LETTER

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Aharonov-Bohm effect on the generalized Dirac oscillator in a cosmic dislocation space-time

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Abstract – In this work, we investigate the spin-half relativistic particles described by the Dirac equation on the topological defect background induced by the cosmic string and torsion with an internal magnetic field. We derive the general expression of the generalized Dirac oscillator on the topological defect background and analyze the analogue of the Aharonov-Bohm effect for the Dirac oscillator with function $f(\rho)$ considered as the Cornell potential, and we explain the influence of related parameters on the energy levels of the studied system.

Introduction. – We study the influence of gravitational effect on quantum systems that have attracted extensive attention and have been widely studied such as topological defects in static or rotating cosmic string [1–9], domain wall [10] and global monopole [11]. Note that topological defects associated with cosmic string appear due to symmetry breaking phase transition in the initial universe [12–14], Applications of topological property on quantum systems may construct a subtle connection for microscopic quantum theories and macroscopic scales [15–17]. Research showed topological defects of curvature and torsion called as dislocation in condensed matter systems in recent years [18–20]. In particular, topological defects associated with dislocation have been finished in crystalline solids via the differential geometry method [21,22]. Topological defect associated with dislocation have been studied in quantum systems. The relativistic quantum systems such as the spiral dislocation have been investigated in a scalar field in a non-inertial frame [23], and the screw dislocation has been studied in the KG and Dirac oscillator [24–26]. In contrast, examples of well-known works include the spiral dislocation in harmonic oscillator [27], the screw dislocation in the harmonic and doubly anharmonic oscillator [28–30] and an electric dipole [31] and non-inertial effects on a non-relativistic spin-$\frac{1}{2}$ Dirac particle in non-relativistic quantum systems [32].

Our work is motivated by the Dirac oscillator on the topological defects background with an internal magnetic field. [33]. Carvalho, Furtado and Moraes derived the eigenfuntions and energy eigenvalues of considered systems and analyzed the analogue of the Aharonov-Bohm effect. Further, the generalized Dirac oscillator has been introduced through a generalized momentum operator, which means that the radial coordinate ρ is replaced by a similarly potential function $f(\rho)$ in studied systems, and the generalized Dirac oscillator considered as different potential functions in cosmic string space-time has been investigated [34]. It is noteworthy that the generalized KG oscillator [35–38] and DKP oscillator [39,40] interacting with the topological defects have been widely addressed. Therefore, we are interested in the study of the Dirac oscillator with function $f(\rho)$ to be considered as the Cornell potential on the topological defect background induced by the cosmic string space-time with a space-like dislocation with an internal magnetic field.

The generalized Dirac oscillator in a cosmic dislocation space-time. – We start with the analysis of generalized Dirac oscillator on the topological defect background induced by the cosmic string space-time with a space-like dislocation [41,42], the expression corresponding

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to line element in cylindrical coordinates reads ($c=\hbar=1$)

$$ds^2=-dt^2+d\rho^2+\alpha^2\rho^2 d\phi^2+(dz+\chi d\phi)^2, \quad (1)$$

where $0<\alpha<1$ indicates the cosmic string parameter, and χ represents the dislocation (torsion) parameter. Note that these parameters are associated with the Burgers vector $\mathbf{b}$ via $\chi=(b/2\pi)$ in condensed matter physics [22,41]. We can recognize that the cosmic string and Minkowski flat space are recovered if the related parameters satisfy $\chi\to 0$ and $\chi\to 0$, and $\alpha\to 1$ in metric (1), respectively. Further, the spin-$\frac{1}{2}$ relativistic particle described by the Dirac equation on the topological defect associated with a cosmic string and torsion reads [33]

$$[i\gamma^\mu(x)\partial_\mu-i\gamma^\mu(x)\Gamma_\mu(x)-m]\Psi(t,x)=0, \quad (2)$$

where $\Gamma_\mu(x)$ indicates the spinor affine connection and m denotes the mass of the particle, the covariant Clifford algebra $\gamma^\mu(x)=e^\mu_a(x)\gamma^a$ satisfies the relation $\{\gamma^\mu,\gamma^v\}=2g^{\mu\nu}$. In this case, the basis tetrad e^μ_a is chosen as

$$e^\mu_a(x)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\phi & \sin\phi & 0\\ 0 & -\dfrac{\sin\phi}{\alpha\rho} & \dfrac{\cos\phi}{\alpha\rho} & 0\\ 0 & \dfrac{\chi\sin\phi}{\alpha\rho} & -\dfrac{\chi\cos\phi}{\alpha\rho} & 1\end{pmatrix}. \quad (3)$$

It is worth emphasizing that the tetrad components e^μ_a satisfy the relation $e^a_\mu(x)e^b_v(x)\eta_{ab}=2g_{\mu v}$. Therefore, we can easily know that the Dirac matrices $\gamma^\mu(x)$ are written as

$$\begin{aligned}\gamma^0&=\gamma^t,\quad \gamma^1=\gamma^\rho=\cos\phi\gamma^1+\sin\phi\gamma^2,\\ \gamma^2&=\gamma^\phi=\frac{-\sin\phi\gamma^1+\cos\phi\gamma^2}{\alpha\rho},\end{aligned} \quad (4)$$

$$\begin{aligned}\gamma^3&=\gamma^z-\frac{\chi}{\alpha\rho}\gamma^\phi=\gamma^z-\frac{\chi}{\alpha\rho}(-\sin\phi\gamma^1+\cos\phi\gamma^2),\\ \gamma^\mu\Gamma_\mu(\mathrm{x})&=\frac{1-\alpha}{2\alpha\rho}\gamma^\rho.\end{aligned} \quad (5)$$

In addition, the Dirac oscillator is denoted by the spin-$\frac{1}{2}$ particles described the Dirac equation which interact with the linear interactions. In other words, the Dirac oscillator with the oscillator frequency ω can be presented via the non-minimal substitution [43] $p_\mu\to p_\mu+m\omega\beta x_\mu$, it can be explained as strong-spin orbit coupling term in non-relativistic limit [44,45]. Further, the generalized Dirac oscillator has been defined by replacing momenta [34]

$$p_\mu\longrightarrow p_\mu+m\omega\beta f_\mu(x_\mu)\delta^\rho_\mu, \quad (6)$$

where $f(\rho)$ indicates similarly potential function. We assume the generalized Dirac oscillator on the topological defect background induced by the cosmic string space-time with a space-like dislocation with an internal magnetic field. Beside, the magnetic flux tube in topological defect background described by the line element (1) is associated with the magnetic field $\vec{B}=\frac{\Phi_B\delta(\rho)}{2\pi\alpha\rho}\hat{z}$ [46], and the magnetic vector potential in the Coulomb gauge is considered as $\vec{A}_\mu=\frac{\Phi_B}{2\pi\alpha\rho}\hat{e}_\phi$ [33], which means related flux tube subjected to the cosmic string and the z-axis. In this case, eq. (2) can be re-expressed as

$$\begin{aligned}&\left[-i\gamma^t\partial_t+i\gamma^\rho\left(\partial_\rho+mw\beta f(\rho)+\frac{\alpha-1}{2\alpha\rho}\right)\right]\Psi(t,\vec{r})\\ &+\left[i\frac{\gamma^\phi}{\alpha\rho}\left(\partial_\phi-\chi\partial_z+i\frac{e\Phi_B}{2\pi}\right)+i\gamma^z\partial_z-m\right]\Psi(t,\vec{r})=0.\end{aligned} \quad (7)$$

By assuming radial coordinate ρ without dependence and rotational symmetry for the background around the z-axis, the solution can be chosen as

$$\Psi=e^{-iEt+i(l+1/2-\Sigma^3/2)\phi+ikz}\begin{bmatrix}\zeta_1(\rho)\\ \zeta_2(\rho)\end{bmatrix}, \quad (8)$$

by substituting eq. (8) into eq. (7) via simple calculation, we can obtain

$$\begin{aligned}&\left[\beta^2E+i\beta\gamma^\rho\left(\partial_\rho+\frac{1}{2\rho}+m\omega\beta f(\rho)\right)-\beta m\right]\begin{pmatrix}\zeta_1\\ \zeta_2\end{pmatrix}\\ &-\left[\beta\gamma^\phi\frac{(l+\frac{1}{2}-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha\rho}+\beta\gamma^z k\right]\begin{pmatrix}\zeta_1\\ \zeta_2\end{pmatrix}=0.\end{aligned} \quad (9)$$

In this case, we make use of the property $\gamma^\phi\Sigma^3=i\gamma^\rho$ and combine eqs. (4), (5), it is easy to obtain as a relation

$$\begin{aligned}\beta\gamma^\rho&=\cos\phi\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}+\sin\phi\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix};\\ \beta\gamma^\rho\beta&=\cos\phi\begin{pmatrix}0 & -\sigma^1\\ \sigma^1 & 0\end{pmatrix}+\sin\phi\begin{pmatrix}0 & -\sigma^2\\ \sigma^2 & 0\end{pmatrix};\end{aligned} \quad (10)$$

$$\begin{aligned}\beta\gamma^\phi&=-\sin\phi\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}+\cos\phi\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix};\\ \Sigma&=\begin{pmatrix}\sigma & 0\\ 0 & \sigma\end{pmatrix};\quad \beta\gamma^z=\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}.\end{aligned} \quad (11)$$

With the matrices in eq. (9), we can obtain

$$\begin{aligned}&\left(\Theta_1+\Theta_2-k\sigma^3\right)\left(\Theta_2+\Theta_3-k\sigma^3\right)\zeta_1\\ &-\left(E^2-m^2\right)\zeta_1=0,\end{aligned} \quad (12)$$

$$\begin{aligned}&\left(\Theta_2+\Theta_3-k\sigma^3\right)\left(\Theta_1+\Theta_2-k\sigma^3\right)\zeta_2\\ &-\left(E^2-m^2\right)\zeta_2=0\end{aligned} \quad (13)$$

with

$$\Theta_1=i(\cos\phi\sigma^1+\sin\phi\sigma^2)\left(\partial_\rho+\frac{1}{2\rho}-m\omega f(\rho)\right), \quad (14)$$

$$\Theta_2=-\frac{\left(l+1/2-\chi k+\frac{e\Phi_B}{2\pi}\right)}{\alpha\rho}(-\sin\phi\sigma^1+\cos\phi\sigma^2), \quad (15)$$

$$\Theta_3=i(\cos\phi\sigma^1+\sin\phi\sigma^2)\left(\partial_\rho+\frac{1}{2\rho}+m\omega f(\rho)\right). \quad (16)$$

We eliminate ζ_2 in eqs. (12) and (13) by simple algebraic calculation, the expression corresponding to wave function ζ_1 reads

$$\begin{aligned}&\frac{\mathrm{d}^2\zeta_1}{\mathrm{d}\rho}+\left[E^2-m^2-m^2\omega^2 f^2(\rho)+m\omega\left(\frac{\mathrm{d}f\rho}{\mathrm{d}\rho}\right)-k^2\right]\zeta_1\\&+\frac{1}{\rho}\frac{\mathrm{d}\zeta_1}{\mathrm{d}\rho}-\frac{1}{\rho^2}\left[\frac{1}{4}+i\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}\sigma^1\sigma^2\right]\zeta_1\\&-\frac{1}{\rho^2}\left[\left(\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}\right)^2\right]\zeta_1+\frac{e\Phi_B s}{2\pi\alpha}\sigma^3\frac{\delta(\rho)}{\rho}\zeta_1\\&-2m\omega\frac{f(\rho)}{\rho}\left[i\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}\sigma^1\sigma^2\right]\zeta_1\\&-2m\omega\frac{f(\rho)}{\rho}\left[ik\left(\rho\cos\phi\sigma^1\sigma^3+\rho\sin\phi\sigma^2\sigma^3\right)\right]\zeta_1=0.\end{aligned}\tag{17}$$

In this case, we can know the relation from refs. [33,34]

$$\begin{aligned}&ik\rho\left(\sigma^1\sigma^3\cos\varphi+\sigma^2\sigma^3\sin\varphi\right)\\&+i\sigma^1\sigma^2\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}=-2\vec{s}\cdot\vec{L},\end{aligned}\tag{18}$$

where $\vec{s}=\frac{\vec{\sigma}}{2}$, the value of $\vec{s}\cdot\vec{\sigma}$ indicates $\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{2\alpha}$, and eq. (17) becomes

$$\begin{aligned}&\frac{\mathrm{d}^2\zeta_1}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}\zeta_1}{\mathrm{d}\rho}-\frac{1}{\rho^2}\left[\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}-\frac{1}{2}\right]^2\zeta_1\\&-m^2\omega^2 f^2(\rho)\zeta_1+\left[E^2-m^2+m\omega\left(\frac{\mathrm{d}f\rho}{\mathrm{d}\rho}\right)-k^2\right]\zeta_1\\&+2m\omega\frac{f(\rho)}{\rho}\left[\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}\right]\zeta_1\\&+\frac{e\Phi_B s}{2\pi\alpha}\frac{\delta(\rho)}{\rho}\zeta_1=0.\end{aligned}\tag{19}$$

We use a similar technique as done earlier, the expression corresponding to the wave equation for ζ_2 can be obtained,

$$\begin{aligned}&\frac{\mathrm{d}^2\zeta_2}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}\zeta_2}{\mathrm{d}\rho}-\frac{1}{\rho^2}\left[\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}+\frac{1}{2}\right]^2\zeta_2\\&-m^2\omega^2 f^2(\rho)\zeta_2+\left[E^2-m^2-m\omega\left(\frac{\mathrm{d}f\rho}{\mathrm{d}\rho}\right)-k^2\right]\zeta_2\\&+2m\omega\frac{f(\rho)}{\rho}\left[\frac{(l+1/2-\chi k+\frac{e\Phi_B}{2\pi})}{\alpha}\right]\zeta_2\\&-\frac{e\Phi_B s}{2\pi\alpha}\frac{\delta(\rho)}{\rho}\zeta_2=0.\end{aligned}\tag{20}$$

Note that δ-function terms in eqs. (19), 20) are interpreted as the Zeeman interaction between the spin and the magnetic flux tube [47,48]. In a conical space, the spin-$\frac{1}{2}$ particles coupled to an Aharonov-Bohm potential in the non-relativistic limit are studied by using the self-adjoint extension method [49,50]. The Dirac oscillator considered by the Zeeman interaction in magnetic cosmic string space-time is investigated [51]. We can observe that the generalized Dirac oscillator on the topological defects background is degenerated into the Dirac oscillator if the Dirac oscillator has function $f(\rho)=\rho$ [33]. In the following, we mainly study the effects of the Dirac oscillator with function $f(\rho)$ to be chosen as the Cornell potential on the energy spectrum of the addressed systems.

The Dirac oscillator with the function $f(\rho)$ taken as the Cornell potential. – The Cornell potential is well known for its extensive applications such as heavy quarks and mesons in particle physics [52–57], which contain the short-range Cornell potential characterized quark and gluon interaction and large-distance linear potential. The Cornell potential reads

$$f(\rho)=\frac{\Delta_1}{\rho}+\Delta_2\rho,\tag{21}$$

where the related potential parameter Δ_1 indicates the Coulomb strength, and Δ_2 indicates the string tension from lattice gauge theory associated with the dual string model. In order to facilitate the study for wave equations ζ_1 and ζ_2, all parameters containing $+$ correspond to ζ_1, and all parameters containing $-$ are associated with ζ_2. Based on this principle, by substituting eq. (21) into eq. (19) and eq. (20) we find the related equation

$$\begin{aligned}&\frac{\mathrm{d}^2\zeta^\pm}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}\zeta^\pm}{\mathrm{d}\rho}-\frac{1}{\rho^2}\nu_\pm^2\zeta^\pm+2m\omega\Delta_2\beta^\pm-m^2\omega^2\Delta_2^2\rho^2\zeta^\pm\\&+\left[E^2-m^2-k^2-2m^2\omega^2\Delta_1\Delta_2+\aleph^\pm s\frac{\delta(\rho)}{\rho}\right]\zeta^\pm=0,\end{aligned}\tag{22}$$

with

$$\begin{aligned}\nu_+&=\left[\frac{\left(l+1/2-\chi k+\dfrac{e\Phi_B}{2\pi}\right)}{\alpha}-\frac{1}{2}-m\omega\Delta_1\right],\\\nu_-&=\left[\frac{\left(l+1/2-\chi k+\dfrac{e\Phi_B}{2\pi}\right)}{\alpha}+\frac{1}{2}-m\omega\Delta_1\right],\\\beta^+&=\left[\frac{\left(l+1/2-\chi k+\dfrac{e\Phi_B}{2\pi}\right)}{\alpha}+\frac{1}{2}\right],\quad\aleph^\pm=\pm\frac{e\Phi_B}{2\pi\alpha},\\\beta^-&=\left[\frac{\left(l+1/2-\chi k+\dfrac{e\Phi_B}{2\pi}\right)}{\alpha}-\frac{1}{2}\right].\end{aligned}\tag{23}$$

Our interest is to study the Dirac oscillator with function $f(\rho)$ considered the Cornell potential excluding the $\rho=0$ region. In this case, the term $\aleph^\pm\frac{\delta(\rho)}{\rho}$ disappears in eq. (22), by making a change in variables $\eta = m\omega\Delta_2\rho^2$, the equation corresponding to eq. (22) can be re-expressed as

$$\frac{\mathrm{d}^2\zeta^\pm}{\mathrm{d}\eta^2} + \frac{1}{\eta}\frac{\mathrm{d}\zeta^\pm}{\mathrm{d}\eta} - \frac{1}{4\eta^2}\nu_\pm^2\zeta^\pm - \frac{1}{4}\zeta^\pm + \frac{\Upsilon^\pm}{4m\omega\Delta_2\eta}\zeta^\pm = 0, \tag{24}$$

with $\Upsilon^\pm = [E^2 - m^2 - k^2 - 2m^2\omega^2\Delta_1\Delta_2 + 2m\omega\Delta_2\beta^\pm]$. Generally, eq. (24) can be addressed due to the asymptotic limit at critical points $\eta \to 0$ and $\eta \to \infty$, we can assume that physical solutions $\zeta^\pm$ read

$$\zeta^\pm = \eta^{\frac{|\nu_\pm|}{2}} e^{-\frac{\eta}{2}}\Xi(\eta)^\pm, \tag{25}$$

we substitute the above expression into eq. (24) through simple algebra calculation, and obtain the confluent hypergeometric function as follows:

$$\eta\frac{\mathrm{d}^2\Xi^\pm(\eta)}{\mathrm{d}\eta^2} + (|\nu_\pm| + 1 - \eta)\frac{\mathrm{d}\Xi^\pm(\eta)}{\mathrm{d}\eta} - \frac{1}{2}(|\nu_\pm| + 1)\Xi^\pm(\eta) + \frac{\Upsilon^\pm}{4m\omega\Delta_2}\Xi^\pm(\eta) = 0. \tag{26}$$

Note that the polynomial series terminations can lead to normalized solutions. In other words, the corresponding requirement must be satisfied without the dependent term to a zero or negative integer: $\frac{1}{2}(|\nu_\pm| + 1 - \frac{\Upsilon^\pm}{2m\omega\Delta_2}) = -n$.

So, we make use of the properties of the confluent hypergeometric function, and obtain the related solution $\Xi^\pm(\eta)$

$$\Xi^\pm(\eta) = \Xi_1^\pm\left(\frac{|v_\pm| + 1}{2} - \frac{\Upsilon^\pm}{4m\omega\Delta_2}, |v_\pm| + 1; \eta\right). \tag{27}$$

Further, the corresponding eigenfunction of the generalized Dirac oscillator can be expressed as

$$\begin{aligned}\Psi &= e^{-iEt+i(l+1/2-\Sigma^3/2)\phi+ikz}\begin{bmatrix}\zeta_1(\rho)\\ \zeta_2(\rho)\end{bmatrix}\\ &= e^{+i\left(l+1/2-\Sigma^3/2\right)\phi}(m\omega\Delta_2)^{\left|\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha} - \frac{m\omega\Delta_1}{2}\right|}\\ &\times e^{-iEt+ikz}\rho^{\left|\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{\alpha} - m\omega\Delta_1\right|}\\ &\times\Xi_1^\pm\left(\frac{\left|\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{\alpha} - m\omega\Delta_1\right| + 1}{2} - \frac{\Upsilon^\pm}{4m\omega\Delta_2},\right.\\ &\left.\left|\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{\alpha} - m\omega\Delta_1\right| + 1; m\omega\Delta_2\rho^2\right).\end{aligned} \tag{28}$$

Meanwhile, by simple manipulation, energy levels can be expressed as

$$\begin{aligned}E^2 &= 4m\omega\Delta_2\left|\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha} - \frac{m\omega\Delta_1}{2}\right|\\ &-4m\omega\Delta_2\left[\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha} - \frac{m\omega\Delta_1}{2}\right]\\ &+4m\omega\Delta_2\left(n + \frac{1-s}{2}\right) + k^2 + m^2,\end{aligned} \tag{29}$$

with $n = 0, 1, 2, 3, \ldots, s = \pm 1$. We can find the effective angular momentum $l_{eff} = l - \chi k + \frac{e\Phi_B}{2\pi}$ associated with the magnetic quantum flux Φ_B and torsion parameter χ. Further, we can obtain the energy levels of the generalized Dirac oscillator in the presence of cosmic string space-time if the torsion parameter satisfies $\chi = 0$, the corresponding energy eigenvalues can be written as

$$\begin{aligned}E^2 &= 4m\omega\Delta_2\left[n + \left|\frac{\left(l+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha} - \frac{m\omega\Delta_1}{2}\right|\right]\\ &-4m\omega\Delta_2\left[\frac{\left(l+\frac{e\Phi_B}{2\pi}-\frac{1-s}{2}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha} - \frac{m\omega\Delta_1}{2}\right]\\ &+k^2 + m^2.\end{aligned} \tag{30}$$

In addition, the Dirac oscillator in a cosmic dislocation space-time with an internal magnetic field is presented if the Dirac oscillator with function parameters satisfies $\Delta_1 = 0$ and $\Delta_2 = 1$. In this case, the energy eigenvalue is consistent with the expression in [33]

$$\begin{aligned}E^2 &= 4m\omega\left[n + \left|\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha}\right|\right]\\ &-4m\omega\left[\frac{\left(l-\chi k+\frac{e\Phi_B}{2\pi}\right)+\frac{1}{2}(1\mp\alpha)}{2\alpha} - \frac{1-s}{2}\right]\\ &+k^2 + m^2.\end{aligned} \tag{31}$$

We can observe that the obtained energy levels such as eqs. (29)–(31) obviously depend on the geometric phase [58], this phenomenon shows that the geometric quantum phase modifies the energy levels and gives rise to the analogue effect to the Aharonov-Bohm effect [59–61]. We have that $E_{n,\frac{l_{eff}}{\alpha}}(\Phi_B + \Phi_0) = E_{n,\frac{l_{eff}}{\alpha}\mp\gamma}(\Phi_B)$, $\Phi_0 = \pm\frac{2\pi}{e}\gamma$, where γ takes values 1, 2, 3, 4.

Now we analyze the influence of each parameter on the considered system. We only consider energy spectrum corresponding to wave equation ζ_1, and ignore the energy levels associated with wave function ζ_2. For the sake of convenience, we take the natural unit ($\chi = m = k = e = l = 1$).

On the one hand, in table 1, we have obtained the energies of the $n = 1, 2, 3$ states fixing the value of the parameter $s = 1$, $\Delta_2 = \chi = m = e = l = 1$, $k = \Delta_1 = 2$, to observe the effect of oscillator frequency ω, the deficit angle parameter α and the internal magnetic quantum flux

Table 1: The energy levels E of the generalized Dirac oscillator for different values of α, ω and Φ_B with values of the quantum numbers n.

E (MeV) $s=1$, $\Delta_2=\chi=m=e=l=1$, $k=\Delta_1=2$					
n	ω	α	$\Phi_B=1.0$ tesla	$\Phi_B=1.5$ tesla	$\Phi_B=2$ tesla
1	0.5	0.1	4.100841516	3.901967671	3.692397903
		0.3	3.503184319	3.426628649	3.348323077
		0.5	3.370961321	3.323412454	3.275173347
		0.7	3.312679130	3.278182243	3.243318457
		0.9	3.279852658	3.252782599	3.225485360
2	1	0.1	6.052586412	5.783658307	5.501600181
		0.3	5.248295034	5.146218787	5.042076442
		0.5	5.072155405	5.009005958	4.945050142
		0.7	4.994765864	4.949036030	4.902879687
		0.9	4.951249026	4.915403266	4.879294172
3	1.5	0.1	8.028119544	7.725027839	7.409548355
		0.3	7.128597417	7.016078086	6.901724587
		0.5	6.934705522	6.865508796	6.795607505
		0.7	6.849929128	6.799958562	6.749618051
		0.9	6.802374613	6.763267251	6.723932438

Table 2: The energy levels E of the generalized Dirac oscillator for different values of α, Δ_1 and Φ_B with values of the quantum numbers n.

E (MeV) $s=1$, $\omega=\Delta_2=\chi=m=e=l=1$, $k=2$					
n	Δ_1	α	$\Phi_B=1.0$ tesla	$\Phi_B=1.5$ tesla	$\Phi_B=2$ tesla
1	2	0.1	5.712600308	5.426850229	5.125193123
		0.3	4.852277894	4.741684068	4.628448428
		0.5	4.661197319	4.592400318	4.522556899
		0.7	4.576864215	4.526914803	4.476408071
		0.9	4.529334048	4.490121298	4.450563067
2	4	0.1	6.680853409	6.438222069	6.186081518
		0.3	5.961929282	5.872271094	5.781222609
		0.5	5.807474533	5.752403036	5.696799181
		0.7	5.740007495	5.700259435	5.660232259
		0.9	5.702180892	5.671083606	5.639814857
3	6	0.1	7.525543321	7.310998797	7.089965060
		0.3	6.895259296	6.817885875	6.739624237
		0.5	6.762156494	6.714919261	6.667347367
		0.7	6.704303546	6.670304163	6.636130591
		0.9	6.671946262	6.645388572	6.618724319

Φ_B on these states. When we increase α, the energy levels are found to be slightly decreasing. Further, when we increase the internal magnetic quantum flux Φ_B, the energy levels are found to be quickly decreasing. Note that we can see that the energy eigenvalues of the generalized Dirac oscillator obviously increase with parameters oscillator frequency ω and quantum numbers n. On the other hand, in table 2, we have obtained the energies of the $n=1,2,3$ states, fixing the value of the parameter $s=1$, $\omega=\Delta_2=\chi=m=e=l=1, k=2$, to observe the effect of the generalized Dirac oscillator parameter Δ_1, the deficit angle parameter α and the internal magnetic quantum flux Φ_B on these states. By comparing tables 1 and 2, in particular, we can observe that energy states go down to a wider shift from nearly 6.052586412 MeV ($n=2, \omega=1, \alpha=0.1, \Delta_1=2$ in table 1) to 6.680853409 MeV ($n=2, \omega=1, \alpha=0.1, \Delta_1=4$ in table 2) when the strength of the generalized Dirac oscillator parameter Δ_1 changes from 2 to 4, which shows that the parameter Δ_1 has a non-negligible effect on the studied system.

Conclusion. – We have solved the Dirac oscillator with function $f(\rho)$ considered as the Cornell potential on the topological defect background induced by the cosmic string space-time with a space-like dislocation with an internal magnetic field. The eignfunctions and energy eigenvalues are presented by using the confluent hypergeometric function. The Dirac oscillator in a space-like dislocation with an internal magnetic field is obtained if $f(\rho)$ associated with parameters satisfies $\Delta_1=0$, $\Delta_2=1$, the corresponding energy levels are consistent with the energy level given in ref. [33]. We also analyze the analogue of the Aharonov-Bohm effect and observe that the generalized Dirac oscillator associated with parameter Δ_1 has a non-negligible effect on the studied system.

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