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To cite this article: Yanxuan Shao *et al* 2019 *EPL* **126** 40005

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Evolutionary dynamics of group cooperation with asymmetrical environmental feedback

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received 24 April 2019; accepted in final form 24 May 2019
published online 27 June 2019

PACS 02.50.Le – Decision theory and game theory

PACS 87.23.Ge – Dynamics of social systems

PACS 87.23.Cc – Population dynamics and ecological pattern formation

Abstract – In recent years, there has been growing interest in studying evolutionary games with environmental feedback. Previous studies exclusively focus on two-player games. However, extension to multi-player game is needed to study problems such as microbial cooperation and crowdsourcing collaborations. Here, we study coevolutionary public goods games where strategies coevolve with the multiplication factors of group cooperation. Asymmetry can arise in such environmental feedback, where games organized by focal cooperators may have a different efficiency than the ones by defectors. Our analysis shows that coevolutionary dynamics with asymmetrical environmental feedback can yield oscillatory convergence to persistent cooperation, if the relative changing speed of cooperators' multiplication factor is above a certain threshold. Our work provides useful insights into sustaining group cooperation in a changing world.

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Introduction. – Cooperation is a prominent phenomenon that widely exists in natural systems among various scales, ranging from microbes to human societies [1–4]. Evolutionary game theory is a powerful theoretical approach to study and understand why and on what conditions a persistent cooperation situation would occur [5–11]. In particular, public goods game (PGG) provides valuable insight into group cooperation [12–15], considering the great challenges we face nowadays, like global warming, pollution control and overexploitation of natural sources [16,17].

PGG can be seen as an extension of the Prisoner's Dilemma [18,19]. In the classical PGG model with well-mixed interactions, the population eventually evolves into a mutual defection state where cooperation vanishes, under the assumption that individuals always choose rational strategies based on the incentives [20]. A great amount of works concentrates on solving this well-known

tragedy of the commons [21,22] by taking into account different realistic factors and evolving mechanisms in the game, such as kin selection [23], punishment and reward [24,25], direct and indirect reciprocity theories [26–30], and in particular spatial reciprocity [31–34]. Additionally, a variety of factors describing heterogeneity of players have been incorporated, such as the optional participation mechanism [35], network topology structure [36,37], wealth-based selection [38] and different environment of the population [39]. These studies proved that heterogeneity plays an important role in promoting group cooperation in the real world [40,41].

Recently, a new framework of replicator dynamics with feedback-evolving games has been proposed to characterize the phenomenon that environment and individual behavior coevolves in many social-ecological and psychological-economic systems [35,42–48]. The environmental feedback can result in oscillating dynamics of both environment quality and strategy states [49]. The persistent cycles also occur in asymmetric conditions with a heterogeneous environment [39]. A more general framework

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for eco-evolutionary games shows that the cyclic dynamics only occurs under the condition that the environmental change is slow enough compared to strategy dynamics [50]. These models provide deep insights into the cooperation behavior in coevolutionary systems [51].

With the rapid development of network technology, the crowdsourcing project, which is a new form of on-line collaboration aiming to complete a project by soliciting contributions from a large group of people or online communities, has attracted increasing attention in the recent years [52]. Crowdsourcing has been successfully applied into many fields, such as knowledge discovery and management, crisis mapping, and crowdfunding [53–55]. Interestingly, these online cooperations often happen under the preliminary conditions that there exists an authoritative organizer who leads the game and may decide the global payoff distributions to some extent, which is an important new character. In most crowdsourcing cases, cooperations are encouraged through a higher-payoff structure for cooperators, such as extra incentives in commercial projects or preferential access in knowledge discovery, which causes the emergence of asymmetrical feedback. We then raise an important question: on what conditions will these collaborations form successfully in a general sense? Specifically, from the perspective of collaboration organizers, how to change the synergy effect of group of cooperators to encourage cooperation when the total resource and benefit of the project are restrained?

In this paper, we focus on the scenario where the multiplication factor of cooperators r_c coevolves with the strategies in PGG (similarly we also consider coevolving r_d for defectors, see details in appendix B in the supplementary material [SupplementaryMaterial.pdf](#) (SM)). We would like to see how ratios of total payoffs of cooperators *vs.* defectors affect the evolving adaptive environment. We let the multiplication factor of cooperators, r_c , change in response to the global payoff difference between cooperators and defectors in the system, and in turn the multiplication factor, r_c , affects the evolutionary dynamics of individual cooperation behaviors. In this way, we add in the role of authoritative organizers who aim to organize the collaboration and can enforce the global payoff distributions, as described above. We highlight the conclusion that the feedback-evolving evolution can give rise to oscillating convergence to persistent cooperation in some parameter regime, but only if the relative changing speed of cooperators' multiplication factor exceeds a threshold. This result indicates that this asymmetrical environmental feedback in PGG is effective for group cooperation only when the feedback updates quickly and promptly enough compared to the strategy change. Our work sheds light on how to successfully organize a group collaboration and avoid the traps of social dilemma in projects like crowdsourcing.

Model. – We consider PGG in a well-mixed infinitely large population, with each individual choosing to be a cooperator or defector, who contributes to the public pool

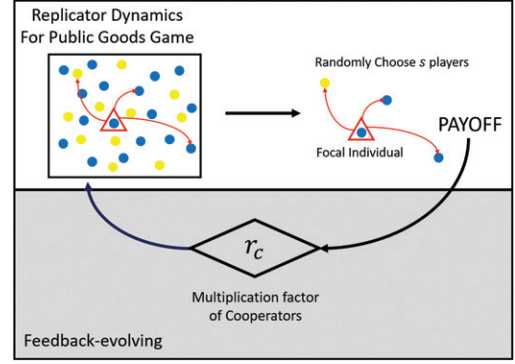


Fig. 1: Schematic of the asymmetrical environmental feedback mechanism of our model. The graph on the top shows how the population evolves according to replicator dynamics, and meanwhile, the total payoff of cooperators and defectors gives the feedback to the cooperator's multiplication factor and in turn modifies the dynamics of strategies.

or not, respectively. In each game, one focal individual randomly chooses other s players to play the game, which means there are in total $s + 1$ participants in one game. In classical PGG, the total contribution is multiplied by a multiplication factor r and distributes to every participant equally. Here to characterize the asymmetrical environmental feedback, we assume that the defector's multiplication factor r_d remains constant, which is relatively low, and the cooperator's multiplication factor r_c keeps changing depending on the influence of the global payoff distribution. In turn, the changing r_c affects the individual's payoff and drives the dynamics of strategies, as illustrated in fig. 1.

We denote x as the frequency of cooperators in the population. For simplicity and without loss of generality, we let the contribution of each cooperator be 1. For a focal individual, the possibility that m out of s selected individuals are cooperators is

$$\binom{s}{m} x^m (1-x)^{s-m}. \quad (1)$$

Thus, the expectation of the focal individual's payoff is

$$\begin{aligned} P_c &= \sum_{m=0}^s \binom{s}{m} x^m (1-x)^{s-m} \left[\frac{(m+1)r_c}{s+1} - 1 \right] \\ &= -1 + \frac{(1+sx)}{s+1} r_c, \end{aligned} \quad (2)$$

$$\begin{aligned} P_d &= \sum_{m=0}^s \binom{s}{m} x^m (1-x)^{s-m} \frac{mr_d}{s+1} \\ &= \frac{sx}{s+1} r_d. \end{aligned} \quad (3)$$

Replicator dynamics are widely used in evolutionary games, which describes the time evolution of the frequency of each strategy. Here the replicator equation for x , the

frequency of cooperators in the population, is

$$\begin{aligned}\dot{x} &= x(P_c - \bar{P}) \\ &= x(1-x) \left(\frac{sx+1}{s+1} r_c - \frac{sx}{s+1} r_d - 1 \right),\end{aligned}\quad (4)$$

where \bar{P} denotes the average payoff of the population.

The other equation describing the dynamics of the cooperator's multiplication factor is

$$\dot{r}_c = \epsilon(r_c - \alpha)(\beta - r_c)f(x, r_c), \quad (5)$$

where $f(x, r_c)$ describes the feedback mechanism of the total payoff in the game interaction with its sign deciding whether r_c increases or decreases. α and β denote the minimum and maximum values of the multiplication factor of cooperators, therefore by the term $(r_c - \alpha)(\beta - r_c)$, r_c will grow logistically and be confined to the range $[\alpha, \beta]$. According to the social dilemma of PGG, we have $1 < \alpha < \beta < s + 1$. ϵ denotes the relative changing speed of r_c compared to x . The multiplication factor of the cooperators, which is characterized by r_c , in turn influences the payoffs as well as the frequencies of different strategies, resulting in a feedback loop. We assume that the multiplication factor of the cooperators is modified by global payoffs due to the limitation of the total rewards for the project and the zero-sum characteristic of resource consumption:

$$f(x, r_c) = -xP_c + \theta(1-x)P_d, \quad (6)$$

where xP_c and $(1-x)P_d$ are the global payoff for cooperator and defector in the population, respectively. $\theta > 0$ denotes the ratio of the increasement rate to the decrease rate of the cooperator's and defector's total payoff expectation in the system. Here, when the resource is adequate, cooperators are rewarded according to their contributions to the public pool, while depletion of resources in the crowdsourcing prevents the cooperator's multiplication factor from increasing infinitely.

Thus, the ODE systems for our model can be written as

$$\begin{cases} \dot{x} = x(1-x) \left(\frac{sx+1}{s+1} r_c - \frac{sx}{s+1} r_d - 1 \right), \\ \dot{r}_c = \epsilon(r_c - \alpha)(\beta - r_c) \left[-x \left(-1 + \frac{r_c(1+sx)}{s+1} \right) + \theta(1-x) \frac{r_d sx}{s+1} \right]. \end{cases} \quad (7)$$

Results. –

Stability of fixed points and thresholds of multiplication factors. There are six possible fixed points of the model: five are on the boundary and the remaining one is an interior fixed point. For the five boundary fixed points, only two of them can be stable: i) ($x^* = 0$), the population is dominated by defectors and is always stable, which also occurs in the classic model of PGG; and ii) ($x^* = 1, r_c = \alpha$),

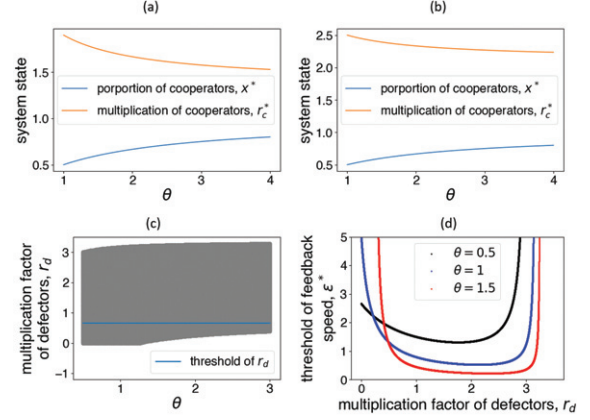


Fig. 2: Impact of varying model parameters on population equilibrium states. Panel (a) and (b) show changes of the interior fixed point (x^* and r_c^*) as θ increases, with $r_d = 0.5$ for (a) and $r_d = 1.5$ for (b). Panel (c) presents the critical boundary of r_d as well as the threshold r_d^* as θ increases. Panel (d) shows the dependence of the threshold of relative changing speed ϵ^* on θ and r_d . In all panels, $s = 3, \alpha = 1.5, \beta = 3.5$.

which is stable only if the multiplication factor of defectors, r_d , is smaller than the threshold $r_d^* = \frac{(s+1)(\alpha-1)}{s}$. This possible fixed point corresponds to the state where the population is dominated by cooperators and the multiplication factor of cooperators r_c is at its minimum value.

Besides, there is one interior fixed point which can be stable:

$$\begin{cases} x^* = \frac{\theta}{1+\theta}, \\ r_c^* = \frac{\theta r_d s + (s+1)(\theta+1)}{\theta s + \theta + 1}. \end{cases} \quad (8)$$

It corresponds to a stable population composed by both cooperators and defectors, with a medium value of cooperators' multiplication factor. Equation (8) indicates that the final position of the interior fixed point is only influenced by θ , the ratio of the increasement rate to the decrease rate of cooperator's and defector's global payoff, which characterizes the nature of the project itself. The detailed impacts of θ on x^* and r_c^* are shown in fig. 2(a), (b). We fix $s = 3, \alpha = 1.5, \beta = 3.5$ and we have $r_d^* = \frac{2}{3}$. Therefore, we set $r_d = 0.5$ for fig. 2(a) and $r_d = 1.5$ for fig. 2(b), respectively. Results show that when θ increases, the stable frequency of cooperators x^* increases, while the cooperator's multiplication factor r_c^* decreases. In order to get an intuitive understanding, we offer an example of a team work. If the team work does not require strong abilities of the workers, like pure labour work, θ increases accordingly, calling for more people participating in the team work for a better outcome. Eventually, there will be a higher proportion of cooperators with a relatively low multiplication factor of cooperators. On the contrary, if the team members are expected to be more skilled, like in scientific collaborations, the decrease of θ asks for people who can make real contributions to the

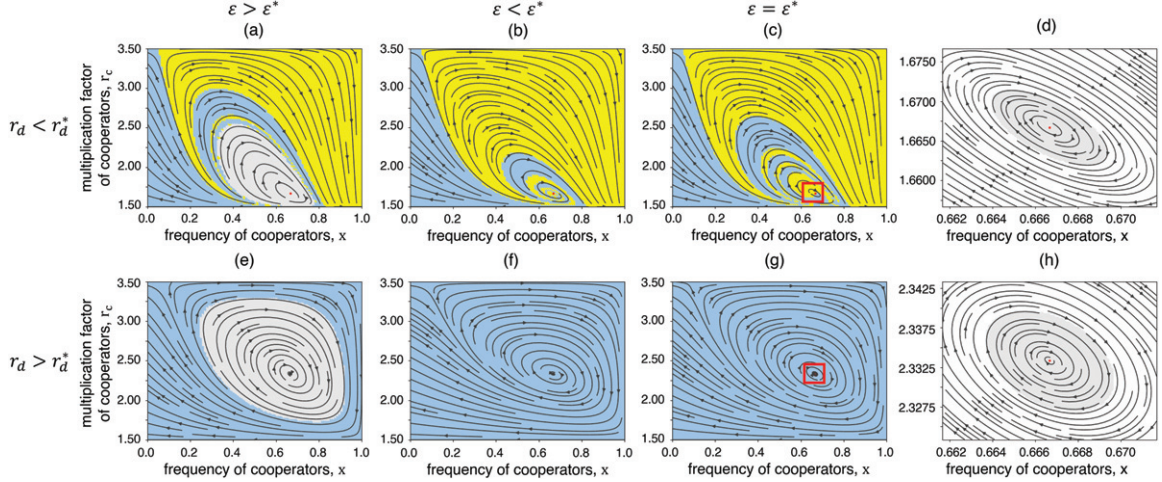


Fig. 3: Phase plane dynamics of the $x - r_c$ system with $s = 3, \alpha = 1.5, \beta = 3.5, \theta = 2$. For the first row, $r_d = 0.5, \epsilon = 1.5, \frac{14}{11}, 1$. For the second row, $r_d = 1.5, \epsilon = 0.4, \frac{2}{9}, 0.2$. Graphs in the last column are magnification of the area near the interior fixed points, highlighted with red rectangles in the third column. Blue, yellow and grey areas show attracting fields of different fixed points ($x^* = 0$), ($x^* = 1, r_c^* = \alpha$) and ($x^* = \frac{\theta}{\theta+1}, r_c^* = \frac{\theta r_d s + (s+1)(\theta+1)}{\theta s + \theta + 1}$), respectively.

project, resulting in a lower frequency of the cooperators with higher multiplication factor of the cooperators.

Since $\alpha \leq r_c \leq \beta$, the interior fixed point is meaningful only when

$$\max \left\{ \frac{\alpha(\theta s + \theta + 1) - (s+1)(\theta+1)}{\theta s}, 0 \right\} \leq r_d \leq \frac{\beta(\theta s + \theta + 1) - (s+1)(\theta+1)}{\theta s}. \quad (9)$$

In fig. 2(c), we show the critical boundary of r_d as well as the position of threshold r_d^* as θ increases.

Finally, this interior fixed point is stable only when $\epsilon > \epsilon^*$, in which ϵ^* depends on other parameters s, r_d, θ, α and β , which reads

$$\epsilon^* = \frac{(1 - x^*)s(r_c^* - r_d)}{(s x^* + 1)(r_c^* - \alpha)(\beta - r_c^*)}. \quad (10)$$

The interior fixed point is the center of the limit cycle ($\epsilon = \epsilon^*$) or is unstable ($\epsilon < \epsilon^*$) otherwise. In fig. 2(d), we set $s = 3, \alpha = 1.5, \beta = 3.5$ and show how ϵ^* varies as θ and r_d change. We choose three values for θ : 0.5, 1 and 1.5, in which condition $0 \leq r_d \leq 3, 0 \leq r_d \leq \frac{19}{6}$ and $\frac{1}{9} \leq r_d \leq \frac{29}{9}$, respectively. ϵ^* firstly goes down sharply followed by mild decreases and a steep increase, caused by the logistic term $(r_c - \alpha)(\beta - r_c)$. In realistic games we concern more about the situations where the multiplication factor of defectors is neither too large nor too small and ϵ^* does not change much. Besides, ϵ^* is larger when θ is smaller.

The detailed proof for the stability of all six fixed points using Jacobian matrices is shown in appendix A in the SM.

Detailed conditions for the emergence of persistent cooperation. In fig. 3, we show how asymmetrical environmental feedback mechanism affects the system state under different situations using phase graphs. We choose a group

of parameters $s = 3, \alpha = 1.5, \beta = 3.5, \theta = 2$ and we have $r_d^* = \frac{2}{3}$ accordingly. Therefore, we set $r_d = 0.5, 1.5$ for two rows separately, where the thresholds of ϵ are $\frac{14}{11}$ and $\frac{2}{9}$, respectively, according to eq. (10). Figures 3(a)–(c) show that a mutual cooperation state can always occur as long as $r_d < r_d^*$, which means that when the defector's multiplication factor is much lower than that of the cooperator, the asymmetrical environmental feedback mechanism can effectively promote the emergence of group collaboration. However, this condition can rarely be satisfied in the real world, especially in PGG where one can hardly control the defector's payoffs. Therefore, we concern more about the emergence of the persistent co-existence of cooperators and defectors, *i.e.*, the stability condition of the interior fixed point. The comparison of fig. 3(a) *vs.* figs. 3(b), (c) as well as of fig. 3(e) *vs.* figs. 3(f), (g) reveals that the relative changing speed of the cooperator's multiplication factor ϵ determines the stability of the interior fixed point. The persistent co-existence of cooperators and defectors can only emerge when ϵ exceeds a threshold ϵ^* . In particular, $\epsilon > \epsilon^*$ is the only chance for breaking social dilemma when $r_d > r_d^*$, as shown in figs. 3(e)–(g). Therefore, we conclude that the asymmetrical environmental feedback in PGG is effective for the emergence of group cooperation only when the feedback updates quickly enough compared to the strategy dynamics.

Furthermore, in fig. 4, we present time evolutions of different system states, represented by the frequency of strategies and the multiplication factor of cooperators, corresponding to the situations shown in fig. 3(a)–(c). Parameters are $s = 3, \alpha = 1.5, \beta = 3.5, \theta = 2, r_d = 0.5$ and $\epsilon = 1.5, \frac{14}{11}, 1$ for figs. 4(a)–(c), respectively. Here $r_d < r_d^*$. In fig. 4(a), $\epsilon > \epsilon^*$, which means the asymmetrical environmental feedback is quickly and promptly enough, the interior fixed point is stable. A population

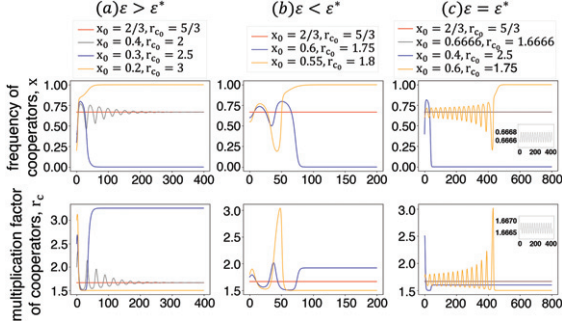


Fig. 4: Time evolutions of the system state, represented by the frequency of strategies and the multiplication factor of cooperators, under different initial conditions. Parameters are $s = 3, \alpha = 1.5, \beta = 3.5, \theta = 2, r_d = 0.5$ and $\epsilon = 1.5, \frac{14}{11}, 1$ for (a)–(c), respectively. The initial conditions are shown in each graph. The inserted small graphs in the third column are the magnification of the dynamics of the initial point $x_0 = 0.6666, r_{c0} = 1.6666$.

with initial conditions near the interior fixed point experiences oscillating convergence to the interior equilibrium state. Other initial states far from the interior fixed point experience rapid oscillation and converge to the boundary, either cooperation-dominated or defector-dominated. In fig. 4(b), when $\epsilon < \epsilon^*$, which indicates that the cooperators are not rewarded in time, all initial states oscillate to the boundary. In fig. 4(c), the interior fixed point becomes the center of the limit cycle when ϵ is exactly at its threshold, in which situation a tiny range of the initial state of the system experiences regular and continuous oscillation around the center. The initial point from the remaining part of the domain ends either in defector-dominated or in cooperator-dominated population.

Conclusion. – In this paper, we extend the two-player evolutionary games with environmental feedback to the multi-player situation where strategies coevolve with the multiplication factor of group cooperation. Using the coevolutionary PGG framework, we study a new form of collaboration in the real world. To describe the existence of asymmetry in the games where focal organizers who aim to organize the collaboration may enforce the global payoff distributions, we differentiate the multiplication factor of cooperators and defectors. On the one hand, cooperators are encouraged in order to avoid the social dilemma by increasing their multiplication factor. While, on the other hand, the resource will limit the number of cooperators when their global payoff is large enough. By adding this asymmetrical environmental feedback mechanism to replicator dynamics, the population can oscillatorily converge to a cooperator-defector coexisting state if the relative changing speed of the cooperator’s multiplication factor exceeds a threshold, breaking the tragedy of the commons in traditional PGG. The final frequency of cooperators is determined only by one parameter: the ratio of the increase rate to the decrease rate of cooperator’s

and defector’s global payoff, involving the limitation of the total resource and the zero-sum characteristic of resource consumption. If the ratio is relatively large, which means cooperation in the project is not that resource-consuming, it appears that there are more cooperators in the population and they get a relatively low multiplication factor. Conversely, there are fewer cooperators who get a higher multiplication factor if the ratio is small, corresponding to projects which are resource-consuming.

This asymmetrical environmental feedback mechanism well describes collaboration situations like crowdsourcing projects, which has potential applications to explain a number of real-world cooperation phenomena. Our work also shows the detailed conditions for the emergence of stable cooperation with resource restraints, thereby shedding light on organizing a successful group collaboration under similar circumstances. While current results focus on linear PGG, a potential direction for further studies is extending our framework to nonlinear PGG, such as threshold PGG which has been successfully used to better understand human behaviors in response to the climate change [56–58].

XW gratefully acknowledges the generous support by the China Scholarship Council. FF gratefully acknowledges the G. Norman Albree Trust Fund, the Dartmouth Faculty Startup Fund, the Neukom CompX Faculty Grant, Walter & Constance Burke Research Initiation Award and NIH Roybal Center Pilot Grant.

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