A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

PERSPECTIVE

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To cite this article: D. Sokolovski 2019 *EPL* **128** 50001

A Letters Journal Exploring the Frontiers of Physics

December 2019

EPL, **128** (2019) 50001
doi: 10.1209/0295-5075/128/50001

www.epljournal.org

**Perspective**

# A minimalist's view of quantum mechanics

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received 18 September 2019; accepted in final form 19 December 2019
published online 5 February 2020

PACS 03.65.Ta – Foundations of quantum mechanics; measurement theory
PACS 03.65.Aa – Quantum systems with finite Hilbert space
PACS 03.65.Ud – Entanglement and quantum nonlocality (*e.g.* EPR paradox, Bell's inequalities, GHZ states, etc.)

**Abstract** – We analyse a proposition which considers quantum theory as a mere tool for calculating probabilities for sequences of outcomes of observations made by an Observer, who him/herself remains outside the scope of the theory. Predictions are possible, provided a sequence includes at least two such observations. Complex valued probability amplitudes, each defined for an entire sequence of outcomes, are attributed to Observer's reasoning, and the problem of wave function's collapse is dismissed as a purely semantic one. Our examples include quantum "weak values", and a simplified version of the "delayed quantum eraser".



> ...there must be a certain conformity
> between nature and our thought.
>
> H. Hertz

Unlike classical mechanics, which has its conceptual issues largely settled by the end of the 19th century [1], quantum theory appears to need an interpretation, which would go beyond mere statement of its mathematical apparatus. One of the reasons for this is the peculiar use of complex valued wave functions or, more generally, amplitudes, needed whenever one wishes to evaluate frequencies (probabilities) with which the observed events would occur under identical circumstances. Present suggestions range from the pragmatic Copenhagen interpretation (see [2] and references therein) to the highly subjective QBism (see [3] and references therein), and include the Bohmian mechanics (see [4] and references therein), Everett's many worlds theory (see [5] and references therein), and the consistent histories approach (see [6] and references therein), to name but a few. Among the issues at stake is the role and place of a conscious Observer, famously brought into the discussion as "Wigner's friend" [7]. Another one is the "collapse" of the wave function (see [8] and references therein), *i.e.*, a sudden change in the observed system's state, apparently not described by the Schroedinger equation. While many of the mentioned interpretations [2–6], each in its own way, aim at a global description of physical world, our objective is somewhat more modest.

The purpose of this paper is to look for the most basic framework, which could unite basic principles of the elementary quantum mechanics, to be tested on a larger scale later. Inevitably, certain general questions need to be addressed first. We start, therefore, by asking what one may expect from quantum theory. A possible answer can be found in Feynman's Lectures [9], and we reproduce it here in full: "*So at the present time we must limit ourselves to computing probabilities. We say "at the present time", but we suspect very strongly that it is something that will be with us forever – that it is impossible to beat this puzzle – that it is the way nature really* is".

In the above quote "we" clearly refers to conscious Observers. The probabilities, on the other hand, tend to be mentioned in the literature in at least three different contexts. Objective probabilities are related to frequencies, with which events occur [10], subjective probabilities describe the degree of one's belief [11], and abstract probabilities, satisfying Kolmogorov's axioms [12], are a mathematical concept. In what follows, we will choose the first option, *i.e.*, assume that the purpose of the theory is to

predict relative frequencies of events, or of series of events, should the same experiment(s) be repeated a large number of times under the same conditions.

What kind of events should then be considered? Physics is an empirical science, so the "events" must refer to objective experiences, *i.e.*, accessible in principle to any number, or to all, conscious Observers. (Such is, for example, observation of the moon in the night sky, whereas one's dream about the moon must fall outside the remit of physical sciences.)

At this point one faces a further choice to be made. Either quantum theory is so universal as to describe the nature of human consciousness, and of life in general, or it is a tool, specifically tailored to and constrained by the limitations of the Observer's perception. The Observer is either a subject of the theory, or its user, whose place is outside the theory's scope.

For an imperfect analogy, consider a community of mobile phone users whose ability, for reasons unknown, is limited to enacting applications on a set's screen. After some trying, the users will be able to compile a rule book, similar to a basic operation manual. But, unable to look inside the set, they will ultimately arrive at the level beyond which no further understanding of telephony's principles is possible. Conversely, although these basic rules will say something about how the users communicate, they will provide little insight into the origin of human consciousness. The analogy is imperfect since, unlike the physical world, a smartphone was made by user's peers, and more detailed descriptions of the set's design, Maxwell's theory, and the network's infrastructure are, in principle, available.

Thus, a choice needs to be made, and in the following, we will opt for the second proposition. An assumption that the make up of the world can be known in its entirety is a strong, and a relatively recent one. Arguably, the fact that a theory inevitably arrives at the level where no further explanation is possible, and the nature simply *is* as it is, may point towards the existence of phenomena, inaccessible to the Observer's experience (cf. the awkward analogy of the previous paragraph). There is little doubt that human observers have only limited perceptive powers, *e.g.*, an ability of directly observing (leave other four senses aside) only surfaces of objects in three-dimensional coordinate space. (Hence the need to equip a measuring device with a pointer, whose spatial displacement encodes the value of the measured variable.)

Another reason for excluding the Observer from the remit of the theory is that no one has so far observed a state of human consciousness, while little is known even about consciousness of ants or trees [9]. (This is not to be confused with observation of physical or biochemical precesses in a live organism, accessible to direct or indirect measurements [13].) Even if the required observational technique could, at some stage, be found, the state of one's consciousness will be accessible to all except the conscious person, caught in a bad progression of being aware of being aware ... of being aware of his/hers own state. This, in turn, contradicts the earlier requirement that physics should deal only with phenomena, accessible to all in equal measure. The old view that "*inner life of an individual is... extra-observational by its very nature*" [13], and quantum mechanics should not try to describe the Observer entirely, is currently regaining its popularity. One example can be found in a recent paper by Frauchieger and Renner [14], although valid critique of the analysis was later given in [15].

Having adopted a view by which quantum theory is *for*, rather than *about* conscious Observers, we can move on to more practical issues. We will do so by analysing the case where the events, perceived by an Observer, are the results of observations, made on a elementary quantum system, with which the theory associates a Hamiltonian operator $\hat{H}$, and a Hilbert space of a finite dimension $N$. An Observer may want to measure a variable $\mathcal{C}$, represented by a Hermitian operator $\hat{C}$, with eigenstates $|c_n\rangle$, and eigenvalues $C_i$, some of which can be degenerate. Quantum theory postulates that an accurate measurement of $\mathcal{C}$ must yield one of the discrete values $C_i$. The outcome of a measurement cannot be predicted with certainty, but the probability of obtaining a $C_i$ is given by

$$P(C_i) = \langle\psi(t)|\hat{\pi}(C_i)|\psi(t)\rangle, \tag{1}$$

where $|\psi(t)\rangle$ is the state in which the system is at the time of measurement, $\hat{\pi}(C_i) = \sum_{n=1}^{N} |c_n\rangle\Delta(C_i - \langle c_n|\hat{C}|c_n\rangle)\langle c_n|$ is the projector onto the state, or a subspace, corresponding to the value $C_i$. (Above we have introduced $\Delta(X-Y)$, which equals 1 if $X = Y$, and 0 otherwise.)

A closer look at eq. (1), which aims to describe a *single* measurement of $\mathcal{C}$, shows that, in fact, it establishes a correlation between *two* Observer's experiences. An Observer must first determine that the system is indeed in $|\psi(t)\rangle$ prior to the measurement, and only then evaluate the odds on having the outcome $C_i$. The first step can be made by preparing the system with the help of an apparatus, controlled by the Observer, or by measuring, at some $t_0 < t$, another variable, $\mathcal{B}$ with non-degenerate eigenvalues $B_i$, so that obtaining a $B_j$ also helps establish that $|\psi(t_0)\rangle = |b_j\rangle$. Either way, with $|\psi(t)\rangle = \hat{U}(t,t_0)|\psi(t_0)\rangle$, where $\hat{U}(t,t_0) = \exp(-i\int_{t_0}^{t}\hat{H}(t')\mathrm{d}t')$ (see footnote [1]), eq. (1) now yields a conditional probability for obtaining first $B_j$ and later $C_i$,

$$P(C_i) = P(C_i \leftarrow B_j) \equiv \sum_{n=1}^{N} \Delta(C_i - C_n)|A(c_n \leftarrow b_j)|^2. \tag{2}$$

A complex valued quantity

$$A(c_n \leftarrow b_j) \equiv \langle c_n|\hat{U}(t,t_0)|b_j\rangle, \tag{3}$$

is a Feynman's transition amplitude [16] for a system which starts in $|b_j\rangle$ at $t_0$ and ends up in $|c_n\rangle$ at $t$.

[1] Understood as a time-ordered product, if $H(t')$ do not commute at different $t'$'s.

Importantly, the sequence $C_i \leftarrow B_j$ cannot be reduced further, *e.g.*, to predicting the statistics of measuring $B_i$ on its own. There are two compelling reasons why the concept of the state of a system, previously not a subject to an Observer's experience, can have little physical meaning. Suppose, Alice receives a spin-1/2 from a completely unknown source. One way to determine the state it is in would be to produce a large number of its identical copies, and perform measurements on the ensemble, created in this manner. But this is forbidden by the no-cloning theorem [17], since the task cannot be performed by means of a unitary evolution, the only kind of evolution allowed by quantum theory. Alternatively, Alice could make a single measurement, but the result will depend on the choice of the measured operator, and cannot, therefore, reveal the true state of the system before Alice's intervention. (See also [18], for a proof that a measured value cannot pre-exist its measurement, and must be *produced* in the course of it.) We are, therefore, encouraged to shift the focus of attention away from the wave function $|\psi(t)\rangle$ in eq. (1) to the transition amplitude $A(c_n \leftarrow b_j)$ in eq. (3), related to the correlations between at least two events, experienced by the Observer.

Furthermore, the two-measurements case (2), (3) is not fully representative of the problem at hand, and we will turn to sequences in which three or more quantities $\mathcal{Q}^\ell$, $\ell = 1, 2, \ldots, L$, are measured times $t_\ell$, $t_{\ell+1} < t_\ell$, with the possible outcomes $Q^1_{i_1}, Q^2_{i_2}, \ldots, Q^L_{i_L}$ (we apologise for the cumbersome notations). To predict the probability of a given series of outcomes, $P(Q^L_{i_L} \ldots \leftarrow Q^1_{i_1})$, one must construct complex valued probability amplitudes for all possible scenarios, add them as appropriate, and take the absolute square of the results [9]. This procedure needs to take into account the degeneracies of eigenvalues $Q^\ell_{i_\ell}$ of the operators $\hat{Q}^\ell$, representing the quantities $\mathcal{Q}^\ell$, and can be summarised as follows.

*I) Virtual (Feynman) paths.* First, one needs to introduce $L$ complete basis sets $\{|q^\ell_{n_\ell}\rangle\}$, $n_\ell = 1, 2, \ldots, N$, in which the operators $\hat{Q}^\ell$ are diagonal. Connecting the states at different times $t_\ell$, yields $N^L$ virtual paths $\{q^L_{n_L} \ldots \leftarrow q^2_{n_2} \leftarrow q^1_{n_1}\}$, each endowed with its own probability amplitude,

$$A(q^L_{n_L} \ldots \leftarrow q^2_{n_2} \leftarrow q^1_{n_1}) = \langle q^L_{n_L}|\hat{U}(t_L, t_{L-1})|q^{L-1}_{n_{L-1}}\rangle \times \ldots \langle q^3_{n_3}|\hat{U}(t_3, t_2)|q^2_{n_2}\rangle\langle q^2_{n_2}|\hat{U}(t_2, t_1)|q^1_{n_1}\rangle. \quad (4)$$

These paths are the elementary building blocks, from which the observable probabilities will later be constructed.

*II) Superposition principle.* We will start with the case where the first measured eigenvalue, $Q^1_{i_1}$, is non-degenerate, thus allowing for one-to-one correspondence $Q^1_{i_1} \leftrightarrow |q^1_{i_1}\rangle$, and return to a more general case in IV) below. Other eigenvalues may, or may not, be degenerate, but different rules apply to the "present", at the last time $t = t_L$, and the "past" at $t = t_\ell$, $1 < \ell < L$. If several eigenstates correspond to a "past" value $\hat{Q}^\ell_{i_\ell}$, one must allow for the interference between the paths, not distinguished by the measurement. In this case, the amplitude for obtaining such a value, and ending up in a state $|q^L_{n_L}\rangle$, is given by

$$A(q^L_{n_L} \ldots \leftarrow Q^\ell_{i_\ell} \ldots \leftarrow Q^1_{i_1}) = \sum_{n_2, n_3, \ldots, n_{L-1}=1}^{n} \prod_{\ell=2}^{L-1} \Delta(Q^\ell_{i_\ell} - \langle q^\ell_{n_\ell}|\hat{Q}^\ell|q^\ell_{n_\ell}\rangle) A(q^L_{n_L} \leftarrow q^{L-1}_{n_{L-1}} \ldots \leftarrow q^1_{i_1}). \quad (5)$$

However, no interference is allowed for the paths, leading to different final states, even if the last observed ("present") value $Q^L_{i_L}$ is degenerate. In this case we have

$$P(Q^L_{i_L} \leftarrow Q^{L-1}_{i_{L-1}} \ldots \leftarrow q^1_{i_1}) = \sum_{n_L=1}^{N} \Delta(Q^L_{i_L} - \langle q^L_{n_L}|\hat{Q}^L|q^L_{n_L}\rangle) |A(q^L_{n_L} \leftarrow Q^{L-1}_{i_{L-1}} \ldots \leftarrow Q^1_{i_1})|^2, \quad (6)$$

which reduces to a simple Born rule for a non-degenerate $Q^L_{i_L}$,

$$P(Q^L_{i_L} \ldots \leftarrow Q^1_{i_1}) = |A(q^L_{i_L} \ldots \leftarrow Q^1_{i_1})|^2. \quad (7)$$

The rule demonstrates, for example, that *at present* a particle cannot be at two different locations in space. Suppose that (we moved from finite-dimensional systems to point particles in one dimension), at $t = t_L$, one measures a projector onto an interval $[a, b]$, $\hat{\pi}_{[a,b]} = \int_a^b |x\rangle\langle x| \mathrm{d}x$. There is no amplitude for being inside $[a, b]$, whose absolute square gives the probability to obtain an eigenvalue 1. Rather, there are probabilities for being at each location inside the interval, whose sum yields the odds on obtaining this eigenvalue. Not so, if $\hat{\pi}_{[a,b]}$ is measured *in the past*, at some $t_\ell < t_L$, where one has to define an amplitude for passing through the entire interval according to eq. (5), and take its absolute square, as prescribed by eq. (6). This is true in every representation, determined by the observations one wishes to make.

We will return to the need for a distinction between the past and the present in the next paragraph, after noting that if Alice wishes to test the theory, she can prepare a statistical ensemble by measuring a $\hat{Q}^1$, selecting those systems, for which the outcome is a non-degenerate eigenvalue $Q^1_{i_1}$, and proceeding to measure the values of the remaining $\mathcal{Q}^\ell$. The gathered statistics will then agree with eq. (6).

*III) Causality and consistency.* Causality ensures that the observations made in the future do not affect the results already experienced. Indeed, it is easy to check that ignoring the outcomes, obtained at $t = t_L$, restores the probabilities (6), for a shorter sequence $\{Q^{L-1}_{i_{L-1}} \ldots \leftarrow Q^1_{i_1}\}$,

$$P(Q^{L-1}_{i_{L-1}} \ldots \leftarrow Q^1_{i_1}) = \sum_{i_L} P(Q^L_{i_L} \leftarrow Q^{L-1}_{i_{L-1}} \ldots \leftarrow Q^1_{i_1}). \quad (8)$$

The rule is also consistent, in the sense that to add one more measurement of $\hat{Q}^{L+1}$ at $t_{L+1} > t_L$, one should

simply relegate the moment $t_L$ to the past, and consider Feynman paths $\{q^{L+1}_{n_{L+1}} \leftarrow q^L_{n_L} \ldots \leftarrow q^1_{n_1}\}$ with the amplitudes

$$A(q^{L+1}_{n_{L+1}} \leftarrow q^L_{n_L} \ldots \leftarrow q^1_{n_1}) = \langle q^{L+1}_{n_{L+1}}|\hat{U}(t_{L+1},t_L)|q^L_{n_L}\rangle \times A(q^L_{n_L} \ldots \leftarrow q^1_{n_1}). \quad (9)$$

Equation (9) helps provide some insight into the meaning of eq. (6). Suppose, at $t_L$ one measures an operator $\hat{Q}^L$, whose eigenvalues are degenerate. It is then possible, without altering the probability of the previous sequence of outcomes, to measure an operator $\hat{Q}^{L+1}$, diagonal in one of the bases, in which $\hat{Q}^L$ is also diagonal. If the eigenvalues of $\hat{Q}^{L+1}$ are all distinct, and the measurement is made immediately after $t_L$, $t_{L+1} \to t_L$, the first factor in the r.h.s. of eq. (9) is a Kronecker delta, $\langle q^{L+1}_{n_{L+1}}|\hat{U}(t_{L+1},t_L)|q^L_{n_L}\rangle \to \langle q^L_{n_{L+1}}|q^L_{n_L}\rangle = \delta_{n_{L+1}n_L}$. Inserting (9) into eq. (5) (with $L$ replaced by $L+1$), applying the Born rule (7), and using (8), yields eq. (6) for the probability of observing a sequence $\{Q^L_{i_L} \leftarrow Q^{L-1}_{i_{L-1}} \ldots \leftarrow Q^1_{i_1}\}$. This illustrates Feynman's assertion [9] that scenarios, which can be distinguished *in principle* (in this case, by a future more detailed measurement), are always *exclusive*. In particular, there can be no interference between the paths leading to orthogonal final states.

*IV) Inconclusive preparation and consistency.* Suppose next that the first measurement yields an $M$-degenerate value $Q^1_{i_1}$, with which one associates an $M$-dimensional sub-space of the system's Hilbert space, spanned by a basis set $|u_m(Q^1_{i_1})\rangle$, $m = 1, 2, \ldots, M$. This information is not sufficient for assigning to the system a particular initial state and Alice, who still wishes to create a statistical ensemble, must make an additional assumption about what the state might be. Consistent with the result $Q^1_{i_1}$, the system is prepared in any of the states $|u_m(Q^1_{i_1})\rangle$. Assuming that the $m$-th choice is made with a probability $\omega_m \geq 0$, $\sum_{m=1}^M \omega_m = 1$, Alice obtains

$$P(Q^L_{i_L} \ldots \leftarrow Q^\ell_{i_\ell} \ldots \leftarrow Q^1_{i_1}) = \sum_{m=1}^M \omega_m P(Q^L_{i_L} \ldots \leftarrow Q^\ell_{i_\ell} \ldots \leftarrow u_m), \quad (10)$$

where $P(Q^L_{i_L} \leftarrow Q^{L-1}_{i_{L-1}} \ldots \leftarrow u_m)$ is the probability (6) for the system, which was prepared in a state $|u_m\rangle$. With all possible choices of $\omega_m$, and of orthonormal bases spanning the $M$-dimensional subspace, Alice has many options. One does, however, stand out. With no other information available, she can decide to give all $|u_m(Q^1_{i_1})\rangle$ equal weights, thus choosing

$$\omega_m = 1/M. \quad (11)$$

Now the probabilities (10) no longer depend on a particular choice of the basis $|u_m(Q^1_{i_1})\rangle$, since $\sum_{m=1}^M |u_m\rangle\langle u_m| = \sum_{m=1}^M |u'_m\rangle\langle u'_m| = \hat{\pi}(Q^1_{i_1})$. (Note that Alice could as well consider *all* states in the subspace to be equally probable. A demonstration is straightforward for $M = 2$, where a state can be parametrised by the polar and azimuthal angles, and the integration of the corresponding projectors over the entire Bloch sphere yields one half of the unity operator, $\hat{I}/2$.)

With the choice (11) made, the rule is consistent in the sense that if $\mathcal{Q}^1$ is a constant quantity, $\hat{Q}^1 = \lambda\hat{I}$, and the first measurement yields no information whatsoever, $P(Q^L_{i_L} \ldots \leftarrow Q^\ell_{i_\ell} \ldots \leftarrow u_m)$ reduces to the probability of a shorter sequence,

$$P(Q^L_{i_L} \ldots \leftarrow Q^\ell_{i_\ell} \ldots \leftarrow Q^1_{i_1} = \lambda) = P(Q^L_{i_L} \ldots \leftarrow Q^\ell_{i_\ell} \ldots \leftarrow Q^2_{i_2}), \quad (12)$$

as if the first measurement, whose outcome is certain, were not made at all.

*V) Composites and separability.* With the help of the above, one can treat observations, made on a system of interest (labelled $S$, with a Hamiltonian $\hat{H}_S$), seen as a part of a larger composite system + environment (labelled $E$, with a Hamiltonian $\hat{H}_E$), whatever this environment might be. The full Hamiltonian is now given by $\hat{\mathcal{H}} = \hat{H}_S + \hat{H}_E + \hat{H}_{int}$, where the last term describes the interaction between the $S$ and $E$. If, for example, the environment is a system in a $K$-dimensional Hilbert space, the eigenvalues $Q_i(S)$ of an operator $\hat{Q}(S)$, representing a system's variable $\mathcal{Q}(S)$, are at least $K$-fold degenerate, and the probability of a series of outcomes of observations made on the $S$ alone are still given by eqs. (4)–(6). It is easy to check that if the system is completely isolated from the environment, so that $\hat{H}_{int} = 0$ and $\hat{U}(t_{\ell+1},t_\ell) = \hat{U}_S(t_{\ell+1},t_\ell) \otimes \hat{U}_E(t_{\ell+1},t_\ell)$, after summing over the degeneracies one recovers eqs. (4)–(6) for the system only, *i.e.*, with $\hat{U}(t_{\ell+1},t_\ell) = \hat{U}_S(t_{\ell+1},t_\ell)$. Note that so far we have also assumed that at $t = t_1$ the result of the first measurement corresponds to a composite's product state $|q^1_{i_1}(S)\rangle \otimes |q^1_{j_1}(E)\rangle$. A measurement of a more general collective quantity, $\hat{Q}(S+E)$, may yield a $Q^1_{i_1}(S+E)$, which would leave the composite in an entangled state $|q^1_{i_1}(S+E)\rangle = \sum_{j_1=1}^N \beta_{i_1 j_1}|q^1_{j_1}(S)\rangle \otimes |\phi_{j_1}(E)\rangle$, $\langle\phi(E)_j|\phi(E)_j\rangle = 1$. If no further interaction between $S$ and $E$ is possible, application of eqs. (4)–(6) to each term of the sum yields

$$\begin{aligned} &P(Q^L_{i_L}(S) \ldots \leftarrow \hat{Q}^\ell_{i_\ell}(S) \ldots \leftarrow Q^1_{i_1}(S+E)) = \\ &\sum_{n_L=1}^N \Delta(Q^L_{i_L}(S) - \langle q^L_{n_L}(S)|\hat{Q}^L(S)|q^L_{n_L}(S)\rangle) \\ &\times \sum_{j,j'=1}^N \beta^*_{i_1 j'}\beta_{i_1 j}\langle\phi(E)_{j'}|\phi(E)_j\rangle \\ &\times A^*(q^L_{n_L}(S) \ldots \leftarrow \hat{Q}^\ell_{i_\ell}(S) \ldots \leftarrow q^1_{j'}(S)) \\ &\times A(q^L_{n_L}(S) \ldots \leftarrow \hat{Q}^\ell_{i_\ell}(S) \ldots \leftarrow q^1_{j}(S)), \end{aligned} \quad (13)$$

which simplifies to

$$P(Q^L_{i_L}(S)\ldots \leftarrow Q^1_{i_1}(S+E)) = \sum_{j=1}^{N} |\beta_{i_1 j}|^2 P(Q^L_{i_L}(S)\ldots \leftarrow q^1_j(S)), \quad (14)$$

in a special case where $|\phi_j(E)\rangle$ are orthogonal, $\langle\phi(E)_{j'}|\phi(E)_j\rangle = \delta_{jj'}$, and the system $(S)$ can be said to start in a state $|q^1_j(S)\rangle$ with a probability $|\beta_{i_1 j}|^2$.

Equations (4)–(6) and (10) can be rewritten in a compact and, perhaps, more familiar form (tr stands for trace):

$$P(Q^L_{i_L}\ldots \leftarrow \hat{Q}^\ell_{i_\ell}\ldots \leftarrow Q^1_{i_1}) = \mathrm{tr}\{\hat{\pi}(Q^2_{i_2},t_2)\ldots\hat{\pi}(Q^L_{i_L},t_L)\ldots\times\hat{\pi}(Q^2_{i_2},t_2)\hat{\rho}(Q^1_{i_1})\}, \quad (15)$$

where, in the Heisenberg representation, $\hat{\pi}(Q^\ell_{i_\ell},t_\ell) \equiv \hat{U}^{-1}(t_\ell,i_1)\hat{\pi}(Q^\ell_{i_\ell})\hat{U}(t_\ell,i_1)$ is the projector onto the eigensubspace, associated with an outcome $Q^\ell_{i_\ell}$ and $\rho(Q^1_{i_1}) = \sum_{m=1}^{M}|u_m\rangle\omega_m\langle u_m|$ is the system's density operator [13]. Similar strings of projectors appear, for example, in the consistent histories approach (CHA) [6], but there are important differences. Firstly, while the CHA aims to be a general theory, which includes observers, we place an Observer outside the theory's scope. Secondly, for us eq. (15) is a derived result, and the primary and most basic quantities are the transition amplitudes (4) and (5).

As an example where this difference is important, consider the case where the system is "pre- and post-selected" in the states $|q^1_{i_1}\rangle$ and $|q^3_{i_3}\rangle$ at $t_1$ and $t_3$, and in eq. (13) the role of environment is played by a von Neumann pointer [13], employed to measure some $\hat{Q}^2$ at $t_1 < t_2 < t_3$. The accuracy of the measurement may vary, yet in every case the information about the system, obtained from the pointer's final position, will have to be expressed in terms of an amplitude $A(q^3_{i_3} \leftarrow Q^2_{i_2} \leftarrow q^1_{i_1})$ in eq. (5) [19]. If $\hat{Q}^2 = |q^2_m\rangle\langle q^2_m|$ is a projector onto a state $|q^2_m\rangle$, and the coupling to the pointer is small (the accuracy of the measurement is poor), the average shift of the pointer, $f$, turns out to be given by

$$\langle f\rangle \approx \mathrm{Re}\left[\frac{A(q^3_{i_3}\leftarrow q^2_m\leftarrow q^1_{i_1})}{\sum_{n=1}^{N}A(q^3_{i_3}\leftarrow q^2_n\leftarrow q^1_{i_1})}\right] = \mathrm{Re}\left[\frac{\langle q^3_{i_3}(t_2)|\hat{Q}^2|q^3_{i_1}(t_2)\rangle}{\langle q^3_{i_3}(t_2)|q^1_{i_1}(t_2)\rangle}\right], \quad (16)$$

where $|q^3_{i_3}(t_2)\rangle \equiv \hat{U}^{-1}(t_3,t_2)|q^3_{i_3}\rangle$ and $|q^1_{i_1}(t_2)\rangle \equiv \hat{U}(t_3,t_2)|q^1_{i_1}\rangle$. The last expression in eq. (16) was first obtained in [20], where the complex valued fraction in brackets was called "the weak value (WV) of the operator $\hat{Q}^2$." Written in this way, a WV looks like a physical variable of a new kind [21], whose physical significance is still discussed in the literature (see, for example [22,23]). However the first expression in the r.h.s. of eq. (16) identifies it with a previously known renormalised Feynman amplitude (or a weighted sum of such amplitudes if a more general $\hat{Q}^2$ is inaccurately measured) [24–29]. The problem is, little is known about the probability amplitudes, apart from their relation to the observable frequencies, discussed above. Until, or, unless a deeper insight into the physical meaning of quantum amplitudes is gained, such an inaccurate "weak" measurement will remain merely an exercise in recovering the values of transition amplitudes from a response of a system to a small perturbation [19].

As a further illustration we revisit, in its simplest version, the "delayed choice quantum eraser experiment" [30]. Figure 1(a) sketches a primitive double-slit experiment, in which a two-level system $(S)$ (a spin-1/2), which an observed outcome $B_1$ has prepared in a state $|b_1\rangle$, is subjected to a later measurement of an operator $\hat{C}(S) = C_1|c_1\rangle\langle c_1| + C_2|c_2\rangle\langle c_2|$ at some $t = t_2$. The final state $|c_1\rangle$ plays the role of a point on the screen, which can be reached by passing, at $t_1 < t_2$, through a pair of orthogonal states $|\uparrow\rangle$ and $|\downarrow\rangle$, representing the two slits. A pair of virtual paths in the two-dimensional Hilbert space (fig. 1(a), solid line) interfere, and the probability to have $C_1(S)$ is given by (spin has no own dynamics)

$$P(C_1 \leftarrow B_1) = |A_1 + A_2|^2, \quad (17)$$
$$A_1 = \langle c_1|\uparrow\rangle\langle\uparrow|b_1\rangle, \quad A_2 = \langle c_1|\downarrow\rangle\langle\downarrow|b_1\rangle.$$

In a different setup, shown in fig. 1(c), initial measurement of a collective variable $\hat{B}(S+E)$ entangles the system with a two-level "environment" $(E)$, whose orthogonal states are $|+\rangle$ and $|-\rangle$. As before, $\hat{C}(S)$ is measured at $t_2$, and then environment's variable $\hat{D}(E) = D_1|d_1\rangle\langle d_1| + D_2|d_2\rangle\langle d_2|$ is measured at $t_3 > t_2$. Four relevant virtual paths in the now four-dimensional Hilbert space (solid lines in fig. 1(b)) are endowed with probability amplitudes ($E$ has no own dynamics either)

$$A_{\mathrm{I}} = \langle d_1|+\rangle A_1, \qquad A_{\mathrm{II}} = \langle d_2|+\rangle A_1,$$
$$A_{\mathrm{III}} = \langle d_1|-\rangle A_2, \qquad A_{\mathrm{IV}} = \langle d_2|-\rangle A_2. \quad (18)$$

Using the rules *I*)–*VI*), for the probabilities of the sequences of outcomes shown in fig. 1(c), we easily find

$$P(D_1 \leftarrow C_1 \leftarrow B_1) = |A_{\mathrm{I}} + A_{\mathrm{II}}|^2,$$
$$P(D_2 \leftarrow C_1 \leftarrow B_1) = |A_{\mathrm{III}} + A_{\mathrm{IV}}|^2 \quad (19)$$

and

$$P'(C_1 \leftarrow B_1) \equiv P(D_1 \leftarrow C_1 \leftarrow B_1) + P(D_2 \leftarrow C_1 \leftarrow B_1) = |A_1|^2 + |A_2|^2. \quad (20)$$

Much of the interest in the above scheme stems from the fact that while there is no interference term $\sim A_1A_2$ in eq. (20), this term reappears in $P(D_1 \leftarrow C_1 \leftarrow B_1) = |A_1 + A_2|^2/2$, *e.g.*, if one chooses $|d_1\rangle = [|+\rangle + |-\rangle]/\sqrt{2}$. It is tempting to conclude that coherence between the paths 1 and 2, apparently lost after measuring $\hat{C}(S)$ at $t_2$, is somehow restored if the second system, $(E)$, is found in $|d_1\rangle$. All the more surprising is that this seems to happen after the outcome $C_1$ has already been observed. However,

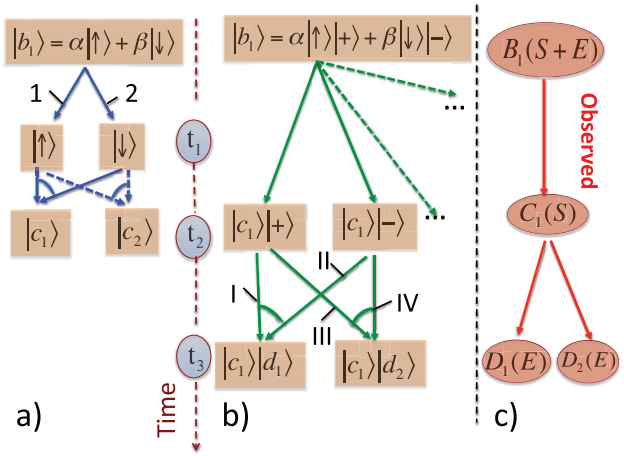


Fig. 1: (a) A primitive "double-slit problem", with four virtual paths connecting states in a two-dimensional Hilbert space of a spin ($S$). No observation is made at $t_1$, and the paths joined by an arc are allowed to interfere. (b) A different "double-slit problem", with virtual paths (only 4 of the 16 are shown) connecting states in a four-dimensional Hilbert space of the composite ($S+E$). A measurement made on the spin at $t_2$ does not destroy interference between paths leading to the same final outcomes. (c) The observed outcomes: what is measured at $t_3$ cannot affect the net probability of obtaining $C_1$ at $t_2 < t_1$.

fig. 1(b) shows that in making this conclusion we are not comparing like with like. In fig. 1(b), the interference term is controlled by the magnitudes of the amplitudes of two virtual paths, I) and II), which connect states in a different four-dimensional Hilbert space of the composite system, and the argument cannot be reduced to a discussion of the individual paths shown in fig. 1(a). Quantum theory does its job of calculating probabilities for the outcomes in fig. 1(c) in an explicitly causal manner, and, we suspect, cannot be asked to do more than that. (For other recent attempts at "demystifying" the delayed eraser experiment we refer the reader to refs. [31,32].)

We can now sum up the "minimalist view", advertised in the title. Quantum theory is a tool, allowing a conscious Observer to predict statistical correlations between the results of two or more of his/hers observations, first of which is needed to "prepare" the system. With a particular series of results in mind, he/she may reason about its likelihood by associating with each outcome a state, or states, in a Hilbert space, including all systems which interact with each other during the time interval considered. A probability amplitude for the *entire* series is then constructed using the prescriptions $I)$–$V)$, and taking its absolute square yields the required probability. Such attributes of the theory as amplitudes, Hilbert spaces, operators, and Hamiltonians are essentially Hertz's "symbols of external objects, formed by ourselves", "whose consequences are always the necessary consequences in the nature of the thing pictured" [1]. Observer's main effort then goes into identifying a system's Hamiltonian $\hat{H}$, and the operators $\hat{Q}^1$, $\hat{Q}^2$, ..., $\hat{Q}^L$, whose spectra contain the possible observational outcomes. As a tool, tailored to Observer's limited abilities, the theory is unable to progress beyond a certain explanatory level, where it must admit, as in the opening quote, that nature simply *is* this way. For the same reason, Observer's conscience is not a valid subject of quantum theory, which must lose its pretence (if any) at explaining the world in its entirety, and give way to other complementary endeavours.

Several other remarks may be in order. Firstly, the symbolic status of probability amplitudes does not prevent that their values can, under certain conditions, be deduced from the measured probabilities [19]. Indeed, this was done, for example, in the experiments reported in [33] and [34].

Secondly, different sets of measurements, *e.g.*, of the quantities $\mathcal{Q}^\ell$ and $\mathcal{Q}'^\ell$, made on the same system, may produce essentially different statistical ensembles even if the operators $\hat{Q}^\ell$ and $\hat{Q}'^\ell$ commmute [28,35]. For example, less detailed probabilities (some of $\hat{Q}^l$'s eigenvalues are degenerate) cannot be obtained by adding the most detailed ones (obtained for a $\hat{Q}'^l$, whose eigenvalues are all non-degenerate). This is, of course, only a more elaborate version of a double-slit experiment, where the price of knowing the way a particle has taken is the loss of the interference pattern on the screen.

Finally, so far no mention has been made of the collapse of the wave function. The possibility of avoiding this issue altogether is precisely the point we intend to make here. An Observer, whose reasoning only requires him/her to evaluate certain matrix elements in an abstract space, may discard the "collapse problem" as a purely semantic one. It is, of course, possible to argue that in eq. (1) the evolution of the state $|q^{L-1}_{n_{L-1}}\rangle$ is mysteriously interrupted at $t = t_L$, but it is equally possible not to enter into this discussion at all. Conceptual economy from not having to worry about the fate of the wave function can be significant. One avoids dealing with a universe which splits every time a measurement is made, as it happens, for example, in the Everett's many worlds (MW) picture [5]. Curiously, in 1995 Price [36] polled physicists wishing to determine the level of support for the MW approach, and counted Feynman among its supporters. We note that Feynman's support must have been lukewarm at best. In [35] one reads: "*Somebody mumbled something about a many-world picture, and that many-world picture says that the wave function $\psi$ is what's real, and damn the torpedoes if there are so many variables, $N^R$. All these different worlds and every arrangement of configurations are all there just like our arrangement of configurations, we just happen to be sitting in this one. It's possible, but I'm not very happy about it*".

To conclude, we note that the proposed viewpoint imposes strict limits and, if adopted, is likely to have implications for such concepts as the "universal wave function" [37], for attempts to construct a quantum theory where no special role is given to an Observer [4–6], or for collapse-related theories of quantum mind (see [38] and

references therein). None of these matters are trivial, and cannot be dismissed out of hand. The format of this letter does not allow for detailed comparisons, so our purpose here was to articulate a maximally reduced view, which can later be extended, modified, or abandoned. For instance, it is possible that the simple model, used to illustrate it, will fail when dealing with extremely large or complex systems, or where the relativistic effects of various kinds need to be taken into consideration. It is also possible that, contrary to the Feynman's quote at the beginning of this article, the quantum method has not yet arrived at its explanatory limit. If so, a more sophisticated theory will have to provide a further insight into the meaning of the transition probability amplitudes, which so far have played the role of basic elements of a quantum mechanical investigation. For now, we argue, a path analysis, similar to the one shown in fig. 1, is the best "explanation" which quantum mechanics is able to offer in cases like the "Hardy's paradox" [39], the "quantum Cheshire cat effect" [23], or indeed in other situations, involving additional measurements made on pre- and post-selected quantum systems [28].

* * *

Financial support of MCIU, through the grant PGC2018-101355-B-100(MCIU/AEI/FEDER, UE) and the Basque Government Grant No. IT986-16 is acknowledged by DS.

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