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Modeling the decentralized optimization of communicative energy

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Abstract – This paper integrates two different ways to face the emergence of communication systems. First, information-theoretic models argue for the emergence of Zipfian properties based on the minimization of communicative energy, combining speaker and hearer efforts. Second, decentralized agent-based models focus on the emergence of a shared communication system in a population of individuals. The main aim here is to explore how a decentralized agent-based model of language formation can exhibit purely from local speaker-hearer interactions the minimization of communicative energy. Numerical simulations show an evolution towards the minimum of communicative energy for populations communicating with Zipfian languages. Our results suggest thus a new way to understand energy-based approaches to the formation of human language.

Introduction. – Is it possible to define an energy-like quantity describing how individuals negotiate word-meaning associations over linguistic interactions? To (partly) answer this question, we consider an idealized speaker-hearer interaction: given some meaning to transfer, the speaker must choose some word-form (from her vocabulary) to convey such meaning. This simple decision involves a fascinating phenomenon: in a selfish scenario, the speaker prefers to minimize her memory costs, and thus she tries to use the least number of words. On the contrary, the hearer prefers to minimize disambiguation costs. Put differently, speakers and hearers exhibit competing interests: while speakers prefer a one-word inventory, hearers prefer a one-to-one word-meaning mapping.

The trade-off between speaker and hearer efforts presented in the previous communicative scenario defines the so-called *least effort principle* [1,2]. G. Zipf introduced a competition between two pressures, *ambiguity* and *memory*: speakers prefer to minimize memory costs; whereas hearers prefer to minimize disambiguation costs. Strikingly, several information-theoretical works have proposed that Zipfian vocabularies appear as a phase transition at a critical stage for both competing pressures [3–7]. The key aspect of the Zipf-type behavior exhibited by those works, as explained in [8,9], is the emergence of the phase transition between referentially useless one-word systems and one-to-one word-meaning reference systems. More precisely, the mentioned information-theoretic interpretation of the least-effort communication principle is not sufficiently strong for generating power laws (Zipf's law) at the critical effort stage. Our approach focuses therefore on the emergence of a Zipf-type phase transition, instead of Zipf's power law. With this, in our work the word *Zipfian* must be considered as a sign of a *phase transition* between two extreme cases of communication systems.

The information-theoretical interpretation attempts to explain, in turn, the appearance of the Zipf-type behavior seeing communication as a global minimum of the so-called *communication energy* $\Omega(\lambda)$ (where λ is a parameter for the relative costs of speakers and hearers), defined by the interplay between maximization of the information transfer and minimization of the entropy of signals [3,10]. In its simplest form, the algorithm proposed by [3] assumes that only one word-meaning mapping (one binary matrix formed by meaning-rows and word-columns) is modified by randomly modifying word-meaning pairs. The new matrix is accepted if $\Omega(\lambda)$ is lowered. The algorithm hypothesizes that Zipfian properties appear at $\lambda \approx 0.5$, where speaker and hearer efforts have a similar contribution to $\Omega(\lambda)$.

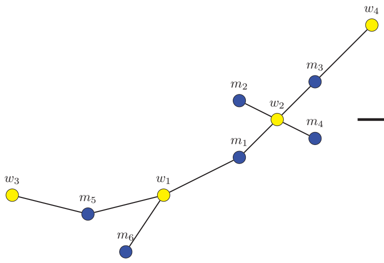
A key feature of the information-theoretic accounts for the appearance of Zipfian properties in human language is the lack of population structure. Contrasting with this, a *language game* is a cooperative and decentralized solution to the emergence of communication systems in a population of individuals [11–14]. It involves a situated dialog, in opposition to the isolated sentences, that is commonly used in formal linguistics [15]. We extend, in our context, an isolated lexical matrix (of information-theoretic accounts) to a situated dialog between individuals equipped with lexical matrices. A language game defines thus simpler models of linguistic interactions within populations of artificial agents, endowed with minimal human cognitive features, negotiating pieces of a common language. In the simplest language game, the *naming game* [16,17], at a discrete time step a pair of players (typically one speaker and one hearer) interacts towards agreement on word-meaning associations.

The main aim of this paper is therefore to integrate two different ways to face the emergence of Zipfian features in communication systems: information-theoretic and decentralized approaches. To do this, we address a decentralized approach (based on a previous proposal [18]) to the minimization of communicative energy $\Omega(\lambda)$, while Zipfian properties in a human-like language tend to arise at some intermediate level of speaker and hearer efforts. Our methodology is mainly based on the characterization of the evolution of speaker and hearer efforts over linguistic interactions, using simple statistical mechanics tools. We run numerical simulations over simple population topologies.

The model. – We first review some concepts of graph theory, in order to propose a precise definition of word-meaning mappings as bipartite graphs. Next, we introduce the language game rules, in which a population of individuals negotiate a common bipartite word-meaning mapping (based on [18]). Finally, to mathematically define the cost of communication for both speakers and hearers, we recall some information-theoretical measures [3,10].

Vocabularies as bipartite graphs. A graph is a pair $G = (V, E)$, where V is a set of nodes and $E \subseteq V \times V$ is the set of edges. A *bipartite graph* is defined by two requisites: i) its vertex set is formed by two disjoint subsets of V, denoted $\top$ and $\perp$ and ii) edges only are defined between $\top$ and $\perp$. More precisely, a *bipartite graph* is a triple $B = (\top, \perp, E)$, where $\top$ and $\perp$ are two mutually disjoint set of nodes, and $E \subseteq \top \times \perp$ is the set of edges of the graph. Here, $\top$ represents the set of *word nodes*, whereas $\perp$ represents the set of *meaning nodes*. The *neighbors* of $u \in \top$ are the nodes connected to u: $N(u) = \{v \in \perp\colon uv \in E\}$ (if $u \in \perp$ the definition is analogous). The degree $d(u)$ of the node u is simply defined by $d(u) = |N(u)|$.

A classical useful tool is the matrix representation of (bipartite) graphs. Let us denote by $A = (a)_{wm}$ the adjacency matrix for the (bipartite) graph B. From the bipartite sets $\top$ and $\perp$, representing respectively word and meaning nodes, we define the rows of A as word nodes, and the columns as meaning nodes, where $(a)_{wm} = 1$ if the word w is joined with the meaning m, and 0 otherwise (see fig. 1).



$$\longrightarrow \begin{array}{cc} \begin{array}{cccccc} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} w_1 \\ w_2 \\ w_3 \\ w_4 \end{array} \end{array}$$

Fig. 1: Adjacency matrix for a bipartite graph. For the example graph, its adjacency matrix representation is exhibited. The graph is formed by two disjoint set of nodes: $\top = \{w_1, w_2, w_3, w_4\}$ and $\perp = \{m_1, m_2, m_3, m_4, m_5, m_6\}$. In the adjacency matrix, rows represent words, whereas columns represent meanings.

Basic ingredients of the language game. The language game is played by a finite population of individuals $P = \{1, \ldots, p\}$, sharing both a set of words $W = \{1, \ldots, n\}$ and a set of meanings $M = \{1, \ldots, m\}$. Each player $k \in P$ is endowed with a bipartite word-meaning mapping $B^k = (\top^k, \perp^k, E^k)$. B^k is formed by two disjoint sets: $\top^k \subseteq W$ (word nodes) and $\perp^k \subseteq M$ (meaning nodes). Each player $k \in P$ only knows its own graph B^k.

Two technical terms are introduced. First, we say that a player $k \in P$ *knows* the word $w \in W$ if $w \in \top^k$. Clearly, this definition is equivalent to the existence of the edge $wm \in E^k$, for some $m \in \perp^k$. Second, the *ambiguity* of the word w, denoted $a(w)$, is defined as its node degree $d(w)$.

Language game rules. The dynamics of the language game is based on pairwise speaker-hearer interactions at discrete time steps. At $t \geqslant 0$, a pair of players is selected uniformly at random: one plays the role of *speaker* s and the other plays the role of *hearer* h, where $s, h \in P$. Each speaker-hearer communicative interaction is defined by two successive phases. The first phase involves the selection of a meaning and a word to transmit them. Next, the hearer receives the word-meaning association and both speaker and hearer behave according to either *repair* or *alignment* strategies.

To start the communicative interaction, the speaker s selects the topic of the conversation: one meaning $m^* \in M$. To transmit the meaning m^*, she needs to choose some word, denoted w^*. If she does not know a word with the meaning m^*, she chooses a word at random from her vocabulary and adds this word-meaning pair to her lexicon B^s.

Then, she *calculates* w^* based on her interests. She behaves according to the *ambiguity parameter* $\wp \in [0, 1]$: with probability $1 - \wp$, she calculates w^* as the least ambiguous word

$$w^* = \min_{w \in \perp^s} a(w),$$

while with probability $\wp$, she calculates w^* as the most ambiguous word

$$w^* = \max_{w \in \perp^s} a(w).$$

She transmits the word w^* to the hearer.

In turn, the hearer behaves as in the *naming game*. On the one hand, if there is a mutual speaker-hearer agreement (the hearer knows the word w^*), *alignment* strategies appear [17]. On the other hand, a speaker-hearer disagreement (if the hearer does not know the word w^*) involves a *repair* strategy in order to increase the chance of future agreements (that is, for $t' > t$). More precisely, if the hearer knows the word w^*, both speaker and hearer remove all word-meaning pairs wm^* from their vocabularies, where w respectively belongs to $\top^s \setminus \{w^*\}$ and $\top^h \setminus \{w^*\}$. On the contrary, if the hearer does not know the word w^*, she adds the word-meaning pair w^*m^* to her vocabulary B^h.

Information-theoretic measures: communicative energy. Within an information-theoretic approach, a series of works [3,10] defines explicitly the compromise between speakers and hearers interests as the so-called *communicative energy*, formed by the combination of two terms:

$$\Omega_\wp = \wp H(R|S) + (1-\wp)H(S), \tag{1}$$

where $\wp$ is a parameter in $[0,1]$, $H(R|S)$ is the effort for the hearer and $H(S)$ is the effort for the speaker. In its original form, $\wp$ weights the contribution of each term. Here, $\wp$ is also the *ambiguity* parameter as explained in the section "Language game rules".

The effort for the speaker $H(S)$ is measured by the entropy of words, that is

$$H(S) = -\sum_{i=1}^{n} p(w_i)\log_n p(w_i). \tag{2}$$

From this definition, if a single word is used for every meaning, the speaker's effort is minimal and $H(S)=0$; on the contrary, when all words are associated to one meaning (the smallest positive frequency), the frequency effect is in the worst case, and thus $H(S)=1$. The frequency of the word w_i is defined as

$$p(w_i) = \sum_{j=1}^{m} p(w_i, m_j). \tag{3}$$

According to the Bayes theorem and assuming that $p(m_j) = \frac{1}{m}$, we have

$$p(w_i, m_j) = p(m_j)p(w_i|m_j) = \frac{1}{m}p(w_i|m_j); \tag{4}$$

$p(w_i|m_j)$ is defined as

$$p(w_i|m_j) = a_{w_i m_j}\frac{1}{\sum_{i=1}^{n} a_{w_i m_j}}, \tag{5}$$

where $\sum_{i=1}^{n} a_{w_i m_j}$ indicates the number of synonyms associated to the meaning m_j.

The effort for the hearer $H(R|S)$ is defined as the average noise for itself, that is

$$H(R|S) = \sum_{i=1}^{n} p(w_i)H(R, w_i), \tag{6}$$

where the noise for the hearer when the word w_i is heard, is defined as the entropy of the distribution of meanings, given w_i:

$$H(R|w_i) = -\sum_{j=1}^{m} p(m_j|w_i)\log_m p(m_j|w_i). \tag{7}$$

Methods. – The population of agents is located on the vertices of a complete graph of size $|P| = 100$ (the *mean field* approximation; from now MF). See the "Results" section for simple variations of this initial topology. The population shares both a set of $n = |W| = 128$ words and a set of $m = |M| = 128$ meanings. Starting from an initial condition in which each player $k \in P$ is associated to a vocabulary B^k, where each word-meaning pair appears with probability 0.5, the dynamics performs a speaker-hearer interaction at discrete time steps $t \geqslant 0$. The vocabularies B^s and B^h are then reevaluated according to communicative success.

To describe previous results about the drastic formation of phases in language formation, first proposed in [3], we consider the (effective) lexicon size at time step t, $V(t)$ [3]:

$$V(t) = \frac{1}{n|P|}\sum_{k\in P}|\top^k|,$$

where $V(t)=1$ if $|\top^k| = n$, while $V(t)=0$ if $|\top^k|=0$. For this measure, all results consider averages over 10 initial conditions and 3×10^5 time steps. We denote by t_f the final time step. The ambiguity parameter $\wp$ is varied from 0 to 1 with an increment of 1%.

To describe the minimization of communicative energy, we measure over language game dynamics three information-theoretical quantities: $\Omega_\wp(t)$, $H(S)$ and $H(R|S)$. We remark that language game rules are not influenced by the minimization (or maximization) of any of such three quantities. In this case, due to the intensive computational work we focused on the evolution of one randomly chosen lexical matrix. This simplification is justified by the small standard deviation of $V(t_f)$, as shown in fig. 3.

Results. – This section focuses on several aspects of language game dynamics. First, we verify that the population (for any value of the parameter $\wp$) reaches a common bipartite word-meaning mapping. Next, we describe the appearance of drastic transitions in language formation. The third subsection presents a decentralized way to minimize communicative energy. In the fourth section, we study the role of topology on the minimization of communicative energy. Finally, we outline the evolution of speaker and hearer costs.

Measuring consensus on a bipartite word-meaning mapping. Language games aim to model how a population of individuals reaches an agreement on simple versions of communication systems. Here, individuals negotiate with

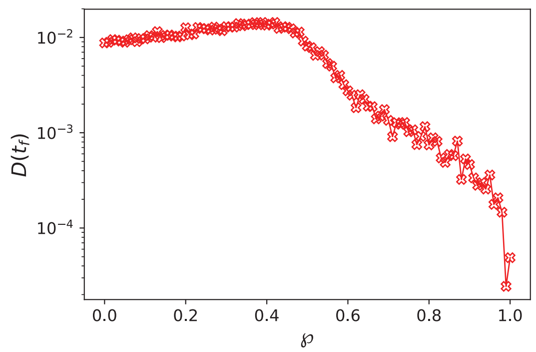


Fig. 2: Consensus function $D(t_f)$ *vs.* ambiguity parameter ℘. $D(t_f)$ *vs.* ℘: after t_f speaker-hearer interactions. ℘ is varied with an increment of 1%. Averages over 10 initial conditions. Vertical axis is in log scale.

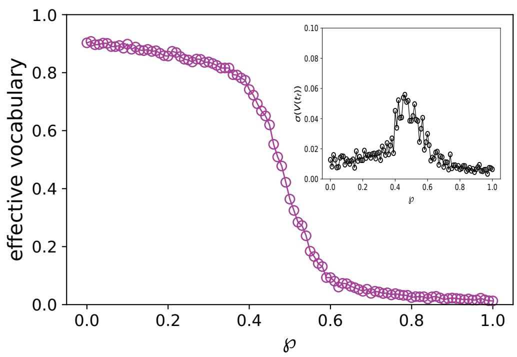


Fig. 3: Effective vocabulary $V(t_f)$ *vs.* ambiguity parameter ℘. Purple circles display the average behavior of $V(t_f)$ *vs.* ℘, after t_f speaker-hearer interactions. Black circles indicate the standard deviation of $V(t_f)$. Three phases appear clearly for language formation: (almost) full vocabularies ($℘ < 0.4$), Zipfian vocabularies ($0.4 < ℘ < 0.6$) and single-word vocabularies ($℘ > 0.6$).

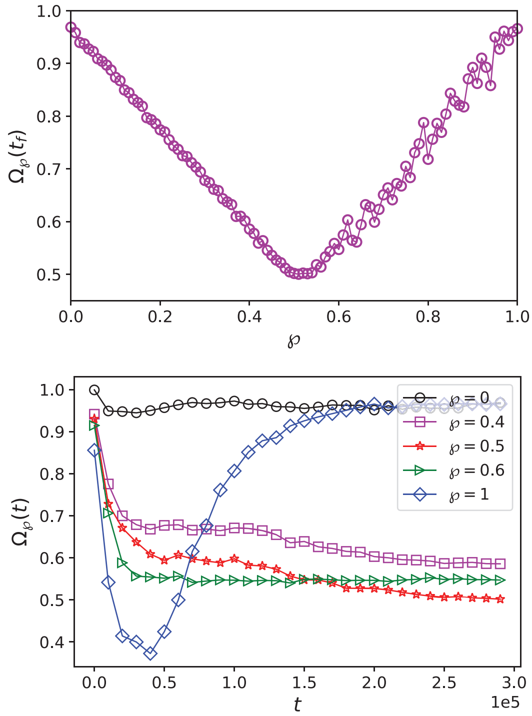


Fig. 4: $\Omega_℘(t)$ *vs.* ambiguity parameter ℘. Left: communicative energy $\Omega_℘(t_t)$ for one initial condition, after $t_f = 3 \times 10^5$ speaker-hearer interactions. ℘ is varied with an increment of 1%. Right: $\Omega_℘(t)$ over time. Points indicate measurements every 10^4 speaker-hearer interactions (starting from $t = 0$). ℘ is varied from $\{0, 0.4, 0.5, 0.6, 1\}$.

each other towards a common vocabulary. To measure this process, we define a simple consensus quantity after t_f speaker-hearer interactions: for each player $k \in P$, we extract the adjacency matrix A^k from its vocabulary B^k. We define then the average adjacency matrix $\bar{A} = \frac{1}{nm}\sum_{k\in P} A^k$. For each matrix A^k, we count the number of (w, m) positions in which A^k and $\bar{A}$ differ. The consensus function $D(t_f)$ denotes therefore the average distance between each matrix A^k and the average matrix $\bar{A}$. With this, $D(t_f) \approx 0$ means that a common word-meaning mapping is reached by the entire population. Figure 2 shows that for all values of the ambiguity parameter ℘, $D(t_f)$ varies from 10^{-2} to 10^{-4}.

Evidence of a drastic transition in language formation. As suggested by previous works (see [3] and [18] for different approaches), three clear domains can be noticed in the behavior of the *effective vocabulary* $V(t)$ *vs.* ℘, at t_f, as shown in fig. 3 (purple circles). First, full vocabularies are attained also for $℘ < 0.4$. Next, a drastic transition appears at the critical domain $℘^* \in (0.4, 0.6)$, in which $V(t_f)$ shifts abruptly towards 0. A small peak of the standard deviation $\sigma(V(t_f))$ is also found at the critical domain. Finally, single-word languages dominate for $℘ > 0.6$.

It is remarkable, in turn, that a Zipfian word-meaning mapping tends to appear at the critical domain $℘^* \in (0.4, 0.6)$, where the communicative costs are shared by both speakers and hearers.

Decentralized minimization of the communicative energy. Figure 4 displays the variation of $\Omega_℘(t)$ *vs.* the ambiguity parameter ℘. First, after t_f speaker-hearer interactions fig. 4 (top) shows that $\Omega_℘(t_f)$ is minimized at the critical domain $(0.4, 0.6)$. There is a clear linear growth of $\Omega_℘(t_f)$ on either side of the minimum energy point $℘^* \approx 0.5$, where the efforts of speakers and hearers are equivalent.

Secondly, fig. 4 (bottom) displays the evolution of $\Omega_℘(t)$ over time t, for different values of ℘. We notice that for values at the critical domain $(0.4, 0.6)$ curves converge smoothly to stationary values. Remarkably, the curves defined by $℘ \in \{0, 1\}$ tend to maximize the communicative energy $\Omega_℘(t)$ towards 1. For $℘ = 1$, the curve first reaches a minimum and then converges to 1. Intriguingly, this speaker-centered scenario exhibits a local minimum

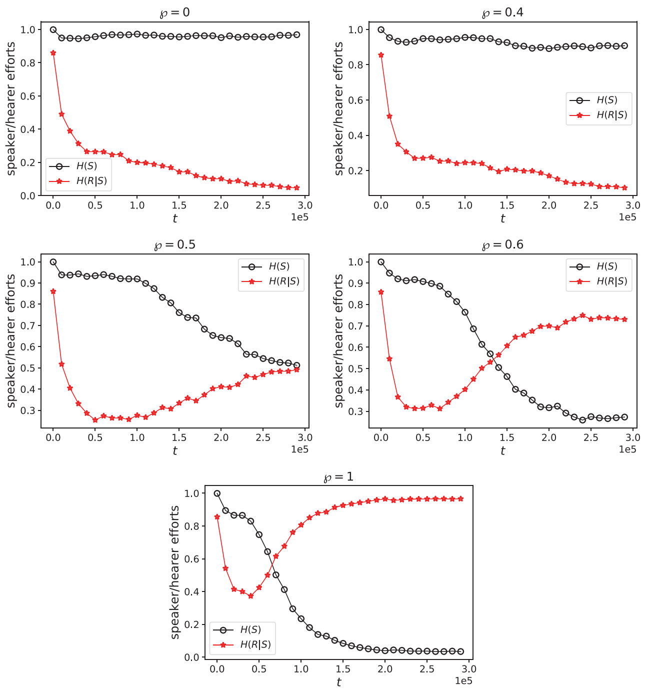


Fig. 5: Evolution of speaker and hearer costs. For $\wp \in \{0, 0.4, 0.5, 0.6, 1\}$), panels display the evolution of hearer (red stars) and speaker (black circles) costs: respectively, $H(R|S)$ and $H(S)$. Points indicate measurements every 10^4 speaker-hearer interactions (starting from $t = 0$).

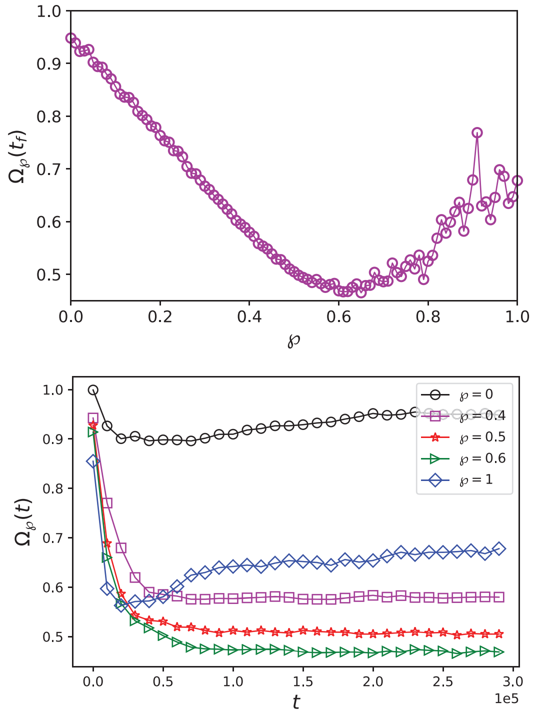


Fig. 6: $\Omega_\wp(t)$ *vs.* ambiguity parameter $\wp$ for radius 2. Left: communicative energy $\Omega_\wp(t_t)$ for one initial condition, after $t_f = 3 \times 10^5$ speaker-hearer interactions. $\wp$ is varied with an increment of 1%. Right: $\Omega_\wp(t)$ over time. Points indicate measurements every 10^4 speaker-hearer interactions (starting from $t = 0$). $\wp$ is varied from $\{0, 0.4, 0.5, 0.6, 1\}$.

of energy. For $\wp = 0$, the curve shows a stationary value around 1.

Evolution of speaker and hearer costs. Figure 5 shows the evolution of hearer $H(R|S)$ and speaker $H(S)$ costs over linguistic interactions. As we can observe in the different panels, the dynamics for $\wp \in \{0, 1\}$ exhibits opposite behaviors. As expected, for $\wp = 0$ hearer costs $H(R|S)$ are minimized, while $H(S)$ remains approximately constant. For $\wp = 1$, $H(R|S)$ and $H(S)$ curves converge respectively to 1 and 0. At the critical parameter $\wp^* \sim 0.5$ both $H(R|S)$ and $H(S)$ tend to converge very slowly to 0.5, which is equivalent to $\Omega_\wp(t_f) = 0.5H(R|S) + 0.5H(S) \sim 0.5$.

Influence of topology on the minimization of communicative energy. Consensus emerges from pairwise interactions between speakers and hearers. In line with [19], it is natural thus to ask to what extent the "form" of agent's neighborhood influences the formation of Zipfian properties. We consider agents located on the vertices of a one-dimensional ring, where speaker-hearer interactions occur at radii 2, 25 and 50. With this, short-range and long-range influences are studied. Figures 6, 7 and 8 exhibit the role of radius on the evolution of $\Omega_\wp(t)$. The convergence time T_c becomes an essential quantity to explore the consensus process on different topologies for the agent-based model proposed here. Indeed, the consensus is reached approximately in a time $T_c^{radius=2} \sim 2 \times 10^5$, $T_c^{radius=25} \sim 0.75 \times 10^5$, $T_c^{radius=50} \sim 0.5 \times 10^5$ and $T_c^{MF} \sim 3 \times 10^5$. At the same time, the results show that T_c is inversely related to the appearance of a drastic linear growth on either side of the critical energy point $\wp^*$. This fact suggests that these topologies induce a trade-off between optimized convergence times T_c and the appearance of Zipfian properties. For the scenarios over a ring (with radius 2, 25 and 50), the consensus process is optimized regarding T_c, while $\Omega_\wp(t_f)$ shows a smoother transition. The analysis of more complex topologies (*e.g.*, two-dimensional lattice or random graphs), inspired by [19], exceeds the goals of this paper and involves high computational costs.

Discussion. –

Decentralized solution to the emergence of Zipfian properties. In this paper, we proposed a novel way to understand the emergence of Zipfian properties in language, through a decentralized approach in which pairs of agents negotiate bipartite word-meaning mappings. Agents are able to select one word to express one meaning according to lexical constraints measured by the parameter $\wp$. We extended in some sense previous information-theoretic accounts for the appearance of a phase transition between two extreme cases of communication systems, only focused on the optimization of one word-meaning mapping.

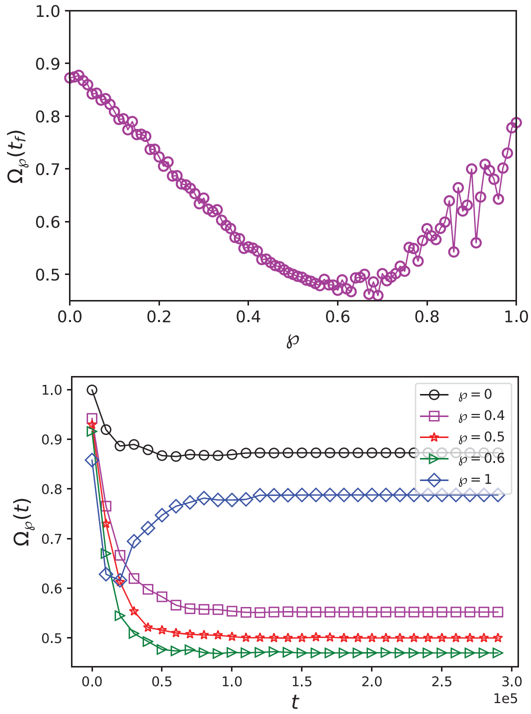


Fig. 7: $\Omega_\wp(t)$ *vs.* ambiguity parameter $\wp$ for radius 25. Left: communicative energy $\Omega_\wp(t_t)$ for one initial condition, after $t_f = 3 \times 10^5$ speaker-hearer interactions. $\wp$ is varied with an increment of 1%. Right: $\Omega_\wp(t)$ over time. Points indicate measurements every 10^4 speaker-hearer interactions (starting from $t = 0$). $\wp$ is varied from $\{0, 0.4, 0.5, 0.6, 1\}$.

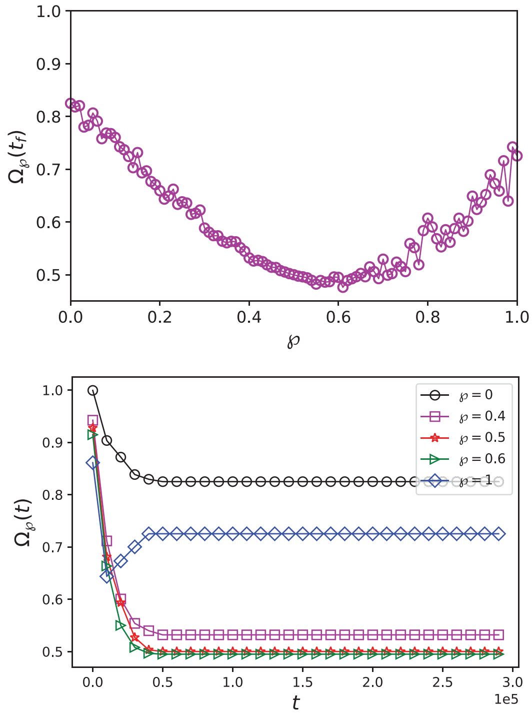


Fig. 8: $\Omega_\wp(t)$ *vs.* ambiguity parameter $\wp$ for radius 50. Left: communicative energy $\Omega_\wp(t_t)$ for one initial condition, after $t_f = 3 \times 10^5$ speaker-hearer interactions. $\wp$ is varied with an increment of 1%. Right: $\Omega_\wp(t)$ over time. Points indicate measurements every 10^4 speaker-hearer interactions (starting from $t = 0$). $\wp$ is varied from $\{0, 0.4, 0.5, 0.6, 1\}$.

This paper reconciles therefore information-theoretic and decentralized frameworks including a simple, but important, novel ingredient: pairwise speaker-hearer interactions between agents calculating their own lexical efforts.

Despite the fact that this paper proposes a decentralized approach to the emergence of Zipfian properties, three main issues require further work. First, the ambiguity constraint is implemented only on the speaker side of the interactions. This should be counteracted by some other factor on the hearer side. One interesting example could be a limit on the number of words an agent can remember (a kind of memory), so that we do not force to end up with a one-to-one word-meaning mapping. Second, in our model the *alignment* strategy only penalizes multiple words for the same meaning, and does not penalize having different words for each meaning. In precise terms, this is a bias against *synonymy*, not *ambiguity*. Third, the parameter $\wp$ works as a probability to select the most-ambiguous word. Future work could compare different ways to define $\wp$. For example, it would be natural to choose w^* uniformly at random.

Emergence of human-like properties in agent-based models. From the above discussion, some crucial questions appear: What properties of human language may emerge as a consequence of decentralized processes? Is there a causal relationship between the proposal of agent-based versions of a human-like language feature and the decentralized "existence" of such feature? How does our decentralized approach to Zipf-type behavior provides positive arguments in favour of this causal relationship? The answers are not obvious. One possible answer arises from a fascinating aspect of human language: individuals are not isolated "talking heads", but rather they belong to communities, in which word-meaning negotiations may occur.

Another ingredient of the answer is the debate opened by [20], about the consequences of the emergence of Zipfian properties for syntax and symbolic reference. According to this debate, the Zipfian phase transition is a necessary precondition for full syntax, and for going beyond simple word-meaning mappings. Moreover, the appearance of syntax has been as abrupt as the transition between the two previously discussed cases of communication systems. This is a key aspect: only in a community of individuals the appearance of Zipf-type behavior, and its consequences for syntax, have a communicative value.

Minimization of $\Omega_{\wp(t_f)}$. How to explain the linear growth of $\Omega_\wp(t_f)$ on either side of the central turning point $\wp^* \approx 0.5$ (as shown in fig. 4)? Why is this behavior linear? A first observation relies on the fact that around $\wp^*$

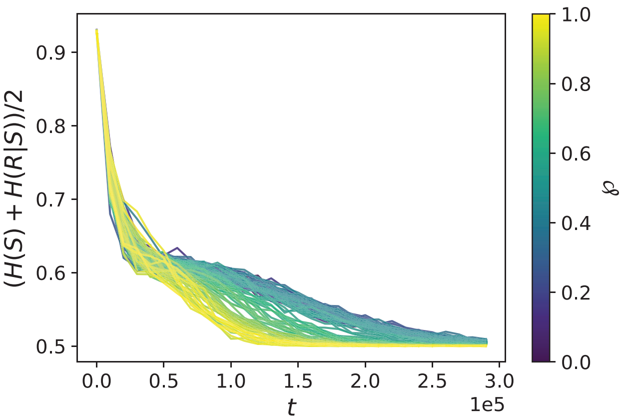


Fig. 9: Evolution of pseudo-energy function. For several values of $\wp$ (varying from 0 to 1 with an increment of 1%), the figure displays the evolution of the pseudo-energy function $(H(S) + H(R|S))/2$ over time t. Points indicate measurements every 10^4 speaker-hearer interactions (starting from $t = 0$).

$\Omega_{\wp(t_f)}$ is symmetric. To study in detail this phenomenon, fig. 9 displays the evolution over time of the *pseudo-energy* function, simply defined by $(H(S) + H(R|S))/2$. Different colors represent different values of the parameter $\wp$. A central observation is that pseudo-energy curves decrease smoothly towards the global minimum 0.5.

Our results suggest that the model converges to a scenario in which speaker constraints and decodification efforts for the hearer have a similar contribution to $\Omega_{\wp}(t_f)$. At this scenario, the dynamics tends to the same final value of $(H(S) + H(R|S))/2 \approx 0.5$ (fig. 9). This fact strongly suggests that the population of individuals reaches a shared word-meaning mapping that conserves the sum of $H(S)$ and $H(R|S)$.

* * *

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