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LETTER

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Steering and focusing of fundamental shear horizontal guided waves in plates by using multiple-strip metasurfaces

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Abstract – We proposed a metasurface consisting of multiple parallel strips with varied widths which can steer and focus the fundamental shear horizontal (SH_0) wave in plates. Firstly, the group velocity of the quasi-SH_0 wave in different-width strips was calculated. Then, a multiple-strip metasurface was designed which was realized by through-thickness grooving in an aluminum plate. Finite-element simulations show that this metasurface can steer the SH_0 wave to a prescribed angle. Later, experiments were conducted in which thickness-shear (d_{15}) piezoelectric strips were used as SH_0 wave exciters and receivers, and experimental results well reproduced the simulation results. Finally, another multiple-strip metasurface was designed to focus the SH_0 wave at a designated point. Both simulations and experiments were conducted, and results validated the proposed design. This work may provide guidance for the design of SH wave metasurfaces.

Metasurface, whose definition is in analogy to the metamaterials [1], is an artificial interface typically consisting of subwavelength structures and can steer or reflect the incident wave in an anomalous way [2]. The concept of metasurface was firstly raised in optics [3–8], then extended to acoustics [9–14], and was recently applied to elastic waves [15–23]. The working principle of all the metasurface is based on the Generalized Snell's Law (GSL) which indicates that when there exists a phase gradient at the interface, abnormal reflection and refraction may appear.

Unlike the optical or acoustical metasurfaces which are mostly for bulk waves, the elastic metasurfaces are typically for guided waves in finite-dimension solids. Yan *et al.* firstly proposed and numerically validated a metasurface (which they called surface bonded elastic metameterial) by bonding a two-dimensional planar array of small lead discs on an aluminum plate with silicone rubber to bend and focus low-frequency flexural A_0 Lamb waves [21]. Zhu and Semperlotti shifted the phase of incoming wave by locally resonant toruslike tapers and realized anomalous refraction of symmetric (S_0) or antisymmetric (A_0) Lamb waves [24]. Liu *et al.* proposed a zigzag, 3-D printed metasurface which can shift and split a point source via forming phase gradient for the flexural Lamb wave (A_0) [15]. Lee *et al.* proposed a high transmission metasurface by mass-stiffness substructuring and successfully realized anomalous steering and focusing of the in-plane symmetric Lamb wave (S_0) [18]. Cao *et al.* proposed a 3-D metasurface consisting of multiple slender pillars for full phase shift and high transmission, which can realize abnormal deflection and negative refraction of flexural Lamb wave (A_0) [17]. They also realized asymmetric flexural wave (A_0) transmission based on dual-layer elastic wave gradient metasurfaces [16]. Recently, Zhu *et al.* realized total internal reflection of the flexural Lamb wave (A_0) based on elastic metasurface consisting of three types of unit cells and applied it to structural vibration isolation [23]. Besides the above-mentioned metasurfaces for Lamb waves, metasurfaces for Rayleigh wave [20], Love wave [19] and bulk SH wave [25] were also proposed and simulated, but experimental validation is lacking.

In this letter, we proposed a metasurface for a guided fundamental shear horizontal (SH_0) wave which consists of several parallel strips with varied width and was fabricated by simply grooving on an aluminum plate. Firstly, simulation results show that the group velocity of the quasi-SH_0 wave in the strips increases steadily with the increasing strip width. Then, a multiple-strip metasurface was proposed with the strip width increasing monotonically. Simulations indicate that when a plane incident SH_0 wave

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passes through this metasurface, a phase gradient will be generated and the SH_0 wave will be steered. Later, experiments were conducted on an aluminum plate using thickness-shear (d_{15}) piezoelectric strips for SH_0 wave excitation and reception, and the experimental results validated the proposed design. Finally, another multiple-strip metasurface was proposed to focus the SH_0 wave, both simulations and experiments validated the proposed design.

According to the generalized Snell's law (GSL) [26], when there exists a phase gradient ($\mathrm{d}\phi/\mathrm{d}y$) of the incident wave at the interface between two media, the relationship between the refraction angle θ_t and the incident angle θ_i turns to be

$$\frac{\sin(\theta_t)}{\lambda_t} - \frac{\sin(\theta_i)}{\lambda_i} = \frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}y}, \qquad (1)$$

where λ_i and λ_t are the wavelengths in the incident medium and the transmission medium, respectively.

Obviously, based on eq. (1), when an interface (metasurface) is introduced in a uniform medium which can cause the phase gradient of the incident wave, abnormal refraction will occur, *i.e.*, the incident wave will be steered in the uniform medium by the metasurface. In this work, the phase gradient is caused by the different group velocities in strips of different widths.

Based on the early work by Mindlin and Fox [27], Cegla calculated the dispersive curve of the quasi-SH_0 wave in an infinite-long, large aspect ratio rectangular strip and found that at high frequency above 200 kHz, the phase (group) velocity decreases (increases) with increasing strip width [28]. In this work, the working frequency of the proposed metasurface is set as 100 kHz and the aspect ratio of the rectangular strip cross-section cannot be very large, thus Cegla's results [28] cannot be employed here. We firstly simulated the guided wave propagation in 30 mm long, 2 mm thick aluminum strips with varied widths using COMSOL. In the simulation, a 100 kHz plane SH_0 wave was excited by a 100 mm long shearing line force on a large aluminum plate, then propagated into the strip and finally propagated to another large aluminum plate, as shown in the insert of fig. 1(a). Results show that the SH-type guided wave is dominant in the strip, see fig. S1 in the supplementary material `Supplementarymaterial.pdf` (SM), and here it is called quasi-SH_0 wave.

The group velocity of the quasi-SH_0 in strips with different frequency-width products was calculated and the results were plotted in fig. 1(a) in which the phase shift was also presented with the 200 kHz · mm strip case as the reference.

As the group velocities of the quasi-SH_0 wave in a strip is dependent on the frequency-width product, the design of the metasurface (width of the strips) needs to be changed accordingly for different frequencies.

Considering the case of 100 kHz, it can be seen from fig. 1(a) that the group velocity of the quasi-SH_0 in the

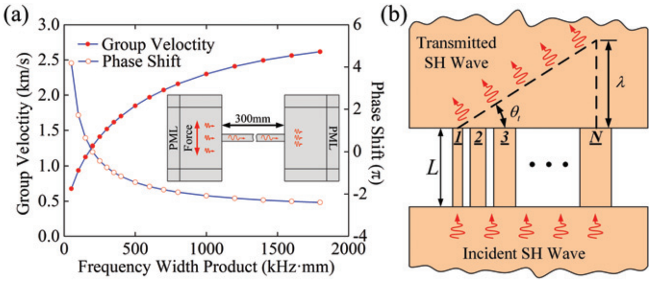

Fig. 1: (a) Simulated quasi-SH_0 wave propagation in 30 mm long, 2 mm thick aluminum strips with varied frequency-width products. Dots: group velocity; circles: phase shift referring to the 200 kHz.mm strip. (b) Illustration of the working principle of the multiple-strip metasurface.

strip increases steadily and nonlinearly with the increasing width. The phase shift referring to the 2 mm wide strip can vary from -2π to 4π in the width range [0.5 mm, 18 mm]. For practical design, the width range [2 mm, 8 mm] was selected and the phase shift can cover a full 2π range. Here to design a metasurface for steering the SH_0 wave by an angle θ_t, multiple parallel 30 mm long strips were proposed with the strip width increasing steadily from 2 mm to 8 mm, as shown in fig. 1(b). The multiple parallel strips were fabricated by simply through-thickness grooving on a large aluminum plate. The gap between all adjacent strips was set to the same, and denoted by W_{gap}. The design process of the multiple-strip SH wave metasurface is as follows. As seen in fig. 1(b), the width of the left, 1st strip is $W_1 = 2$ mm. The width of the 2nd strip, W_2, should satisfy the following implicit equation:

$$\frac{(W_1/2 + W_{gap} + W_2/2)}{\Delta\phi(W_2)}\tan(\theta_t) = \frac{\lambda_{SH}}{2\pi}, \qquad (2)$$

where the function of $\Delta\phi(W)$ can be obtained by curve fitting of the calculated phase shift in fig. 1(a).

W_2 can be obtained by solving eq. (2) numerically. Similarly, the width of the i-th strip, W_i, can be obtained in sequence.

Simulations and experiments. – Finite-element (FEM) simulations were then conducted by using the commercial software COMSOL to predict the performances of the design metasurface. Here the steering angle of the SH_0 wave is set to be 18.5°, and the designed metasurface consists of 17 strips with the width increasing from 2 mm to 8 mm. For the convenience of fabrication, the gap between adjacent strips was set to be 2 mm. The length of the metasurface is about 94 mm and the width of each strip is provided in the SM. The schematic of the FEM model was shown in fig. 2(a), in which the 100 kHz plane SH_0 wave was excited by using 10 pieces of 20 mm long d_{15} piezoelectric strips bonded on the aluminum plate in one row with a gap of 4 mm. The PML (Perfectly Matched Layers) were used at all boundaries to remove the boundary reflections. The simulated in-plane Y displacement of

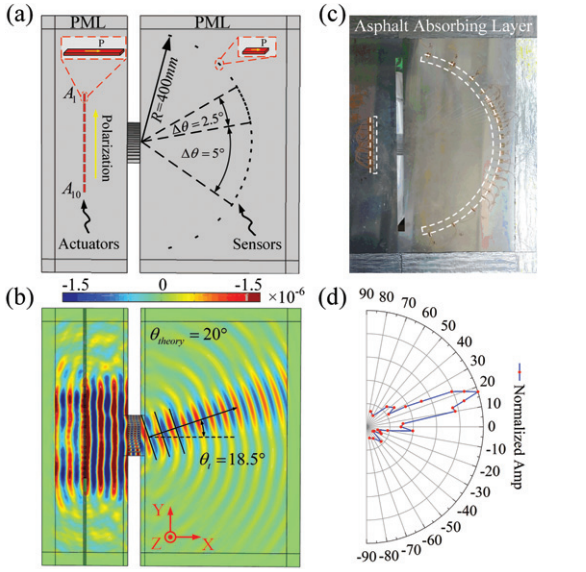

Fig. 2: Steering of the SH_0 wave in an aluminum plate using the proposed multiple-strip metasurface. (a) Schematic of finite-element simulation and experimental testing; (b) photo of the experimental setup; (c) simulated in-plane displacement of the SH_0 wave; (d) amplitude of the steered SH_0 wave received by d_{15} piezoelectric strips.

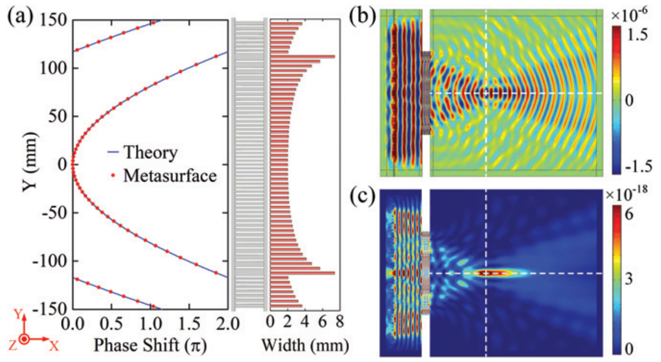

Fig. 3: Simulated focusing of the SH_0 wave using a multiple-strip metasurface. (a) Phase shift corresponding to the strip width; (b) simulated in-plane Y displacement of the SH_0 wave showing the focusing point; (c) simulated SH_0 wave energy highlighting the focusing point.

the incident SH_0 wave and that of the transmitted SH_0 wave through the metasurface were shown in fig. 2(b). It can be seen that the designed metasurface can indeed steer the SH_0 wave by an angle of 18.5°. The energy transmission rate by the designed metasurface is about 65%, which is fairly high. The energy loss is mainly caused by the relatively large gap (2 mm) between the adjacent strips, which will reflect a considerable amount of SH_0 wave energy. If the gap were reduced, the energy transmission rate can be considerably increased.

To further validate the design of the metasurface, experiments were conducted on a $1200\times1200\times2\,\text{mm}^3$ aluminum plate on which the multiple-strip metasurface was fabricated by simply through-thickness grooving. By applying asphalt on the upper and lower surfaces of the four edges of the aluminium plate (15 mm width asphalt sticker is used in this experiment), the SH wave can be effectively absorbed and the effect of "absorbing layer" can be realized. The SH_0 wave was excited by using 10 pieces of 20 mm long d_{15} piezoelectric strips, the same as those in the simulation. The steered SH_0 wave was received by 8 mm long d_{15} piezoelectric strips distributed non-uniformly on a 400 mm radius arc with the middle point of the metasurface as the circle center, as shown in fig. 2(a) and (c), where the best angle resolution in measurement is 2.5°. Figure 2(d) shows the measured amplitude of the steered SH_0 wave distributed along the arc sensor array. It can be seen that, as expected, the SH_0 wave was steered by the designed metasurface. The maximum amplitude appears at 17.5°. Considering the measurement resolution of 2.5°, the measured steering angle should be 16.25°–18.75°, which is consistent with the designed steering angle of 18.5°. This confirmed the validity of the designed metasurface in the steering of the SH_0 wave.

Based on the above SH_0 wave steering results, we further proposed a multiple-strip metasurface for the SH wave focusing with the focal length of 200 mm. The designed metasurface for a 100 kHz SH_0 wave consists of 61 strips and is illustrated in fig. 3(a) in which the calculated and theoretical phase shift was also presented. The length of the focusing metasurface is about 300 mm and the detailed width info is provided in the SM. Also for the convenience of fabrication, the gap between adjacent strips is set to 2 mm.

Finite-element simulations were firstly conducted to predict the performance of the designed metasurface in SH wave focusing. In the simulation, the 100 kHz SH_0 wave was excited by using 20 pieces of 20 mm long d_{15} piezoelectric strips in one row bonded on the aluminum plate. Figure 3(b) shows the simulated in-plane Y displacement of the incident SH_0 wave and that of the transmitted SH_0 wave via the metasurface. It can be seen that the SH_0 wave was focused by the designed metasurface with a focal length of 200 mm. After propagating across the focal point, the SH_0 wave turns to be divergent. The simulated energy distribution of the SH_0 wave was shown in fig. 3(c), from which it can be seen more clearly that the wave energy was focused at the designed focal point.

Experiments were then conducted to check the performance of the designed focusing metasurface. In the same way as that in the simulation, the incident 100 kHz SH_0 wave was excited by 20 pieces of 20 mm long d_{15} piezoelectric strips. To reduce the effect of the bonded sensors on the received SH wave signals, two sets of d_{15} strip sensor arrays were bonded in sequence around the designed focal point, *i.e.*, along the X-direction (fig. 4(a)) and Y-direction (fig. 4(c)) of the designed focal point, respectively. Figure 4(b) and fig. 4(d) show the measured SH_0 wave energies along the X-direction and Y-direction

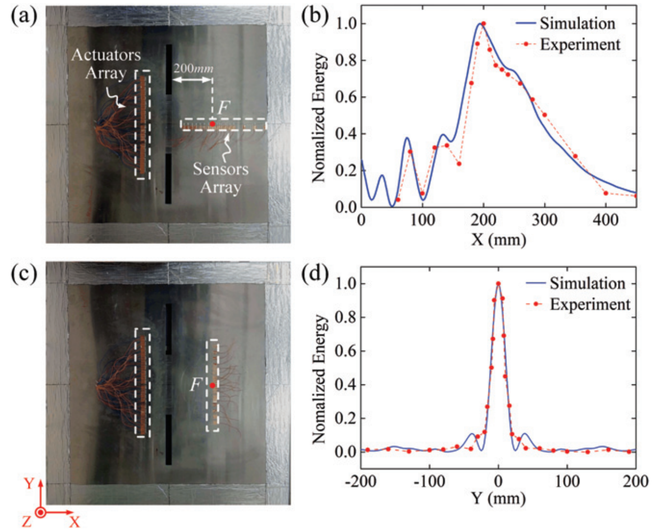

Fig. 4: Experimental verification of the SH_0 wave focusing using a metasurface. Left: layout of the experimental setup showing the different sensor arrays around the designed focal point, (a) along the X-direction, (c) around the Y-direction; Right: Measured and simulated SH_0 wave signals in different sensor arrays, (b) along the X-direction, (d) along the Y-direction.

of the focal point, respectively. It can be seen that the experimental results are consistent with the simulation results for both cases, *i.e.*, the SH_0 wave energy reaches its maxima at the designed focal point along both directions. From fig. 4, it can be concluded that the designed metasurface can effectively focus the SH wave at the designed focal point.

It should be noted that in above simulations and experiments, the excitation waves are always continuous waves. Actually, this metasurface can also work under pulse excitations. To show this, we have simulated the performance of the proposed metasurface under pulse excitation using the semi-analytical finite-element method (SAFE) [29,30] and the results were presented in fig. S3 and fig. S4 in the SM. However, due to the dispersion of the quasi-SH_0 wave in the strips, there exist a quasi-SH wave with different group velocities in the same strip under pulse excitation signals with a certain frequency bandwidth, which will affect the performance of the metasurface and bring difficulties to the design. How to make this metasurface work better under pulse excitations requires further investigations.

In summary, we proposed a design of metasurface for the SH_0 wave steering and focusing in plates based on multiple parallel strips. The metasurface is based on the generalized Snell's law and the rule that the group velocity of quasi-SH_0 in strips increases steadily with the strip width. Both simulations and experimental measurement validated the designed metasurfaces in SH_0 wave steering and focusing. The proposed multiple-strip metasurface is easy to fabricate and the transmission rate can be fairly high. This work provides a convenient solution for manipulating the SH wave in plate-like structures.

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