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LETTER

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Focus Article

Dynamical localization of interacting ultracold atomic kicked rotors[(a)]

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Abstract – We study a system of two atomic quantum kicked rotors with hard-core interaction. This system shows different dynamical behavior depending on the value of the kick period. In particular, we find that for periods close to resonance, the system shows a crossover from quantum resonance to dynamical localization. We characterize this crossover by the analysis of momenta distribution and density probability function in the configuration space, and discuss the role of the hard-core interaction on the dynamical localization by comparing it to the free-bosons case. In particular we note that dynamical localization of the center of mass persists even in the presence of strong interaction among the atoms. Some experimental proposals are also discussed.

Introduction. – The Quantum Kicked Rotor (QKR) is a prototypical model for both theoretical and experimental understanding of emerging phenomena like quantum resonance and dynamical localization. In particular, it can be considered an outstanding quantum simulator [1,2] for the exploration of transport in disordered quantum systems. Since the first realization of an atomic kicked rotor by the Raizens group, consisting of a cloud of laser-cooled atoms submitted to a periodically pulsed laser standing wave (SW) [3], many experimental results in different fields such as, *e.g.*, quantum transport [4,5] and quantum metrology [6], have followed. In particular, the quantum resonant regime of a kicked rotor model has been realized also in different systems, *e.g.*, in linear molecules kicked by periodic trains of laser pulses [7–9].

While the dynamical localization is a robust phenomenon, the quantum resonance is a rather sensitive effect since being exactly at resonance in an experiment is very challenging. Furthermore less is known about the quantum dynamics when the kicking period is close to, but not exactly at resonance [10].

On the other hand, while the single-rotor systems have been extensively studied and are well understood (see [11–14] for a comprehensive presentation of the subject), the situation is less clear for two or more kicked rotors (see, *e.g.*, [15]). In fact, it was believed that interactions between rotors generally destroy localization [16–24]. However Qin *et al.* [25] have recently studied the dynamical localization of two quantum kicked rotors with contact interaction. They have claimed that the dynamical localization is destroyed for relative momentum, while being preserved for the center-of-mass momentum. Also other recent studies have shown that the interplay of quantumness and interactions dramatically modifies the system dynamics of N kicked rotors inducing a transition between energy saturation and unbounded energy increase depending on the kick strength [26,27]. Consequently the understanding of systems formed by two or more QKR is still challenging.

This article aims to study the interplay between quantumness and interaction of two kicked rotors made

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of two bosons both in the non-interacting (free) and hard interacting (Tonks) limits, in the quasi-resonance and in the dynamical localization regime. Moreover the obtained results in Tonks limit are compared with the case considered in [25] of two bosons interacting through a contact potential.

We show that a dynamical localization persists in the momentum space when looking at the center-of-mass momentum, while a spreading of the distribution of the relative momenta is visible. In particular, we focus on a quasi-resonant regime for $T = 2\pi n - \delta$, where δ measures the distance from resonance, showing a crossover to a dynamically localized regime both in the case of interacting and free particles.

The paper is organized as follows. We firstly introduce the model and discuss its solution in the non-interacting and Tonks cases, respectively. We successively show the results of the system dynamics in the quasi-resonant regime, and discuss the differences between the non-interacting and strongly interacting regime (Tonks case) by analyzing the momentum distribution and the probability density function in the configuration space. Finally we draw the conclusions and discuss the possible experimental implications.

Resonant and quasi-resonant regime in the QKRs. – Starting from the QKR model (see [11] for further details), we explicitly calculate the wave function for two QKRs, both for the case of particles interacting via an infinite contact interaction (Tonks' particles) with anti-periodic boundary conditions (APBC), and in the case of the non-interacting particles (free bosons) with periodic boundary conditions (PBC).

The Hamiltonian of the system of two QKRs for a generic contact interaction is given by the following expression:

$$H = \sum_{j=1,2} H_j + H_{int}, \tag{1}$$

where $H_j, j = 1, 2$ is the Hamiltonian for the j-th particle:

$$H_j = H_j^{free} + H_j^{kick}, \tag{2}$$

with

$$\begin{cases} H_j^{free} = \dfrac{\partial^2}{\partial \vartheta_j^2}, \\ H_j^{kick} = g \displaystyle\sum_{j=1,2} \cos(\vartheta_j) \sum_n \delta(t - nT) \end{cases}$$

and H_{int} is the interaction term:

$$H_{int} = \lambda \delta(\theta_1 - \theta_2). \tag{3}$$

In the last equations $g = \xi T$, ξ and T being, respectively, the strength and the periodicity of the kicks, ϑ_j is the angular position, and n is the number of kicks. The interaction $\cos(\vartheta_j)$ has been chosen having in mind experiments on laser control of ultracold atoms [28–30]. Furthermore, we have expressed the energy in units of $\frac{\hbar^2}{2m}$ (m being the mass of a single particle) and the time in units of $\frac{2m}{\hbar}$. Finally, since we consider exclusively the two limit cases of free and hardcore bosons, for the strength interaction appearing in eq. (3) we have to consider respectively the limits $\lambda \to 0$ and $\lambda \to \infty$.

We note that, due to the different nature of particles, the symmetry of the wave function at any time depends on the interaction regime: in the free and strong interacting case the wave function will be, respectively, symmetric and antisymmetric.

Consequently, for free bosons, we can expand the initial wave function in plane waves as

$$\psi(\vartheta_1, \vartheta_2, 0)_B = \sum_n \frac{1}{\sqrt{2(1 + \delta_{k_1,k_2})}} [\varphi_{k_1,k_2}(\vartheta_1, \vartheta_2) + \varphi_{k_2,k_1}(\vartheta_1, \vartheta_2)], \tag{4}$$

where $\varphi_{k_i,k_j}(\vartheta_i, \vartheta_j) = \frac{1}{\sqrt{2\pi}} e^{i(k_i\vartheta_i + k_j\vartheta_j)}$, $n = n(k_1, k_2)$ is the energy level identified by the momenta (k_1, k_2), with $k_1, k_2 \in \mathbb{Z}$, $\delta_{k_1 k_2}$ is the Kronecker delta, and $\frac{1}{\sqrt{2\pi}}$ is the normalization factor that comes from the PBC conditions.

For the hardcore bosons case, we have to consider the APBC conditions. Furthermore it is useful to resort to the Bose-Fermi mapping [31], which relates the wave function of hardcore bosons to that of non-interacting spinless fermions in the same periodic interaction potential. Consequently, starting from the wave function for free fermions,

$$\psi(\vartheta_1, \vartheta_2, 0)_F = \frac{1}{\sqrt{2}} \sum_n [\varphi_{k_1,k_2}(\vartheta_1, \vartheta_2) - \varphi_{k_2,k_1}(\vartheta_1, \vartheta_2)], \tag{5}$$

we obtain through the mapping the following expression for the wave function:

$$\psi(\vartheta_1, \vartheta_2, 0)_{HB} = A\, \psi(\vartheta_1, \vartheta_2, 0)_F, \tag{6}$$

with

$$A = \mathrm{sgn}(\vartheta_1 - \vartheta_2),$$

where "sgn" is the sign function and A is the unit antisymmetric function, which ensures that $\psi(\vartheta_1, \vartheta_2, t)_B$ has the proper symmetry under the exchange of two bosons. We note that the APBC force the momenta to be half-integer ($k_i = m_i + 1/2, \forall\, m_i \in \mathbb{Z}$). Finally we choose for the two systems the ground state as the initial state.

Concerning the system evolution, even if the single-particle Hamiltonian equation (2), is explicitly time-dependent due to the kick interaction, the latter is only of importance for times $t = nT$, while the system evolves freely between kicks. Resorting to standard methods [11] the system evolution can then be obtained via a combination of free evolution and kick interaction:

$$\psi_i(\theta_1, \theta_2, nT) = U^n(T) \psi_i(\theta_1, \theta_2, 0), \quad i = B, HB, \tag{7}$$

with

$$U(T) = U_\delta(T) U_f(T) = e^{-i \sum_{j=1}^2 H_j^{kick} T} e^{-i \sum_{j=1}^2 H_j^{free} T}.$$

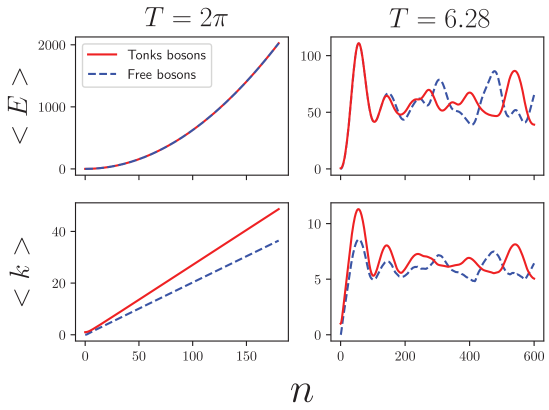

Fig. 1: Behavior of the mean energy $\langle E \rangle$ and the mean relative momentum $\langle k \rangle$ as a function of the number of kicks in the resonant $T = 2\pi$ and quasi-resonant $T = 6.28$ regimes.

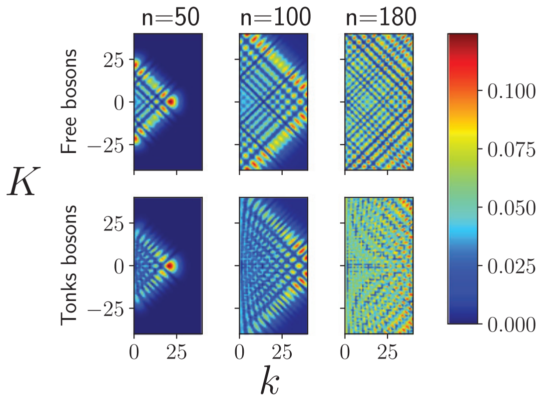

Fig. 2: Density of states in the resonant regime ($T = 2\pi$) in terms of the center of mass momentum K and the relative momentum k as a function of the number of kicks n.

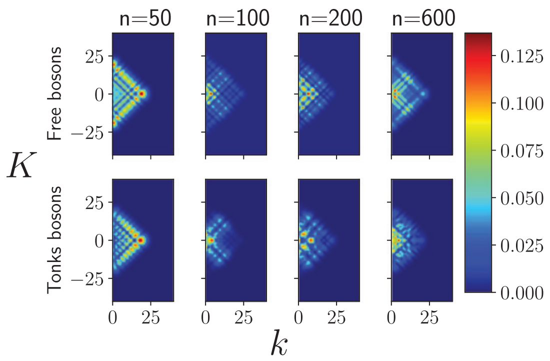

Fig. 3: Density of states in the quasi-resonant regime ($T = 6.28$) in terms of the center of mass momentum K and the relative momentum k as a function of the number of kicks n.

The quantum resonant regime is a constructive interference phenomenon, characterized by the fact that the wave function acquires the same phase for each kick, namely the effect of kicks adds coherently. This regime can be obtained choosing $T = 2\pi n$ in eq. (7): the evolution operator becomes $U(T)^n = U_\delta^n(T)$, the free evolution operator $U_f(T)$ being identity one.

On the other hand, when the frequency of the kicks is not fine tuned exactly at resonance, decoherence effects generate non-diagonal terms in the free evolution operator U_f, which dramatically change the dynamics of the system. In fact, as we will see below, a localization regime appears above a certain critical time that depends on the strength of the kick.

Results. – Since we compare the resonant and quasi-resonant regimes, we consider the period varying in a neighborhood of $T = 2\pi$. Following the current literature [25] we resort to various quantities whose behaviour is a clear indicator of the resonance, both in the configuration and momentum space.

Mean energy and mean relative momentum. In fig. 1 the behavior of the mean energy $\langle E \rangle$ and the relative momentum $\langle k \rangle$ is reported as a function of the number of kicks, in the resonant $T = 2\pi$ and quasi-resonant $T = 6.28$ regimes. In the former both systems have a diffusive behaviour, highlighted by the ballistic evolution of the mean energy [11] and the linear growth of the mean relative momentum, the different slope in the two cases due to the different interaction strength between particles. Regarding the quasi-resonant regime, after the mean energy and the mean relative momentum have reached a maximum, preceded by a diffusive behaviour, the system enters in an oscillating phase, which is a typical signature of a localized regime [11]. The qualitative trend is the same for both interaction limits.

Center of mass and relative momentum. A more insightful analysis of the resonant and localized phases can be obtained considering the expansion coefficients of the wave function expressed in the basis labeled by the momenta (K, k) [25]:

$$|\psi_i(nT)\rangle = \sum_{K,k} C^{K,k}(nT)|\psi_i^{K,k}(0)\rangle, \quad i = B, HB. \tag{8}$$

Figure 2 shows the amplitude distribution $|C^{K,k}(nT)|$ for different kicks in the resonant regime. We observe that the mean value of the center of mass momentum K remains localized around zero during the evolution, while there is a strong delocalization for the relative momentum k, since the system expands along the k-direction.

In fig. 3 we observe for the quasi-resonant case a crossover from the resonant regime to the localized regime. Indeed, after an initial diffusive phase, characterized by a rapid growth of the amplitude distribution along the k-direction (leftmost panel at $n = 50$ kicks), the system localizes (as shown in the remaining panels), in an interaction strength depending way. In fact the free-bosons system localizes faster: the momenta K and k assume almost the same values. Furthermore, in the long time limit ($n = 600$), they localize towards the ground state ($K = k = 0$). On the other hand in the Tonks case, after many kicks ($n \geq 100$), while the momentum K localizes

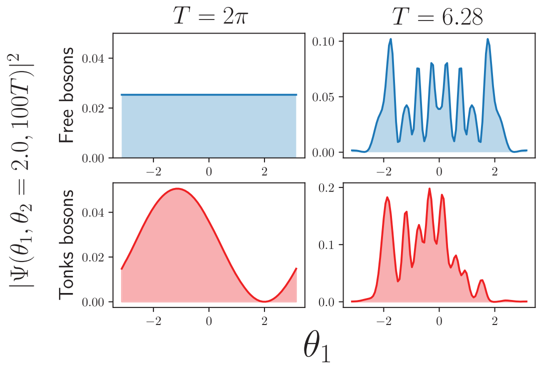

Fig. 4: Probability density in configuration space as a function of the angular position θ_1 of the first particle, the second particle being fixed at $\theta_2 = 2.0$ rad. The number of kicks is fixed to $n = 100$. The behaviour is presented both for the resonant $T = 2\pi$ and quasi-resonant $T = 6.28$ regime.

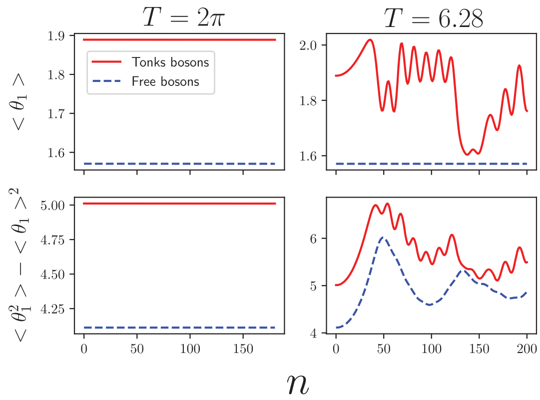

Fig. 5: Behaviour of the mean position $\langle \theta_1 \rangle$ and the variance of the first particle as a function of the number of kicks in the resonant $T = 2\pi$ and quasi-resonant $T = 6.28$ regimes.

at zero, the momentum k expands in the region of excited states. This effect, due to the interaction, is observed also by Qin *et al.* [25], in the Tonks limit the expansion in the k-direction being more restrained than the finite contact interaction case, because the hardcore bosons in momentum space behave as free fermions.

Probability density function and one-particle mean position. The effect of resonant regime in configuration space can be conveniently analyzed through the form of the probability density function fixing one of the rotors (for example the value $\theta_2 = 2.0$ in fig. 4), the mean value position and its variance (fig. 5). Indeed the former can be obtained straightforwardly for both interaction regimes through eqs. (4), (5) and (7). In the case of free bosons it can be shown that the function assumes a constant value ($|\psi(\theta_1, 2, t)|^2 = |\psi(\theta_1, 2, 0)|^2 = 1/(4\pi^2)$) (see footnote [1]) as reported in the left upper panel of fig. 4, while for hardcore bosons, due to the strong interaction, the probability density presents an oscillating behaviour.

In the quasi-resonant regime (the right panels of fig. 4), the form of the probability density function is a clear signature of the localization regime: indeed we observe that both systems are delocalized over all space, as expected since they are localized in momentum space. Furthermore, the effect of the interaction manifests itself through the symmetry: the probability density is symmetric for free bosons, due to the strong attraction between particles, while for Tonks particles it is asymmetric, due to the strong contact interaction.

Finally, in fig. 5 we plot the mean value position and its variance as a function of the number of kicks for one rotor, the other being constrained in the interval $[0, \pi]$. We observe that in the resonance case both the mean value position and its variance are constant, due to the coherently interference of kicks. The effect of the different interaction manifests in the different value of $\langle \theta_1 \rangle$: for Tonks particles the effect of the repulsion translates in a different value of the mean. Furthermore, the constant value for the variance in both cases (left lower panel of fig. 5) indicates the presence of a diffusive regime in momentum space.

A different behaviour for the mean value in instead observed for the two systems in the quasi-resonant regime (right upper panel of fig. 5): while the mean value of the free bosons is constant, the value of the hardcore bosons, after a ballistic initial phase (the parabolic trend), is a rapidly oscillating function. Consequently we deduce that the strong interaction makes the system more sensitive to the change of the kick period with respect to the free-particle ones, that instead localize rapidly. We thus find that the non-interacting boson rotors are more robust against localization phenomenon with respect to the strongly interacting one.

Finally the variance in the quasi-resonant regime (the right lower panel of fig. 5) presents a damped oscillating trend, that corresponds to a semi-diffusive regime. In particular the Tonks position variance presents another oscillation frequency, that is a clear signature of the strongly contact interaction.

Scaling of the mean energy and the mean relative momentum. In fig. 6 we plot the mean energy and the mean relative momentum both for the free bosons and the Tonks particles as a function of $\epsilon = 2\pi - T$, where T is the kick period taken close to resonance.

We observe that both the mean energy and the relative momentum diverge when the system approaches the resonant regime (the limit $\epsilon \to 0$). On the other hand, for $\epsilon \to \infty$, the mean values converge and both systems localize. We thus find that the behaviour of the plotted quantities resembles a localization length for both systems. Indeed this is analogous for systems that interact with a disordered potential: the resonant regime is equivalent to a weakly disordered system (diffusive regime), characterized by the parabolic growth of the mean energy, and by the linear trend of the mean relative momentum. In contrast the choice of a kick period closed to 2π

[1]The wave function is normalized with respect to the variables $\theta_1, \theta_2 \in [0, 2\pi]$.

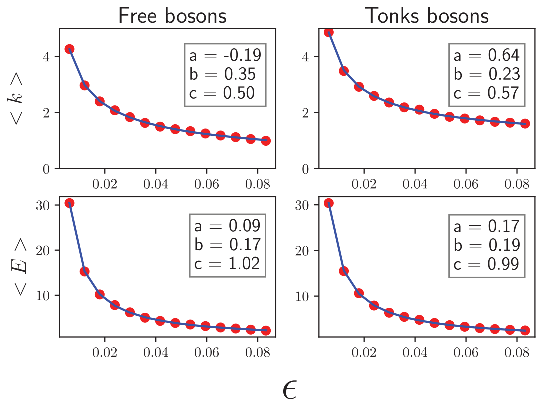

Fig. 6: Scaling of the mean energy and the relative momentum as a function of $\epsilon = 2\pi - T$, where the period $T \in [6.2, 6.277]$. The data was fitted with the function $f = a + b/\epsilon^c$, $a, b,$ and c being the fitting parameters. The error is much smaller than the values obtained and it is not reported.

(non-resonant regime) corresponds to the increase of the disordered strength, that forces the particles not to exchange energy among each other, and the system is localized. This result is a direct evidence of the kicked rotor mapping to the Anderson localization [26].

Conclusions and perspectives. – By analyzing the dynamical behaviour of two quantum atomic kicked rotors we have analyzed the fate of the many-body localization in the presence of strong correlations. We have characterized the behaviour of the free particles *vs.* the interacting ones by looking at the momentum amplitude distribution and the probability density in the configuration space. The former shows that states with small center of mass and relative momentum are preferably occupied by free bosons while the interaction mainly affects the occupation of the states at higher momenta. The results for the dynamical behavior in the configurational space is consistent with that in the momentum space and we have shown that the mean position of a single particle (having fixed the position of the other particle) grows first quadratically in time, then the system dynamically localizes. Moreover, our results show that, irrespectively of the strong interaction, the localization regime persists also for hardcore bosons in agreement with the results of [25] where two atomic kicked rotors with finite contact interaction were considered.

In experimental applications the regime close to resonance can be easily generated by a controlled detuning of the pulse train period from the revival time when acting on a gas of ultracold atoms and the interaction among atoms can be tuned by a Feshbach resonance. In particular one can develop a complex optical setup capable of generating high-intensity trains of femtosecond pulses, that are focused onto a vacuum chamber containing a constant flow of molecules (such as nitrogen gas) or atoms.

To fully determine the fate of dynamical localization, we need to consider many interacting atoms and study the highly complex case of many-body interactions for quantum kicked rotors. While this is still a challenging task, we refer to the Bose-Hubbard model studied in [32] where the persistence of a dynamical localization is shown. Finally, the robustness of a localized regime makes the kicked rotors systems useful for precision quantum sensing and metrology applications.

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REFERENCES

[1] Bloch I., Dalibard J. and Nascimbene S., *Nat. Phys.*, **8** (2014) 267.
[2] Georgescu I. M., Ashhab S. and Nori F., *Rev. Mod. Phys.*, **86** (2014) 153.
[3] Moore F. L., Robinson J. C., Bharucha C. F., Sundaram B. and Raizen M. G., *Phys. Rev. Lett.*, **75** (1995) 4598.
[4] Dana I., Ramareddy V., Talukdar I. and Summy G. S., *Phys. Rev. Lett.*, **100** (2008) 024103.
[5] Sadgrove M., Wimberger S., Parkins S. and Leonhardt R., *Phys. Rev. Lett.*, **94** (2005) 174103.
[6] Ma Z. Y., d'Arcy M. B. and Gardiner S. A., *Phys. Rev. Lett.*, **93** (2004) 164101.
[7] Cryan James P., Bucksbaum Philip H. and Coffee Ryan N., *Phys. Rev. A*, **80** (2009) 063412.
[8] Zhdanovich S., Bloomquist C., Floß J., Averbukh I. Sh., Hepburn J. W. and Milner V., *Phys. Rev. Lett.*, **109** (2012) 043003.
[9] Zahedpour S., Wahlstrand J. K., and Milchberg H. M., *Phys. Rev. Lett.*, **112** (2014) 143601.
[10] Wimberger S., Guarneri I. and Fishman S., *Nonlinearity*, **16** (2003) 1381.
[11] Reichl L. E., *The Transition to Chaos: Conservative Classical Systems and Quantum Manifestations* (Springer-Verlag, New York) 2004.
[12] Chirikov B. V., Izrailev F. M. and Shepelyansky D. L., *Sov. Sci. Rev. C*, **2** (1981) 209.
[13] Casati G., Chirikov B., Izraelev F. and Ford J., *Lect. Notes Phys.*, Vol. **93** (Springer, Berlin, Heidelberg) 1979, pp. 334–352.
[14] Fishman S., Grempel D. R. and Prange R. E., *Phys. Rev. Lett.*, **49** (1982) 509.
[15] Rajak Atanu, Citro Roberta and Dalla Torre Emanuele G., *J. Phys. A: Math. Theor.*, **51** (2018) 465001.
[16] Schomerus H. and Lutz E., *Phys. Rev. A*, **77** (2008) 062113.
[17] Cohen D., *J. Phys. A*, **27** (1994) 4805.
[18] Drittich T. and Graham R., *Europhys. Lett.*, **4** (1987) 263.
[19] Brambila D. S. and Fratalocchi A., *Sci. Rep.*, **3** (2013) 2359.
[20] Artuso R. and Rebuzzini L., *Phys. Rev. E*, **66** (2002) 017203.

[21] KLAPPAUF B. G., OSKAY W. H., STECK D. A. and RAIZEN M. G., *Phys. Rev. Lett.*, **81** (1998) 1203; **82** (1999) 242.

[22] GADWAY B., REEVES J., KRINNER L. and SCHNEBLE D., *Phys. Rev. Lett.*, **110** (2013) 190401.

[23] TOLOUI B. and BALLENTINE L. E., arXiv:0903.4632v2 (2009).

[24] KESER A. C., GANESHAN S., REFAEL G. and GALITSKY V., *Phys. Rev. B*, **94** (2016) 085120.

[25] QIN P., ANDREANOV A., PARK H. C. and FLACH S., *Sci. Rep.*, **7** (2017) 41139.

[26] NOTARNICOLA S., IEMINI F., ROSSINI D., FAZIO R., SILVA A. and RUSSOMANNO A., *Phys. Rev. E*, **97** (2018) 022202.

[27] TOIKKA L. A. and ANDREANOV A., arXiv:1901.09362v1 (2009).

[28] BAYFIELD J. E., CASATI G., GUARNERI I. and SOKOL D. W., *Phys. Rev. Lett.*, **63** (1989) 364.

[29] KOCH P. M., MOORMAN L., SAUER B. E., GALVEZ E. J., VAN LEEUWEN K. A. H. and RICHARDS D., *Phys. Scr.*, **T26** (1989) 51.

[30] MOORE F. L., ROBINSON J. C., BHARUCHA C., WILLIAMS P. E. and RAIZEN M. G., *Phys. Rev. Lett.*, **73** (1994) 2974.

[31] YUKALOV V. I. and GIRARDEAU M. D., *Laser Phys. Lett.*, **2** (2005) 375.

[32] AGARWALA A. and SEN D., *Phys. Rev. B*, **95** (2017) 014305.