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Current noise spectrum in a solvable model of tunneling Fermi-edge singularity

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Abstract – We consider tunneling of spinless electrons from a single-channel emitter into an empty collector through an interacting resonant level of the quantum dot (QD). When all Coulomb screening of sudden charge variations of the dot during the tunneling is realized by the emitter channel, the system is mapped onto an exactly solvable model of a dissipative qubit. The qubit density matrix evolution is described with a generalized Bloch equation which permits us to count the tunneling electrons and find the charge transfer statistics. The two generating functions of the counting statistics of the charge transferred during the QD evolutions from its stationary and empty state have been expressed through each other. It is used to calculate the spectrum of the steady current noise and to demonstrate the occurrence of the bifurcation of its single zero-frequency minimum into two finite-frequency dips due to the qubit coherent dynamics.

The Fermi-edge singularity (FES) resulting [1,2] from the reconstruction of the Fermi sea of conduction electrons under a sudden change of a local potential has been primarily observed [3,4] as a power-law singularity in X-ray absorption spectra. A similar phenomenon of the FES in the transport of spinless electrons through a quantum dot (QD) was predicted [5] in the perturbative regime when a localized QD level is below the Fermi level of the emitter in its proximity, the collector is effectively empty (or in an equivalent formulation through the particle-hole symmetry) and the tunneling rate of the emitter is sufficiently small. Then, the subsequent separated in time electron tunnelings from the emitter vary the localized level charge and generate sudden changes of the scattering potential leading to the FES in the *I-V* curves at the voltage threshold corresponding to the resonance. In reality the threshold singularity is smeared out due to a finite lifetime of the electrons in the localized state, which brings forth the level broadening [5], and it is also smeared by the temperature [6]. A further complication for the direct observation of the perturbative results in experiments [6–11] is caused by poorly controlled variations of the tunneling parameters by the applied bias voltage. Therefore, it has been suggested [12] that the true FES nature of a threshold peak in the *I-V* dependence can be verified through observation of the oscillatory behavior of a corresponding time-dependent transient current. Indeed, in the FES theory [1,2] the appearance of such a threshold peak signals the formation of a two-level system of the exciton electron-hole pair or qubit in the tunneling channel at the QD. The qubit undergoes dissipative dynamics characterized [13, 14], in the absence of the collector tunneling, by the oscillations of the levels occupation. By studying an exactly solvable model of the FES we earlier demonstrated that for a wide range of the model parameters the qubit dynamics also manifest themselves through the resonant features of the *a.c.* response [15] and in the oscillating behavior [12] of the collector transient current, in particular, when the QD evolves from its empty state. Although a possible observation of these oscillations would give the most direct verification of the nature of the *I-V* threshold peaks, in the recent experiments [9,16] the low-temperature noise measurements have been considered for this purpose.

For this reason, in this work we study quantum fluctuations of the steady tunneling current into the collector in the exactly solvable model of the FES. This model describes a simplified, but still realistic system of spinless electrons tunneling from a single channel emitter into an empty collector through an interacting resonant level of QD, when all Coulomb screening of sudden variations of the charge in the QD is realized by the emitter channel. In order to see traces of the qubit dynamics in the shot noise spectrum of the current we apply methods of the full counting statistics by describing the qubit density matrix evolution with a generalized Bloch equation, which permits us to count the tunneling electrons and find the charge transfer statistics. The two generating functions of statistics of the charge transferred during the QD evolution from its stationary and empty states are expressed through each other. This relation is further used to establish the direct connection between the spectrum of the steady current shot noise and Fourier transformation of the time-dependent transient current produced in the process of the QD empty state evolution.

The final expression for the current noise spectrum is analyzed to conclude how the spectral features reflect the oscillating behavior of the time-dependent transient current. In particular, we show that its oscillating behavior in the wide range of the model parameters results in the bifurcation of the spectrum single zero-frequency minimum into two finite-frequency dips, which presents a fingerprint of the true FES nature of a threshold peak in the *I*-*V* dependence.

Model. – The system we consider below is described by the Hamiltonian $\mathcal{H} = \mathcal{H}_{res} + \mathcal{H}_C$ consisting of the one-particle Hamiltonian of the resonant tunneling of spinless electrons and the Coulomb interaction between instant charge variations of the dot and electrons in the emitter. The resonant tunneling Hamiltonian takes the following form:

$$\mathcal{H}_{res} = -\epsilon_d d^+ d + \sum_{a=e,c} \mathcal{H}_0[\psi_a] + w_a(d^+\psi_a(0) + \text{h.c.}), \quad (1)$$

where the first term represents the resonant level of the dot, whose energy is $-\epsilon_d$. Electrons in the emitter (collector) are described with the chiral Fermi fields $\psi_a(x), a = e(c)$, whose dynamics is governed by the Hamiltonian $\mathcal{H}_0[\psi] == -i\int \mathrm{d}x\psi^+(x)\partial_x\psi(x)(\hbar = 1)$ with the Fermi level equal to zero or drawn to $-\infty$, respectively, and w_a are the corresponding tunneling amplitudes. The Coulomb interaction in the Hamiltonian $\mathcal{H}$ is introduced as

$$\mathcal{H}_C = U_C\psi_e^+(0)\psi_e(0)(d^+d - 1/2). \quad (2)$$

Its strength parameter U_C defines the scattering phase variation δ for electrons in the emitter channel and, therefore, the change of the localized charge in the emitter $\Delta n = \delta/\pi$ ($e = 1$), which we assume provides the perfect screening of the QD charge: $\Delta n = -1$.

After implementation of the bosonization of the emitter Fermi field $\psi_e(x) = \sqrt{\frac{D}{2\pi}}\eta e^{i\phi(x)}$, where η denotes an auxiliary Majorana fermion, D is the large Fermi energy of the emitter, and the chiral Bose field $\phi(x)$ satisfies $[\partial_x\phi(x), \phi(y)] = i2\pi\delta(x-y)$, and further completion of a standard rotation [17], under the above screening assumption we have transformed [12] $\mathcal{H}$ into the Hamiltonian of the dissipative two-level system or qubit:

$$\begin{aligned}\mathcal{H}_Q =& -\epsilon_d d^+ d + \mathcal{H}_0[\psi_c] + w_c(\psi_c^+(0)e^{i\phi(0)}d + \text{h.c.}) \\ &+ \Delta\eta(d - d^+),\end{aligned} \quad (3)$$

where $\Delta = \sqrt{\frac{D}{2\pi}}w_e$ and the time-dependent correlator of the electrons in the empty collector $\langle\psi_c(t)\psi_c^+(0)\rangle = \delta(t)$ is further used to cancel contributions from the bosonic exponents of the third term on the right-hand side in (3).

Charge counting Bloch equation for the qubit evolution. – We use this Hamiltonian to describe the dissipative evolution of the qubit density matrix $\rho_{a,b}(t)$, where $a, b = 0, 1$ denote the empty and filled levels, respectively. In the absence of the tunneling into the collector at $w_c = 0$, $\mathcal{H}_Q$ in eq. (3) transforms through the substitutions of $\eta(d - d^+) = \sigma_1$ and $d^+d = (1 - \sigma_3)/2$ ($\sigma_{1,3}$ are the corresponding Pauli matrices) into the Hamiltonian $\mathcal{H}_S$ of spin 1/2 rotating in the magnetic field $\mathbf{h} = (2\Delta, 0, \epsilon_d)^T$ with the frequency $\omega_0 = \sqrt{4\Delta^2 + \epsilon_d^2}$. Then the evolution equation follows from

$$\partial_t\rho(t) = i[\rho(t), \mathcal{H}_S]. \quad (4)$$

To incorporate in it the dissipation effect due to tunneling into the empty collector we apply the diagrammatic perturbative expansion of the S-matrix defined by the Hamiltonian (3) in the tunneling amplitudes $w_{e,c}$ in the Keldysh technique [18]. This permits us to integrate out the collector Fermi field in the following way. At an arbitrary time t each diagram ascribes indexes $a(t_+)$ and $b(t_-)$ of the qubit states to the upper and lower branches of the time-loop Keldysh contour. This corresponds to the qubit state characterized by the $\rho_{a,b}(t)$ element of the density matrix. The expansion in w_e produces two-leg vertices in each line, which change the line index into the opposite one. Their effect on the density matrix evolution has been already included in eq. (4). In addition, each line with index 1 acquires two-leg diagonal vertices produced by the electronic correlators $\langle\psi_c(t_\alpha)\psi_c^+(t'_\alpha)\rangle$, $\alpha = \pm$, which enforce $t_\alpha = t'_\alpha$ and hence the mutual cancellation of the corresponding bosonic exponents of the Hamiltonian (3). These vertices result in the additional contributions to the density matrix variation: $\Delta\partial_t\rho_{10}(t) = -\Gamma\rho_{10}(t)$, $\Delta\partial_t\rho_{01}(t) = -\Gamma\rho_{01}(t)$, $\Delta\partial_t\rho_{11}(t) = -2\Gamma\rho_{11}(t)$, $\Gamma = w_c^2/2$. Next, to count the electron tunnelings into the collector we ascribe [19] the opposite phases to the collector tunneling amplitude $w_c\exp\{\pm i\chi/2\}$ along the upper and lower Keldysh contour branch, respectively. These phases do not affect the above contributions, which do not mix the amplitudes of the different branches. Then there are also vertical fermion lines from the upper branch to the lower one due to the non-vanishing correlator $\langle\psi_c(t_-)\psi_c^+(t'_+)\rangle$, which lead to the variation affected by the phase difference as follows: $\Delta\partial_t\rho_{00}(t) = 2\Gamma w\rho_{11}(t)$, $w = \exp\{i\chi\}$. Note that the corresponding bosonic exponents at time $t_- = t'_+$, though separated in time on the Keldysh contour, can be safely dropped out since the exponents $e^{\pm i\phi(t)}$ between them always appear in pairs. Incorporating these additional terms into eq. (4) we come to the modified quantum master equation

$$\begin{aligned}\partial_t \rho(t,w) \,=\, & i[\rho, \mathcal{H}_S] - \Gamma|1\rangle\langle 1|\rho - \Gamma\rho|1\rangle\langle 1| \\ & + 2w\Gamma|0\rangle\langle 1|\rho|1\rangle\langle 0|. \end{aligned} \tag{5}$$

for the qubit density matrix evolution and also counting the charge transfer. Here the vectors $|0\rangle = (1,0)^T$ and $|1\rangle = (0.1)^T$ describe the empty and filled QD, respectively. This equation is of a type known in the theory of open quantum systems as the GKSL equations [20,21]. It is exact in our model with the Hamiltonian (3) that takes into account the many-body interaction of the QD with the emitter Fermi sea. Contrary to the general case $\delta \neq -\pi$, the Lindbladian evolution defined by the ordinary differential equation (5) does not have quantum memory. The physical reason for this behavior originates from combination of the instant tunneling of electrons into the empty collector and the perfect screening by the emitter of the QD charge variations, which leave no traces in the Fermi sea after each electron jump. Therefore, the evolution of the system obeys the Born-Markov description [22], which permits to introduce the full counting statistics (FCS) by different methods (see, for example, [23,24]) with the same final result.

Solving eq. (5) with some initial $\rho(0)$ independent of w at $t = 0$, we find the generating function $P(w,t)$ of the FCS of the charge transfer by calculating the trace of the density matrix: $P(w,t) = Tr[\rho(w,t)] = \sum_{n=0}^{\infty} P_n(t) w^n$.

Making use of the four-component Bloch vector $\mathbf{a}(t,w)$ we represent the trace non-conserving density matrix as $\rho(t,w) = [a_0(t,w) + \sum_l a_l(t)\sigma_l]/2$, where the additional component $a_0 = P(w,t)$ evolves from its initial value $a_0(0) = 1$ and stays equal to one at $w = 1$, but as a function of w it gives us the generating function of charge transfer during the process time t. The substitution of this density matrix representation into eq. (5) results in the following evolution equation for the Bloch vector $\mathbf{a}(t,w)$:

$$\partial_t \mathbf{a}(t,w) = M(w) \cdot \mathbf{a}(t,w), \tag{6}$$

where $M(w)$ stands for the matrix

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\Gamma & -\epsilon_D & 0 \\ 0 & \epsilon_D & -\Gamma & -2\Delta \\ 2\Gamma & 0 & 2\Delta & -2\Gamma \end{pmatrix} + (w-1)\Gamma|\mathbf{e}_E\rangle\langle\mathbf{e}_F| \tag{7}$$

and the ket and bra vectors $|\mathbf{e}_E\rangle = (1,0,0,1)^T$, $\langle\mathbf{e}_F| = (1,0,0,-1)$ define the empty and filled QD state, respectively.

The general solution to eq. (6) describing the evolution of the Bloch vector starting from its value $\mathbf{a}(0)$ independent of w at zero time can be found through the Laplace transformation in the following form:

$$\mathbf{a}(t,w) = \int_C \frac{\mathrm{d}z e^{zt}}{2\pi i}[z - M(w)]^{-1}\mathbf{a}(0), \tag{8}$$

where the integration contour C coincides with the imaginary axis shifting to the right far enough to have all poles of the integral on its left side. Writing the inverse matrix in the standard form $[z - M(w)]^{-1} = [z - M(w)]_A / \det[z - M(w)]$, where $[z - M(w)]_A$ denotes the corresponding matrix of the algebraic complements, we conclude that these poles are equal to four roots of its determinant $\det[z - M(w)] \equiv p_4(z+\Gamma)$, which is

$$p_4(x) = x^4 + (4\Delta^2 + \epsilon_d^2 - \Gamma^2)x^2 - 4\Delta^2\Gamma w x - \Gamma^2\epsilon_d^2. \tag{9}$$

Then the general form of the generating function follows from (8) as

$$P(t,w) = \int_C \frac{\mathrm{d}z e^{zt}}{2\pi i}\frac{g_a(z+\Gamma,w)}{p_4(z+\Gamma)}, \tag{10}$$

where $g_a(z+\Gamma,w) \equiv \langle\mathbf{e0}|[z - M(w)]_A|\mathbf{a}(0)\rangle$ contrary to $p_4(x)$ depends also on the initial Bloch vector and $\langle\mathbf{e0}| = (1,0,0,0)$. We are interested in considering the process starting from the stationary Bloch vector $\mathbf{a}(0) = \mathbf{a}^{st}$ defined by $M(1)\mathbf{a}^{st} = 0$. As we show below, this process, in fact, is determined by the Bloch vector evolution starting from the empty QD.

Solving $M(1)\mathbf{a}^{st} = 0$ with $M(1)$ from eq. (7) and $a_0^{st} = 1$ we find the stationary Bloch vector $\mathbf{a}^{st} = [1, \mathbf{a}_\infty^T]^T$, where

$$\mathbf{a}_\infty = \frac{[2\epsilon_d\Delta, -2\Delta\Gamma, (\epsilon_d^2 + \Gamma^2)]^T}{(\epsilon_d^2 + \Gamma^2 + 2\Delta^2)}. \tag{11}$$

In general, an instant tunneling current $I(t)$ into the empty collector directly measures the diagonal matrix element of the qubit density matrix [25] through their relation

$$I(t) = 2\Gamma\rho_{11}(t,1) = \Gamma[1 - a_3(t,1)]. \tag{12}$$

It gives us the stationary tunneling current as $I_0 = 2\Gamma\Delta^2/(2\Delta^2 + \Gamma^2 + \epsilon_d^2)$. Since in our model ϵ_d is equal to the bias voltage applied to the emitter, the current $I_0(\epsilon_d)$ specifies a symmetric threshold peak in the *I-V* dependence smeared by the finite tunneling rates and exhibiting the power decrease as ϵ_d^{-2} far from the threshold. At $\Gamma \gg \Delta$ this expression coincides with the perturbative results of [5,11] and shows the considerable growth of the maximum current $I_0(0) = w_e^2(D/\pi\Gamma)$ due to the Coulomb interaction.

The substitution of $M(w)$ from eq. (7) into the denominator of the integrand on the right side of eq. (8) and further its expansion in $(w-1)$ bring up the following expression for the Bloch vector evolution:

$$\begin{aligned}\mathbf{a}(t,w) \,=\, & \int_C \frac{\mathrm{d}z}{2\pi i}\frac{e^{zt}}{[z - M(1)]} \\ & \times \sum_{n=0}\Big[(w-1)\Gamma|\mathbf{e}_E\rangle\langle\mathbf{e}_F|\,(z - M(1))^{-1}\Big]^n \mathbf{a}(0). \end{aligned} \tag{13}$$

For the initial vector $\mathbf{a}(0) = \mathbf{a}^{st}$ this expression transforms into

$$\mathbf{a}^{st}(t,w) = \mathbf{a}^{st}(0) + (w-1)I_0 \int_C \frac{\mathrm{d}z}{2\pi i z} \frac{e^{zt}}{[z - M(1)]} \mathbf{e}_E \times \sum_{n=0} \left(\left\langle \mathbf{e}_F \left| \frac{(w-1)\Gamma}{z - M(1)} \right| \mathbf{e}_E \right\rangle \right)^n \quad (14)$$

due to the properties of the stationary Bloch vector discussed above. On the other hand, for the evolution from the empty QD and the choice $\mathbf{a}(0) = \mathbf{e}_E$ eq. (13) can be rewritten as

$$\mathbf{a}^E(t,w) = \int_C \frac{\mathrm{d}z}{2\pi i} \frac{e^{zt}}{[z - M(1)]} \mathbf{e}_E \times \sum_{n=0} \left(\left\langle \mathbf{e}_F \left| \frac{(w-1)\Gamma}{z - M(1)} \right| \mathbf{e}_E \right\rangle \right)^n. \quad (15)$$

From comparison of these two expressions we find the relation between the two Bloch vectors:

$$\mathbf{a}^{st}(t,w) = \mathbf{a}^{st}(0) + (w-1)I_0 \int_0^t \mathrm{d}\tau \mathbf{a}^E(\tau, w). \quad (16)$$

The relation (16) between the zero components of the Bloch vectors shows that the generating function $P^{st}(t,w)$ of the charge transfer statistics in the process starting with QD in the stationary state can be found from the generating function for the process starting from the empty QD. By differentiating it with respect to time one can rewrite this relation as

$$\partial_t P^{st}(t,w) = (w-1)I_0 P(t,w)\theta(t), \quad (17)$$

where the Heavyside step function $\theta(t)$ starts counting the charge transfer at $t = 0$. It is straightforward to see from eq. (17) that in the steady process $\langle I \rangle_{st} = \partial_w \partial_t P^{st}(t,1) = I_0$ and similarly one can relate higher current correlators. Therefore, it suffices below to focus our study on the generating function $P(t,w)$ for the process starting from the empty QD. In this case $g_E(z+\Gamma, w) \equiv \langle \mathbf{e0} | [z - M(w)]_A | \mathbf{e}_E \rangle$ is calculated as

$$g_E(x) = x^3 + \Gamma x^2 + (4\Delta^2 + \epsilon_d^2)x + \Gamma \epsilon_d^2, \quad (18)$$

which does not depend on w.

Spectrum of the current noise. – In order to calculate the current noise spectrum $S(\omega)$ defined as the Fourier transformation of the real part of the current-current correlator

$$S(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t} \langle \{I(t), I(0)\}_+ \rangle = \mathrm{Re}\left[\int_0^{\infty} \mathrm{d}t e^{i\omega t} \langle \{I(t), I(0)\}_+ \rangle \right] \quad (19)$$

we need to express the time-dependent correlator $\langle \{I(t), I(0)\}_+ \rangle = Sp(\{I(t), I(0)\}_+ \rho_{st}(0))$ in terms of the generating function $P(t,w)$. This can be done through the following relation:

$$\partial_t \langle N^2(t) \rangle_{st} = \int_0^t \mathrm{d}t' \langle \{I(t'), I(0)\}_+ \rangle. \quad (20)$$

Since its left-hand side is equal to $\partial_t (w\partial_w)^2 P^{st}(t,w)$ at $w = 1$, we obtain the expression in question making use of eq. (17) in the following form:

$$\langle \{I(t), I_0\}_+ \rangle = I_0 \delta(t) + 2I_0 \partial_w \partial_t P(t,w)|_{w=1}. \quad (21)$$

The derivative of the generating function on the right-hand side of eq. (21) coincides with the transient current $\langle I(t) \rangle_E$ in the tunneling process, which starts from the empty QD state. Its oscillating behavior has been suggested [12] as an observable manifestation of the qubit dynamics at the FES. Therefore, the current noise spectrum relates to the spectral decomposition of this transient current as follows:

$$S(\omega) = I_0 + 2I_0 \int_0^{\infty} \mathrm{d}t \cos(\omega t) \langle I(t) \rangle_E \quad (22)$$

and should reflect its oscillatory features.

Substituting here the $P(t,w)$ derivative expression through the inverse Laplace transformation in eq. (10) and taking the time integral and then the contour integral after closing the contour in the right half-plane we come to

$$S(\omega) = I_0 + 2I_0 \mathrm{Re}[-i\omega g_E(\Gamma - i\omega) \partial_w p_4^{-1}(\Gamma - i\omega, w)]|_{w=1}, \quad (23)$$

where the functions p_4 and g_E are specified in eqs. (9), (18). Making use of their explicit expressions we calculate the right-hand side in eq. (23) and write the final result in the normalized form $S(\omega)/I_0 \equiv F_2(\omega)$ of the frequency-dependent Fano factor,

$$F_2(\omega) = 1 - \frac{8\Gamma^2\Delta^2(3(\Gamma^2 + \omega^2) - \epsilon_d^2)}{4\Gamma^2(\Gamma^2 + \epsilon_d^2 + 2\Delta^2 - 2\omega^2)^2 + \omega^2(\omega^2 - \epsilon_d^2 - 5\Gamma^2 - 4\Delta^2)^2}. \quad (24)$$

Its zero-frequency limit reduces to

$$F_2(0) = 1 + \frac{2\Delta^2(\epsilon_d^2 - 3\Gamma^2)}{(\epsilon_d^2 + \Gamma^2 + 2\Delta^2)^2} \quad (25)$$

and shows in fig. 1 the clear border given by $\epsilon_d^2 = 3\Gamma^2$ between the sub-Poissonian distributions of the current fluctuations near the resonance at $\epsilon_d = 0$ and the super-Poissonian ones far from it. As follows from (22) the difference between the two regimes of the current fluctuations is defined by the extra charge accumulated in the collector during the transient process, which is positive in e units in the case of the super-Poissonian and negative otherwise. The Fano factor $F_2(0)$ takes its smallest values at the resonance, where it reaches its minimum $F_2 = 1/4$ at

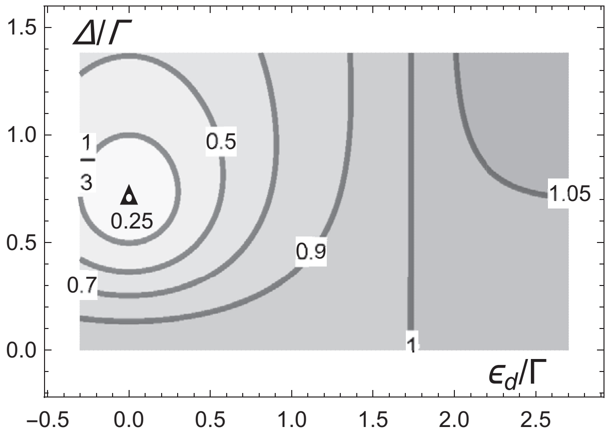

Fig. 1: Contour plot of the Fano factor $F_2(0)$ in eq. (25) as a function of the ϵ_d/Γ and Δ/Γ. The white point in the black triangle corresponds to the absolute minimum of $F_2(0)$.

$\Delta = \Gamma/\sqrt{2}$. The frequency dependence of $F_2(\omega)$ at the resonance follows from eq. (24) as

$$F_2 = 1 - \frac{24\Delta^2\Gamma^2}{4\Gamma^4 + (16\Delta^2 + 5\omega^2)\Gamma^2 + (\omega^2 - 4\Delta^2)^2} \qquad (26)$$

and is depicted in the upper panel of fig. 2. Its single zero-frequency minimum at small Δ splits with the increase of Δ into two minima located at the finite frequencies $\pm\omega_M$, $\omega_M = 2\sqrt{\Delta^2 - 5\Gamma^2/8}$, when $\Delta \geq \sqrt{5/8}\Gamma \approx 0.79\Gamma$, which are equal to

$$\min_{\omega} F_2(\omega) = \frac{12\Delta^2 - 9\Gamma^2/4}{36\Delta^2 - 9\Gamma^2/4} \leq \frac{1}{3}. \qquad (27)$$

This two-dips split in the frequency dependence of $F_2(\omega)$ signals the occurrence of the oscillations in the time-dependent transient current [12], but with the higher frequency [15] $\omega_I = 2\sqrt{\Delta^2 - \Gamma^2/16}$ than ω_M.

Moving ϵ_d out of the resonance one finds the increase of ω_M and that more minimum positions at smaller Δ split and shift away from the zero frequency as illustrated by the medium and lower panels of fig. 2 and the main part of fig. 3. The minimization of the right-hand side of eq. (24) defines the parametric region of $\omega_M = 0$ with the inequality

$$F(\epsilon_d, \Delta, \Gamma) \equiv \epsilon_d^6 + \left(51\Gamma^4 + 32\Gamma^2\Delta^2 + 16\Delta^4\right)\epsilon_d^2 + \left(3\Gamma^2 + 8\Delta^2\right)\epsilon_d^4 + 24\Gamma^4\Delta^2 - 15\Gamma^6 \leq 0. \qquad (28)$$

It is depicted in the inset in fig. 3 as the grey area that covers the black one corresponding [12] to the non-oscillating transient current behavior. The equality in eq. (28) defines the grey-area boundary serving as a bifurcation line, on crossing of which outward the single zero-frequency minimum of $F_2(\omega)$ splits into the two dips. These dips are located at $\pm\omega_M$, where ω_M^2 coincides with the real positive

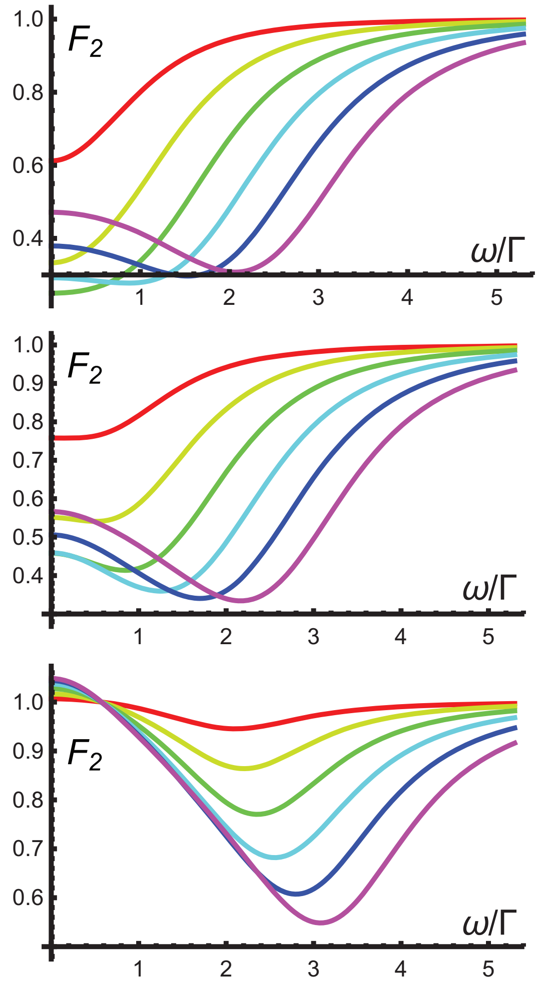

Fig. 2: Plot of the Fano factor $F_2(\omega)$ in eq. (24) as a function of ω. The red, yellow, green, light blue, blue and purple lines correspond to the parameter $\Delta/\Gamma = 0.3$, 0.5, 0.7, 0.9, 1.1 and 1.3. The upper panel corresponds to $\epsilon_d = 0$, the medium panel corresponds to $\epsilon_d/\Gamma = 0.5$ and the lower panel corresponds to $\epsilon_d/\Gamma = 2$.

root of the cubic equation:

$$6X^3 + 3(9\Gamma^2 - 3\epsilon_d^2 - 8\Delta^2)X^2 + 4(3\Gamma^2 - \epsilon_d^2) \times \left(3\Gamma^2 - \epsilon_d^2 - 4\Delta^2\right)X - F(\epsilon_d, \Delta, \Gamma) = 0. \qquad (29)$$

Near the bifurcation line ω_M^2 is small and reduces to

$$\omega_M^2 = \frac{\theta(F)F(\epsilon_d, \Delta, \Gamma)}{4(3\Gamma^2 - \epsilon_d^2)\,(3\Gamma^2 - \omega_0^2)}, \qquad (30)$$

whereas far from the bifurcation line it is given through the asymptotic expansion in small Γ/ω_0 by

$$\omega_M^2 = \omega_0^2 - 10\Gamma^2\frac{\Delta^2}{\omega_0^2} + 15\Gamma^4\frac{\omega_0^4 + 6\omega_0^2\Delta^2 - 40\Delta^4}{4\omega_0^6}. \qquad (31)$$

Note that in this limit ω_M^2 can be larger than ω_0^2 if $\epsilon_d^2 \gg \Gamma^2 > \Delta^2$. The substitution of the asymptotics (31) into eq. (24) gives us the Fano factor asymptotics as

$$F_2(\omega_M) = \frac{2\Delta^2 + \epsilon_d^2}{6\Delta^2 + \epsilon_d^2} + O\left(\frac{\Gamma^2}{\omega_0^2}\right). \qquad (32)$$

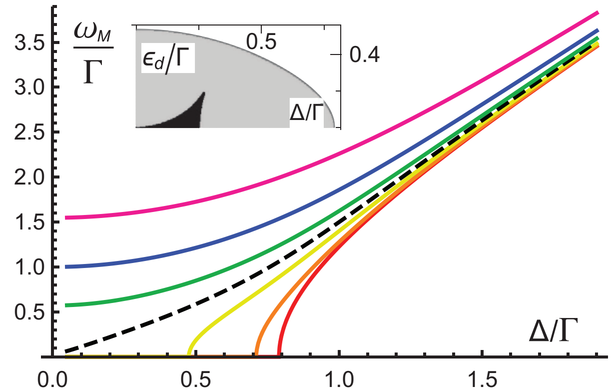

Fig. 3: Plot of the minimum location ω_M of the Fano factor $F_2(\omega)$ in eq. (24) as a function of Δ. The red, orange, yellow, green, blue and purple lines correspond to the parameter $\epsilon_d/\Gamma=$ 0, 0.2, 0.4, 0.7, 1, and 1.5. The black dashed curve corresponds to the bifurcation point $\epsilon_d/\Gamma \approx 0.54$. Inset: the grey area corresponds to the single Fano factor minimum at $\omega_M = 0$, the black area from [12] shows where the transient current does not oscillate.

It varies from 1/3 at the resonance to 1 at large ϵ_d^2. We also compare the asymptotics (31) with [12,15]:

$$\omega_I^2 = \omega_0^2 - \Gamma^2 \frac{4\Delta^2(\Delta^2+\epsilon_d^2)}{\omega_0^4} + \omega_0^2 O\left(\frac{\Gamma^4}{\omega_0^4}\right). \qquad (33)$$

Both frequencies, the minimum location ω_M and the oscillation frequency ω_I being some transformation of the initial qubit frequency ω_0 by the tunneling-produced dissipation, are different and approach one another only asymptotically at small Γ.

A similar feature of the finite frequency dip, but of a smaller depth, in the current noise spectrum has been predicted [26,27] in transport through a Coulomb-blockaded double quantum dot under the condition that the two dots are not occupied by more than a single electron. This dip is produced by the displacement part of the total current, which realizes the Coulomb screening of the dots in this model due to finite capacitances between the dots and the leads. This mechanism of screening does not change the tunneling particles dynamics and the particle currents. Hence, it does not affect the low-frequency noise and no spectrum bifurcation has been found in that system.

Conclusion. – The quantum fluctuations of the current of spinless electrons tunnelling through an interacting resonant level of a QD into an empty collector have been studied in the especially simple, but realistic model, in which all sudden variations in charge of the QD are effectively screened by a single tunneling channel of the emitter. Making use of the exact solution to this model, we have derived a general expression for the counting statistics of the charge transfer and found a simple relation between the two statistics for the processes of the QD evolution from its stationary and empty states. This relation has allowed us to obtain the spectrum of the steady current shot noise through the calculation of the Fourier transformation of the time-dependent transient current produced in the process of the QD empty state evolution.

The oscillating behavior of this current results from the emergence of the qubit built of electron-hole pair at the QD and its coherent dynamics in the wide range of the model parameters [13] and can be used for the identification of the nature of the FES observed in the tunneling current dependence on voltage. We have demonstrated that the current noise spectrum being the spectral decomposition of the transient current in eq. (22) can also be used for the identification of the FES. Indeed, its frequency dependence normalized by the mean current is characterized by the dips whose positions reflect the oscillating behavior of the time-dependent transient current: A single zero-frequency minimum in the normalized spectral dependence occurring for the large collector tunneling rate Γ splits with the decrease of Γ into the two resonant dips located at finite frequencies $\pm\omega_M$ when either the emitter tunneling coupling Δ or the absolute value of the resonant level energy $|\epsilon_d|$ becomes large enough in comparison with the collector tunneling rate Γ and either $\Delta^2 > 5\Gamma^2/8$ or $\epsilon_d^2 > 0.29\Gamma^2$ holds. Note, these conditions are more restrictive than the ones for the transient current oscillations to appear, which are either $\Delta > \Gamma/4$ or $\epsilon_d^2 > \Gamma^2/27$, respectively.

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