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Black holes in the Einstein-Born-Infeld-Weyl gravity

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Abstract – By basing on the neutral (Schwarzschild and non-Schwarzschild) black holes in Einstein-Weyl gravity, here we construct asymptotically flat black holes of Einstein-Weyl action coupled to a Born-Infeld (BI) gauge field by holding the same horizon radius. Later, we discuss the thermodynamic properties of these black holes in detail, and show that these obey the first law of thermodynamics of black hole.

Introduction. – Though the general relativity (GR) has been extensively tested at the highest achievable experimental precision up to date, gravity is not a renormalizable quantum field theory from the theoretical viewpoint. A possible attempt to solve the problem of the non-renormalizability of general relativity is to include higher-order corrections that become important at higher energy [1]. In four-dimensional spacetime, the most general theory up to the second order in curvature takes the following form [2,3]:

$$\mathcal{I} = \int \mathrm{d}^4x\sqrt{-g}[\gamma R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \eta R^2] \tag{1}$$

without any matter field. Here α, η and γ are constants, and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. In a theory of gravity, black holes can be viewed as the most fundamental objects, and provide powerful probes for studying some of the more subtle global aspects of the theory. In pure gravity or with traceless matter stress tensor, the no-go theorem discussed in refs. [2,3] demonstrated that R must vanish for a black hole, and hence the ηR^2 term has no contribution to the equations of motion. With this setting, the non-Schwarzschild black hole (NSBH) solutions with $\eta = 0$ were obtained in four [2–4] and higher [5] dimensional Einstein-Weyl gravity, even the generalizations of AdS [6,7] and charged solutions [8,9] in four-dimensional Einstein-Weyl gravity. The Hawking radiation in the vicinity of non-Schwarzschild black hole was discussed in ref. [10]. The quasinormal modes of this NSBH have been also investigated under the test scalar field perturbation [11,12].

Besides the curvature terms, one would also expect higher derivative gauge field contributions to the action. Its Lagrangian $\mathcal{L}(\mathcal{F})$ is

$$\mathcal{L}(\mathcal{F}) = 4\beta^2\left(1 - \sqrt{1 + \frac{F^{\mu\nu}F_{\mu\nu}}{2\beta^2}}\right), \tag{2}$$

where the constant β is the Born-Infeld (BI) parameter, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is electromagnetic tensor field and A_μ is the vector potential. The BI theory was originally introduced to get a classical theory of charged particles with finite self-energy [13]. Hoffmann [14] was the first one in relating general relativity and the BI electromagnetic field, and derived a solution of the Einstein equations for a point-like BI charge, which is devoid of the divergence of the metric at the origin that characterizes the Reissner-Nordström (RN) solution [15]. In the Einstein gravity coupled to a BI electromagnetic field, asymptotically flat BI black holes have been presented in three- [16] and four-dimensional [17,18] spacetime. Later, the dynamical stability and thermodynamical properties of BI black holes have been discussed in refs. [19–21]. Notice that when the BI term is added, its stress tensor is not traceless, and hence R will not be zero. In this case the ηR^2 term will have non-trivial effect on the solution. In this paper, we construct BI black holes based on the two "seed" solutions (Schwarzschild black hole (SBH) and non-Schwarzschild black hole (NSBH)) in the Einstein-Born-Infeld-Weyl gravity. Hence, we still choose $\eta = 0$ so that this new action can uncover the neutral black holes (SBH and NSBH) when the BI term vanishes. We will further

$$\frac{2h}{r^2}\left[1+\frac{1}{f}\left(1+4r^2\beta^2-rf'-\frac{2\beta(Q^2+2r^4\beta^2)}{\sqrt{Q^2+r^4\beta^2}}\right)\right]-\frac{h'^2}{2h}+\frac{(4f+rf')h'}{2rf}+h''=0,$$
$$(2h-rh')f''-\frac{3hf'^2}{2f}-f'\left(-h'-\frac{rh'^2}{2h}+\frac{h}{rf}\left(2+8r^2\beta^2-2f-\frac{4\beta(Q^2+2r^4\beta^2)}{\sqrt{Q^2+r^4\beta^2}}\right)\right)$$
$$+\frac{fh'^2(rh'-3h)}{2h^2}+h\left(\frac{12+16r^2\beta^2}{3r^2}-\frac{3+16\alpha\beta^2+r^2(6+64\alpha\beta^2)\beta^2}{3\alpha f}-\frac{4f}{r^2}+\frac{1}{\alpha}\right)$$
$$-\frac{8r^6\beta^5}{3(Q^2+r^4\beta^2)^{3/2}}+\frac{2\beta h}{3r^2f}\left(\frac{4(r^4\beta^2+4r^6\beta^4-3Q^2f)}{\sqrt{Q^2+r^4\beta^2}}+\frac{\sqrt{Q^2+r^4\beta^2}(4\alpha+r^2(3+16\alpha\beta^2))}{\alpha}\right)=0,$$
$$A_t'+\frac{Q}{\sqrt{Q^2/\beta^2+r^4}}\sqrt{\frac{h(r)}{f(r)}}=0, \tag{8}$$

discuss the thermodynamic properties of these BI black holes, and verify the first law of thermodynamics of black holes.

This paper is organized as follows. In the next section, we introduce the BI field into the action of Einstein gravity with additional quadratic curvature terms and derive an asymptotically flat black hole solution in Einstein-Born-Infeld-Weyl gravity. Then, some related thermodynamic properties of these BI black holes will be explored in the third section. We end the paper with concluding remarks in the fourth section.

Solutions in Einstein-Born-Infeld-Weyl gravity. – The action of Einstein-Weyl gravity in the presence of a nonlinear BI electromagnetic field can be written as

$$\mathcal{I}=\int \mathrm{d}^4x\sqrt{-g}[R-\alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}+\kappa\mathcal{L}(\mathcal{F})]. \tag{3}$$

In the limit $\beta\to\infty$, $\mathcal{L}(\mathcal{F})$ (2) reduces to the standard Maxwell form

$$\mathcal{L}(\mathcal{F})=-F^{\mu\nu}F_{\mu\nu}+\mathcal{O}(F^4). \tag{4}$$

By varying the action (3) with regard to the gauge field A_μ and metric $g_{\mu\nu}$, these corresponding equations of motion can be written as

$$\begin{aligned} R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-4\alpha B_{\mu\nu}-2\kappa T_{\mu\nu}&=0,\\ \partial_\mu\left(\frac{\sqrt{-g}F^{\mu\nu}}{\sqrt{1+\frac{F^2}{2\beta^2}}}\right)&=0, \end{aligned} \tag{5}$$

where the trace-free Bach tensor $B_{\mu\nu}$ and the energy-momentum tensor of the BI field $T_{\mu\nu}$ are defined as

$$\begin{aligned} B_{\mu\nu}&=\left(\nabla^\rho\nabla^\sigma+\frac{1}{2}R^{\rho\sigma}\right)C_{\mu\nu\rho\sigma},\\ T_{\mu\nu}&=\frac{F_{\mu\lambda}F_\nu{}^\lambda}{\sqrt{1+\frac{F_{\mu\lambda}F_\nu{}^\lambda}{2\beta^2}}}+\frac{1}{4}g_{\mu\nu}\mathcal{L}(\mathcal{F}). \end{aligned} \tag{6}$$

We assume a static and spherically symmetric metric ansatz

$$\mathrm{d}s^2=-h(r)\mathrm{d}t^2+\frac{1}{f(r)}\mathrm{d}r^2+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right), \tag{7}$$

and substitute this ansatz into the field equations (5)

see eq. (8) *above*

where we set $\kappa=1$, the prime ($'$) denotes differentiation with respect to r, and Q denotes electric charge.

If $Q\to 0$, the equations of motion (8) reduce to those found in refs. [2,3], and recover the same solutions: the Schwarzschild black holes (SBH) and the non-Schwarzschild black holes (NSBH). It is interesting to note that a BI black hole in the general relativity (GR) usually possesses more than one horizon because of the electric charge in the metric. However, this BI metric was not a solution in this Einstein-Born-Infeld-Weyl gravity ($\alpha\neq 0$). In addition, that the spacetime has only one horizon will make the numerical calculation easier. Therefore, we regard the neutral black holes (SBH and NSBH), rather than the BI metric in the GR as the background solutions to construct BI black holes in the Einstein-Born-Infeld-Weyl gravity. From now on we shall take $\alpha=1/2$ without loss of generality.

Moreover, those numerical BI black holes in the Einstein-Born-Infeld-Weyl gravity can be divided into two groups: Group I is a charged generalization of the higher derivative curvature for SBH; Group II is a higher derivative curvature charged generalization of NSBH. The NSBH has been constructed in the Einstein-Weyl gravity [2,3,9], where there exist a bound of $0.363<r_0<1.143$ for a horizon radius r_0 of NSBH when taking $\alpha=1/2$. Here 0.363 and 1.143 correspond to the disappearance of temperature T and mass M of NSBH, respectively.

We assume $h(r)=f(r)e^{-2\delta(r)}$ and $f(r)=1-\frac{2m(r)}{r}$, and take the expansions of $m(r)$, $\delta(r)$ and $A_t(r)$ around the event horizon r_0 of the corresponding neutral

$$
\begin{aligned}
m_2 &= \frac{3r_0(\beta\sqrt{Q^2+r_0^4\beta^2}-m_1-r_0^2\beta^2)}{8\alpha(2m_1-1)} - \frac{\beta\left(4r_0^4\beta^2(2r_0^2\beta^2+3m_1-1)+Q^2(4r_0^2\beta^2+4m_1-1)\right)}{2r_0(2m_1-1)\sqrt{Q^2+r_0^4\beta^2}} \\
&+ \frac{Q^2r_0^3\beta^3}{2(Q^2+r_0^4\beta^2)^{3/2}} + \frac{m_1(6r_0\beta^2-1+2m_1)+4r_0^4\beta^4}{r_0(2m_1-1)} - \frac{Q^2(Q^2-4r_0^2)\beta^2}{2r_0(2m_1-1)(Q^2+r_0^4\beta^2)}, \\
\delta_1 &= \frac{r_0(\beta\sqrt{Q^2+r_0^4\beta^2}-m_1-r_0^2\beta^2)}{2\alpha(2m_1-1)^2} - \frac{8\beta^2r_0(m_1+2r_0^2\beta^2)}{3(2m_1-1)^2} - \frac{2Q^4\beta^2}{3r_0(2m_1-1)^2(Q^2+r_0^4\beta^2)} \\
&+ \frac{8r_0\beta^3(Q^2+m_1r_0^2+2r_0^4\beta^2)}{3(2m_1-1)^2\sqrt{Q^2+r_0^4\beta^2}} + \frac{2Q^2\beta(Q^2+2m_1r_0^4\beta^2)}{3(2m_1-1)^2(Q^2+r_0^4\beta^2)^{3/2}}, \\
A_{t1} &= \frac{e^{-\delta_0}Q\beta}{\sqrt{Q^2+r_0^4\beta^2}}. \qquad (10)
\end{aligned}
$$

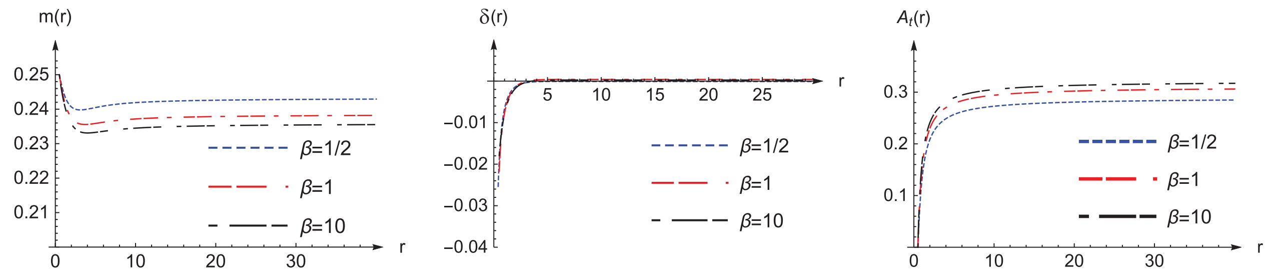


Fig. 1: Numerical solutions of Group I for $m(r)$, $\delta(r)$ and $A_t(r)$ with $Q = 0.16$ and $r_0 = 0.5$. The values of expansion coefficients (δ_0, m_1) are $(-0.027, -0.010)$, $(-0.022, -0.015)$ and $(-0.019, -0.018)$ for $\beta = 1/2, 1$ and 10, respectively. Moreover, the mass function $m(r_0)$ equals $\frac{r_0}{2} = 0.25$.

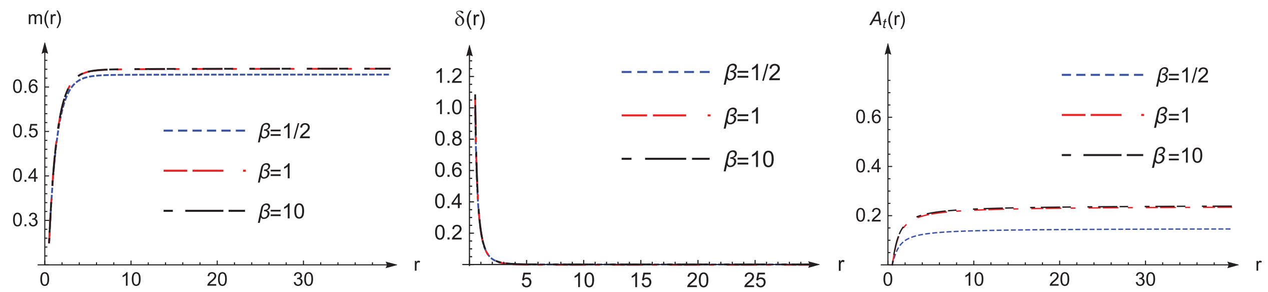


Fig. 2: Numerical solutions of Group II for $m(r)$, $\delta(r)$ and $A_t(r)$ with $Q = 0.16$ and $r_0 = 0.5$. The values of expansion coefficients (δ_0, m_1) are $(0.983, 0.399)$, $(1.064, 0.410)$ and $(1.086, 0.410)$ for $\beta = 1/2, 1$ and 10, respectively. Moreover, the mass function $m(r_0)$ equals $\frac{r_0}{2} = 0.25$.

(Schwarzschild and non-Schwarzschild) black holes

$$
\begin{aligned}
m(r) &= \frac{r_0}{2} + m_1(r-r_0) + m_2(r-r_0)^2 + \dots, \\
\delta(r) &= \delta_0 + \delta_1(r-r_0) + \delta_2(r-r_0) + \dots, \\
A_t(r) &= A_{t1}(r-r_0) + A_{t2}(r-r_0)^2 + \dots, \qquad (9)
\end{aligned}
$$

and substituting these expansions into (5), the coefficients δ_i and A_{ti} for $i = 1$, and m_i for $i = 2$ can be solved in terms of the three non-trivial free parameters r_0, m_1 and δ_0. For example, m_2, A_{t1} and δ_1 can be obtained as

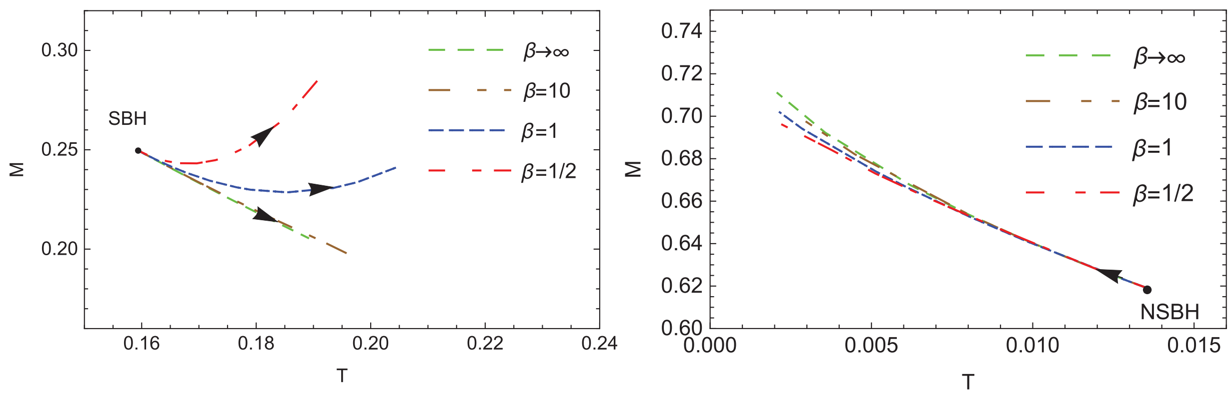
see eq. (10) *above*

At the radial infinity ($r \to \infty$), the metric functions and vector potential can be expanded in power series, this time in terms of $1/r$. Demanding that the metric components reduces to those of the asymptotically flat Minkowski spacetime

$$
m(r) = M - \frac{Q^2}{2r} + \dots, \quad \delta(r) = \frac{2\alpha Q^2}{r^4} + \dots,
$$
$$
A_t(r) = \Phi - \frac{Q}{r} + \dots, \qquad (11)
$$

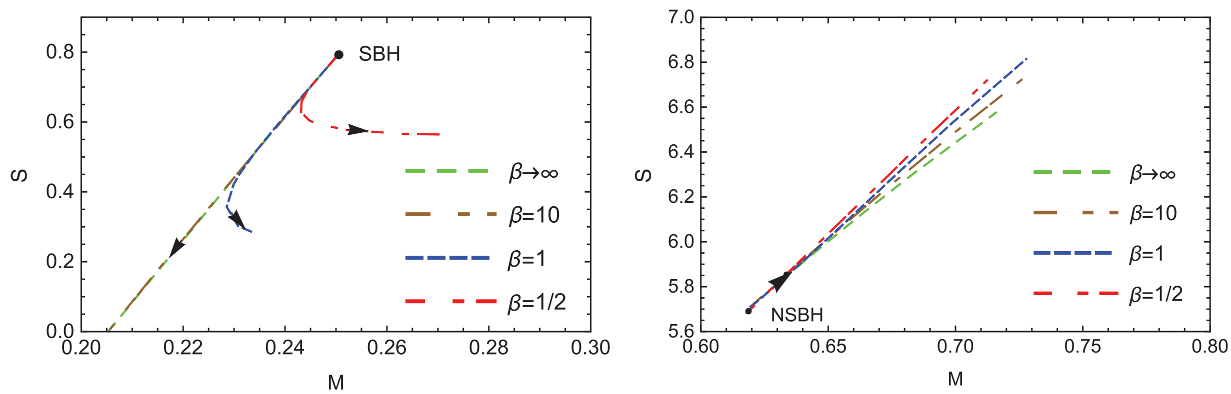
where M and Q are associated with the mass and charge of the black hole, and Φ is the electric potential.

Adopting the expansions (9) up to the $(r-r_0)^6$ order, we assume the initial values of the parameters δ_0, m_1 at a radius $r_i = r_0 + \frac{1}{1000}$ just outside the horizon r_0, and then use numerical routines in Mathematica to integrate the equations out to large radius, so that these interpolation



(a) Group I (b) Group II

Fig. 3: M *vs.* T for BI and charged ($\beta \to \infty$) black holes with the same horizon radius $r_0 = 0.5$. Here the stating points (black spots) in (a) and (b) correspond to $T = 0.159, M = 0.25$ (SBH) and $T = 0.0136, M = 0.619$ (NSBH) with $r_0 = 0.5$, respectively. The arrow indicates the increase of charge Q.



(a) Group I (b) Group II

Fig. 4: S *vs.* M for BI and charged ($\beta \to \infty$) black holes with the same horizon radius $r_0 = 0.5$. Here the stating points (black spots) in (a) and (b) correspond to $M = 0.25, S = 0.785$ (SBH) and $M = 0.619, S = 5.71$ (NSBH) with $r_0 = 0.5$, respectively. The arrow indicates the increase of charge Q.

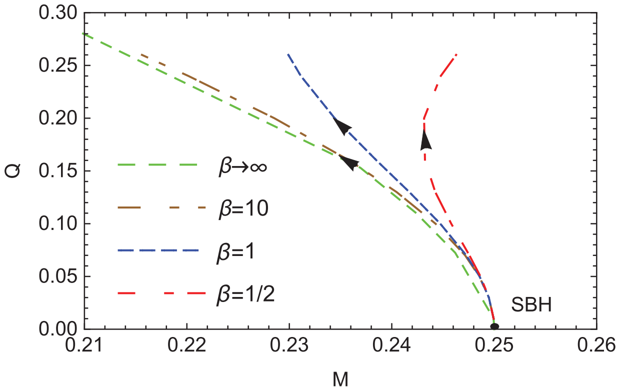
functions of metric functions $m(r)$ and $\delta(r)$ and vector potential $A_t(r)$ satisfy the boundary condition (11). Finally, we can obtain the numerical solutions for $m(r)$, $\delta(r)$ and $A_t(r)$ with proper initial values of δ_0 and m_1 for different β. For example, we set $\alpha = 1/2$, $Q = 0.16$ and $r_0 = 0.5$ to derive numerical solutions of metric functions $m(r)$ and $\delta(r)$, and vector potential $A_t(r)$ for different values of β in the Groups I and II as shown in figs. 1 and 2.

Thermodynamic properties of Born-Infeld black holes. – Having established the existence of the BI black holes, it is instructive to investigate the thermodynamic properties of BI black holes in the Einstein-Born-Infeld-Weyl gravity.

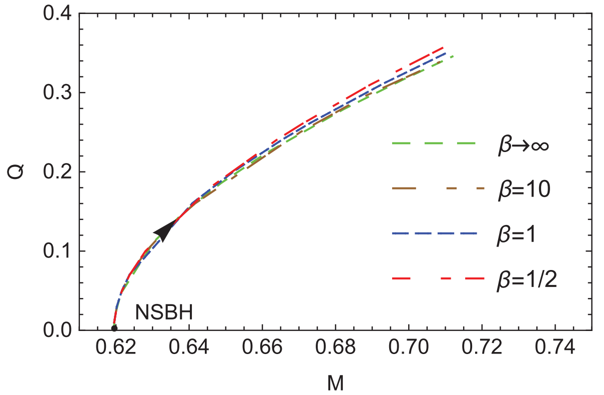
Starting from SBH and NSBH with fixed horizon radius $r_0 = 0.5$, we can separately construct a sequence of BI black hole solutions for $\beta = \infty$, 10, 1 and 1/2 with increasing Q, and then collect the numerical results for these BI black holes in Groups I and II. The relationships between the masses M and temperatures T of these black holes are shown in fig. 3, where the stating points (black spots) correspond to the values of thermodynamical quantities like mass M, temperature T and entropy S of SBH and NSBH with horizon radius $r_0 = 0.5$ [2,3,9], which take the following values $T = 0.159, M = 0.25, S = 0.785$ (SBH) and $T = 0.0136, M = 0.619, S = 5.71$ (NSBH), respectively. From fig. 3, we can find that the $M \sim T$ curve of the BI black hole with larger values of β approaches that of the charged black hole ($\beta \to \infty$).

Since we are dealing with a higher-derivative theory, the entropy is not simply given by one quarter of the area of the event horizon, and equals $S = \pi r_0^2 + 8\pi\alpha m_1$. We then find the entropy S of these BI black holes starting from Schwarzschild and non-Schwarzschild black holes, as a function of mass M with the increase of charge Q for the Groups I and II as shown in figs. 4 and 5.

Now we discuss the first law of thermodynamics of BI black holes. In order to evaluate this law, the discrete values of thermodynamical quantities of mass M, temperature T, entropy S and and potential Φ in the two groups of BI black holes with different charge Q are shown in



(a) Group I



(b) Group II

Fig. 5: Q *vs.* M for BI and charged ($\beta \to \infty$) black holes with the same horizon radius $r_0 = 0.5$. Here the stating points (black spots) in (a) and (b) correspond to $M = 0.25$ (SBH) and $M = 0.619$ (NSBH) with $r_0 = 0.5$, respectively. The arrow indicates the increase of charge Q.

Table 1: The discrete values of thermodynamical quantities M, T, S and Φ for BI black holes with $r_0 = 0.5$ and $\beta = 10$ in Group I (left) and Group II (right).

No.	Q	M	S	T	Φ
1	1/100	0.24994	0.78440	0.15919	0.01992
2	3/100	0.24947	0.77644	0.15949	0.05978
3	5/100	0.24854	0.76060	0.16008	0.09968
4	7/100	0.24716	0.73703	0.16097	0.13966
5	10/100	0.24427	0.68767	0.16283	0.19982
6	13/100	0.24047	0.62241	0.16530	0.26031
7	16/100	0.23584	0.54241	0.16835	0.32118
8	20/100	0.22853	0.41514	0.17324	0.40308
9	24/100	0.22015	0.26731	0.17898	0.48590

No.	Q	M	S	T	Φ
1	1/100	0.61953	5.71125	0.013461	0.015488
2	3/100	0.620252	5.71834	0.01333	0.046434
3	5/100	0.62167	5.73247	0.013079	0.07729
4	7/100	0.62380	5.75345	0.01270	0.10799
5	10/100	0.62827	5.79728	0.01193	0.15364
6	13/100	0.63423	5.85501	0.01095	0.19863
7	16/100	0.64159	5.92537	0.00978	0.24277
8	20/100	0.65345	6.03633	0.00804	0.30000
9	24/100	0.66748	6.16348	0.00621	0.35499

table 1. On the left (BI black hole in Group I), forward differences of mass M entropy S and charge Q can be written as

$$\begin{aligned}
\Delta M &\equiv \frac{M[i+2]-M[i]}{2}, \\
&= \{-0.000699, -0.001157, -0.002135, -0.003344, \\
&\quad -0.004215, -0.005968, -0.007843\}, \\
\Delta S &\equiv \frac{S[i+2]-S[i]}{2}, \\
&= \{-0.011899, -0.019705, -0.036463, -0.057312, \\
&\quad -0.072630, -0.103632, -0.137553\}, \\
\Delta Q &\equiv \frac{Q[i+2]-Q[i]}{2} = \{0.02, 0.02, 0.025, 0.03, 0.03, \\
&\quad 0.035, 0.04\}, \quad i = 1, \ldots, 7.
\end{aligned} \tag{12}$$

Then the expression $\mathrm{d}M - (T\mathrm{d}S + \Phi\mathrm{d}Q)$ in the form of discrete points is given by

$$\begin{aligned}
&\Delta M[i] - (T[i+1]\cdot\Delta S[i] + \Phi[i+1]\cdot\Delta Q[i]) \\
&= \{-2.6*10^{-6}, -3.82*10^{-6}, -0.00024, \\
&6.8*10^{-6}, 0.000019, -0.00023, 0.00013\}, \quad i = 1, \ldots, 7.
\end{aligned} \tag{13}$$

Similarly, we can also calculate $\mathrm{d}M - (T\mathrm{d}S + \Phi\mathrm{d}Q)$ by using the finite difference method

$$\begin{aligned}
&\Delta M[i] - (T[i+1]\cdot\Delta S[i] + \Phi[i+1]\cdot\Delta Q[i]) \\
&= \{2.57*10^{-7}, 1.27*10^{-6}, 0.0001888, -1.71*10^{-6}, \\
&-2.0*10^{-6}, 0.000228, -0.0000146\}, \quad i = 1, \ldots, 7,
\end{aligned} \tag{14}$$

from table 1, right (BI black hole in Group II). Thus the thermodynamical quantities of BI black holes are seen to obey the first law $\mathrm{d}M = T\mathrm{d}S + \Phi\mathrm{d}Q$ to quite a high precision.

Concluding remarks. – In this paper, we have obtained asymptotically flat Born-Infeld black hole solutions for different values of $\beta = \infty$, 10, 1 and 1/2 in Einstein-Born-Infeld-Weyl gravity. Here $\beta \to \infty$ corresponds to the charged black hole in Einstein-Maxwell-Weyl gravity. Moreover, all BI black holes hold the same horizon radius $r_0 = 0.5$ and have been separately constructed starting from Schwarzschild and non-Schwarzschild black holes with increasing of Q. We also discussed thermodynamic proprieties of these BI black holes, and collected the related values for the thermodynamic quantities M, S, Q, T and Φ, so that we found the BI black holes obey the first law of thermodynamics of black holes by using the finite difference method.

Recently, the stability of non-Schwarzschild black holes has been discussed in refs. [11,12]. It is interesting to explore the stability of BI black holes in the Einstein-Born-Infeld-Weyl theory. In addition, (Anti-) de Sitter (AdS/dS) charged black hole solutions in Einstein-Maxwell-Weyl gravity have been also constructed in ref. [22]. We will also extend to recover the AdS/dS BI black hole solutions in the Einstein-Born-Infeld-Weyl theory.

* * *

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