed. The milling-induction effect is stronger than the milling-destruction effect for $0.3 \leq v_1/(r\omega) \leq 1.6$, with a high peak at $v_1/(r\omega) \simeq 0.5$. For $0 < v_1/(r\omega) < 0.3$ and $v_1/(r\omega) > 1.6$ the milling-destruction effect is stronger than the milling-induction effect (fig. 4).

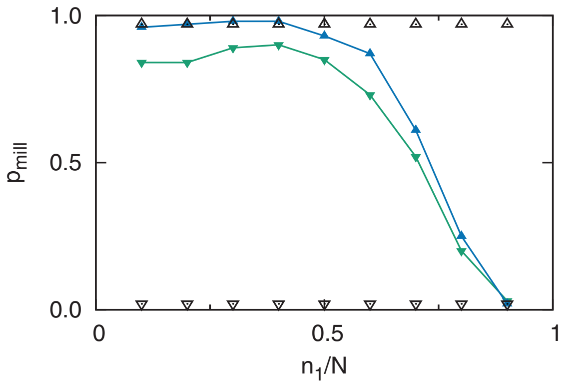

Fig. 5: Milling proportion p_{mill} (computed over 100 runs) as a function of the proportion of particles of type 1, n_1/N ($v_1/(r\omega) = 0.4$, $v_2/(r\omega) = 0.8$, $\phi = 180°$, $\eta/(\omega\Delta t) = 0.5$, $\rho = 2.5$). Empty down-triangles refer to a single-type system of particles of type 1: their milling proportion depends on their speed. Empty up-triangles refer to a single-type system of particles of type 2: their milling proportion does not depend on the speed of particles of type 1; the points are repeated at a constant value as a guide for the eye. Filled symbols refer to the binary mixture: filled down-triangles represent particles of type 1, filled up-triangles represent particles of type 2.

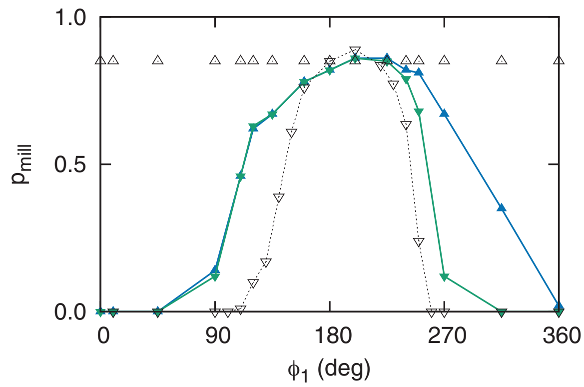

Fig. 6: Milling proportion p_{mill} (computed over 100 runs) as a function of the field of view of particles of type 1 ϕ_1 ($\phi_2 = 180°$, $v/(r\omega) = 1.0$, $\eta/(\omega\Delta t) = 0.5$, $\rho = 2.5$). Empty down-triangles refer to a single-type system of particles of type 1: their milling proportion depends on their speed. Empty up-triangles refer to a single-type system of particles of type 2: their milling proportion does not depend on the speed of particles of type 1; the points are repeated at a constant value as a guide for the eye. Filled symbols refer to the binary mixture: filled down-triangles represent particles of type 1, filled up-triangles represent particles of type 2.

In order to study how many particles of one type are necessary to induce milling in a second type of particles, simulations of binary mixtures with $n_1 \neq n_2$ are performed. Milling proportion is measured as a function of the relative amount of particles of type 1, n_1/N. Particles of type 1 have ratio of speed to maximal angular velocity $v_1/(r\omega) = 0.4$. For particles of type 2 $v_2/(r\omega) = 0.8$, such that, at $n_1/N = 0.5$, particles of type 2 induce milling to particles of type 1. For a larger proportion of particles of type 1 ($n_1/N > 0.5$), the milling proportion decreases linearly, reaching a value close to zero for $n_1/N = 0.9$ (fig. 5).

Results: difference in field of view. – The collective motion of binary mixtures ($n_1 = n_2$) of particles that differ only in their field of view is here studied. The milling proportion p_{mill} of the two particle types is measured varying the field of view of particles of type 1 from $\phi_1 = 0°$ to $\phi_1 = 360°$, and keeping the field of view of particles of type 2 at $\phi_2 = 180°$, a value for which particles in single-type systems show a high milling proportion. The ratio of speed to maximal angular velocity of both particle types is kept at $v/(r\omega) = 1.0$.

For $\phi_1 \leq 45°$ and $\phi_1 = 360°$ particles of type 1 destroy completely the milling of particles of type 2 (fig. 6). For $45° < \phi_1 < 170°$ the two particle types have similar milling proportion, which differs from the milling proportions in single-type systems, showing that milling-induction and milling-destruction are competing and that the two particle types display the same pattern of collective motion (fig. 6). On the other hand, for $240° \leq \phi_1 < 360°$, the milling proportions of the two particle types differ from each other, and differ from the values of single-type systems, indicating both a competition between milling-induction and milling-destruction and a possible difference in collective motion patterns performed by the two particle types (fig. 6). In the interval $170° \leq \phi_1 < 240°$ both particle types already mill in single-type systems ($p_{mill} > 0.75$) and the milling proportion values of the binary mixture remain high and close to each other.

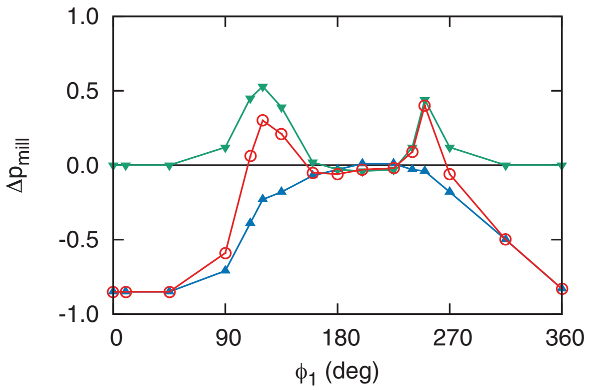

Fig. 7: Difference in milling proportion Δp_{mill} (computed over 100 runs) as a function of the field of view of particles of type 1 ϕ_1 ($\phi_2 = 180°$, $v/(r\omega) = 1.0$, $\eta/(\omega\Delta t) = 0.5$), $\rho = 2.5$). Down-triangles: particles of type 1; up-triangles: particles of type 2; circles: sum of the two differences ($\Delta p_{mill,1} + \Delta p_{mill,2}$).

The difference of the milling proportion in binary mixtures with the milling proportion in single-type systems ($\Delta p_{mill} = p_{mill,b} - p_{mill,s}$) shows that milling-induction is stronger than milling-destruction for $120° \leq \phi_1 < 160°$ and for $240° \leq \phi_1 < 270°$ with two peaks at $\phi_1 \simeq 120°$

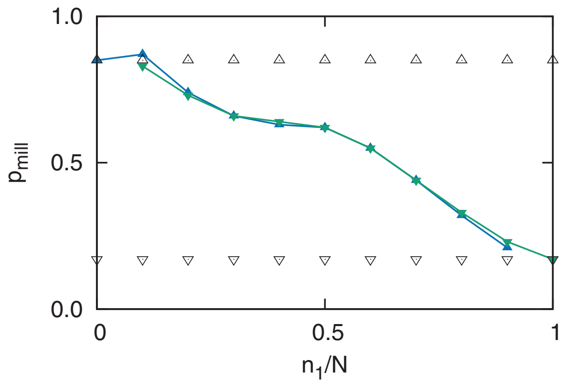

Fig. 8: Milling proportion p_{mill} (computed over 100 runs) as a function of the proportion of particles of type 1, n_1/N ($v/(r\omega) = 1.0$, $\phi_1 = 135°$, $\phi_2 = 180°$, $\eta/(\omega\Delta t) = 0.5$, $\rho = 2.5$). Empty down-triangles refer to a single-type system of particles of type 1: their milling proportion depends on their speed. Empty up-triangles refer to a single-type system of particles of type 2: their milling proportion does not depend on the speed of particles of type 1; the points are repeated at a constant value as a guide for the eye. Filled symbols refer to the binary mixture: filled down-triangles represent particles of type 1, filled up-triangles represent particles of type 2.

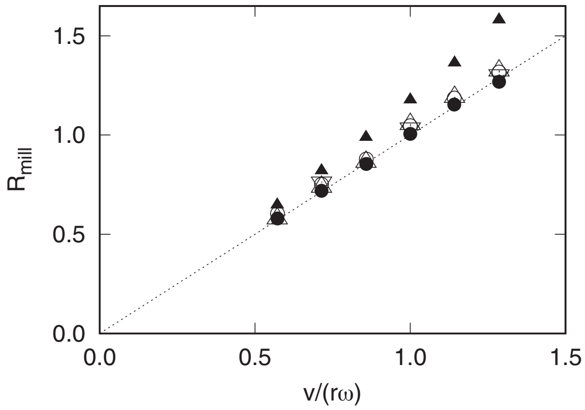

Fig. 9: Radius of mills R_{mill} (computed over 3 runs) as a function of ratio between speed and maximal angular velocity $v/(r\omega)$. Field of view $\phi = 180°$, density $\rho = 2$, noise $\eta/(\omega\Delta t) = 0$ (filled circles), $\eta/(\omega\Delta t) = 1$ (empty circles), $\eta/(\omega\Delta t) = 2$ (filled up triangles). Field of view $\phi = 220°$, density $\rho = 2$, noise $\eta/(\omega\Delta t) = 1$ (empty up triangles). Field of view $\phi = 180°$, density $\rho = 4$, noise $\eta/(\omega\Delta t) = 1$ (empty down triangles). The dashed line is a guide to the eye and represents $R_{mill} = v/(r\omega)$.

and $\phi_1 \simeq 250°$ (fig. 7). On the other hand, for $\phi_1 \leq 90°$ and $\phi_1 > 270°$ milling-induction is weaker than milling-destruction. For $160° \leq \phi_1 < 240°$ the milling proportion of both particle types is already high in single-type systems, such that neither milling-induction nor milling-destruction are taking place (fig. 7).

To study the relative amount of particles that are necessary to induce milling in a second particle type, milling proportion is measured in binary mixtures with $n_1 \neq n_2$ as a function of the relative amount of particles of type 1, n_1/N. Particles of type 1 have field of view $\phi_1 = 135°$, while particles of type 2 have field of view $\phi_2 = 180°$, resulting in particles of type 2 inducing milling to particles of type 1. For every n_1/N, both particle types have the same milling proportion, indicating that both types of particles are performing the same pattern of collective motion. The milling proportion decreases monotonously with relative amount of particles of type 1 (fig. 8).

Conclusions. – A minimal model of collective motion based on only alignment interactions has been used to numerically investigate milling (*i.e.*, collective circular motion) in binary mixtures of self-propelled particles. The existence of milling-induction and milling-destruction effects has been shown for the first time. Particles that do not mill in single-type systems, can either be induced to mill by a second particle type or destroy the milling of the second particle type, depending on parameter values. The emergence of these effects has been investigated as a function of speed, field of view, and the relative amount of the two particle types.

Crucial ingredient for the emergence of mills ($p_{mill} > 0$) in mixtures of self-propelled particles is that parameters of non-milling particles have to be close enough ($\Delta v/(r\omega) \leq 0.25$ and $\Delta\phi \leq 45°$) to values for which milling can emerge ($p_{mill} > 0$) in single-type systems, as shown by the two peaks in figs. 4 and 7. An exception is the case of a mixture of moving and non-moving particles, for which mills of moving particles are not affected by non-moving particles. Moreover, milling-induction increases with the relative amount of inducing-particles, and also a relative low amount of inducing-particles ($n/N = 0.2$) can induce milling ($p_{mill} \simeq 0.25$).

Open questions are if (and how) the reported findings on binary mixtures apply to other models of collective motion, and if (and how) they extend to three-dimensional systems. Since the emergence of milling in single-type systems has been observed also in models of collective motion based on attraction and avoidance, the presented phenomena of milling-induction and milling-destruction may be general phenomena that do not depend on model details.

The presented findings are of theoretical interest and practical relevance, since they give new insights into the mechanisms underlying the spontaneous emergence of circular motion in heterogeneous systems of self-propelled particles, and may help to understand the mechanisms of milling formation also in animal groups.

* * *

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Appendix: size of mills. – In single-type systems, starting from a random configuration, mills emerge if particles have a blind angle in their back ($\phi \simeq 180°$), if their speed and their maximal angular velocity are small enough ($v\Delta t/r \ll 1$ and $v/(r\omega) \simeq 1$), and if the ratio of speed to maximal angular velocity is such that the equation describing circular motion ($v = \omega R$) is satisfied [4]. Every size of mills could be stable, provided that the speed of particles increases with the mill's radius (*i.e.*, satisfying $v = \omega R$ at constant angular velocity ω). However, in the process of milling spontaneously emerging from a random starting configuration, the size of mills is mainly determined by the interaction range r and to a minor extent by the ratio of speed to maximal angular velocity v/ω, the two physical lengths of the system, resulting in both mills' radius and interaction range being of the same order of magnitude (fig. 9). Increasing the speed too much (keeping v/ω constant) will lead to complete mixing and no milling. The radius of mills increases with noise and does not depend on the particle density or field of view (fig. 9). For low noise values ($\eta/(\omega\Delta t) \leq 1$), the radius of mills satisfies the equation $R_{mill} = v/(r\omega)$ (dashed line of fig. 9). Therefore, in binary mixtures (of particles with different speeds) faster particles are on the outer part of the mill (fig. 2(b) and (c)).

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