A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

LETTER

Edge state magnetoresistance of a two-dimensional topological insulator

To cite this article: L. S. Braginsky and M. V. Entin 2021 *EPL* **134** 37001

A LETTERS JOURNAL EXPLORING
THE FRONTIERS OF PHYSICS

May 2021

EPL, **134** (2021) 37001
doi: 10.1209/0295-5075/134/37001

www.epljournal.org

Edge state magnetoresistance of a two-dimensional topological insulator

L. S. BRAGINSKY[(a)] and M. V. ENTIN

Institute of Semiconductor Physics - 630090 Novosibirsk, Russia and
Novosibirsk State University - 630090 Novosibirsk, Russia

received 28 January 2021; accepted in final form 22 March 2021
published online 14 July 2021

Abstract – The magnetoresistance theory of the edge state of a two-dimensional topological insulator is developed. The magnetic field violates the time reversal invariance. Magnetoresistance arises due to the energy gap opened by a magnetic field. The combined action of impurities and magnetic field causes the backscattering of edge electrons. Although impurities are necessary for scattering, a sufficiently strong interaction with impurities leads to backscattering suppression.

Introduction. – The edge states of 2D topological insulators is one of the most inspiring problems of modern solid state physics. It was shown that these states possess the so-called topological protection, preventing an electron from backscattering. At the moment there is a large and rapidly growing number of publications on this issue (see, *e.g.*, [1–4]). The study of the edge states conductance of the two-dimensional topological insulator assumes their topological protection. This makes backscattering prohibited or extremely weak. Therefore, one-dimensional states are collisionless and the conductance of electrons on them is e^2/h.

There were some attempts to realize the mechanisms violating the time reversibility and, therefore, causing backscattering. In particular, refs. [5,6] ascribe backscattering to the transitions between the edges with an opposite direction of the travel of same-spin electrons. The transitions between the opposite edges of a TI strip were considered also in ref. [7].

The present paper was stimulated by the experimental finding [8] of the strong magnetoresistence of the edge state electron conductance. The important experimental fact that has to be explained is the presence of magnetoresistance, as well as its gigantic value and fluctuations with the Fermi level. Unusual is the sensitivity of 1D edge states to the magnetic field, which is absent in other 1D systems. The results of [8] cannot be explained by means of interedge transitions [7] due to a large TI strip width value in the experimental conditions.

[(a)]E-mail: brag@isp.nsc.ru (corresponding author)

The model is based on the one-dimensional Hamiltonian

$$H = \sigma_z vp + V(x) + H_{nd} = \begin{pmatrix} vp + V(x) & \Delta_1 \\ \Delta_1^* & -vp + V(x) \end{pmatrix}. \tag{1}$$

Here p and v are the electron momentum and velocity, σ_i are the Pauli matrices, and $V(x)$ is the impurity potential. Hamiltonian (1) is the simplest one that violates the time reversal invariance in the magnetic field and leads to the gap $2|\Delta_1|$ at $p = 0$.

The gap in the electron spectrum of the 2D topological insulator arises in the magnetic field $\mathbf{B}$ via the off-diagonal Zeeman Hamiltonian [9]

$$H_{nd} = \frac{\mu_B}{2} g_{ij} \sigma_i B_j. \tag{2}$$

Here μ_B is the Bohr magneton. Within the 4×4 **kp** Bernevig-Hue-Zhang Hamiltonian for the grown $(0,0,1)$ CdTe-HgTe-CdTe structure, the non-zero g-factor components are

$$\begin{aligned} g_{xx} &= \frac{1}{2}(g_e^{||} - g_h^{||}), \\ g_{yy} &= \frac{1}{2}(g_e^{||} + g_h^{||}) \frac{-\delta}{\sqrt{\delta^2 + \gamma^2}}, \\ g_{yz} &= \frac{2m_0 \mathcal{A}^2}{\hbar^2} \frac{-\delta\gamma}{(\delta^2 + \gamma^2)^{3/2}}. \end{aligned} \tag{3}$$

Component g_{zz} can be excluded by the gauge transformation. Parameters $\mathcal{A} = 3.6\,\text{eV Å}$, $\mathcal{B} = -68\,\text{eV Å}^2$, $\gamma = 5\,\text{meV}$, $g_e^{||} = -20$, and $g_h^{||} = 0$ are the standard parameters of the bulk HgTe Hamiltonian [10]. The calculations using ref. [9] data yield $g_{xx} = -10$, $g_{yy} = -8.9$, and $g_{yz} = 135$.

We see that the value of g_{yz} essentially exceeds those of g_{xx} and g_{yy}. Taking into account that these components are incorporated into the final result quadratically, we conclude that the sensitivity of the conductivity to the out-of-plane magnetic field is much stronger than that to the in-plane magnetic field.

If $B_z = 0$, then $\Delta_1 = g\mu_B B = \mu_B(g_{xx}B_x + ig_{yy}B_y)$ for the (0,0,1) orientation. Here μ_B is the Bohr magneton, g_{xx} and g_{yy} are the g-factor components in the specimen plane, $g = g_{xx}\cos\theta + ig_{yy}\sin\theta$, where θ is the angle between $\mathbf{B}$ and the edge direction ($0x$).

The gap is

$$2|\Delta_1| = \mu_B\sqrt{g_{xx}^2 B_x^2 + g_{yy}^2 B_y^2}.$$

At a weak in-plane magnetic field the gap is negligible. However, the presence of the off-diagonal part H_{nd} in the Hamiltonian (1), together with the impurity potential, leads the electrons to backscattering. The potential itself does not cause any transitions between the states.

The backscattering time τ is the most important parameter of the electron transport at the edge states both in classical and localization regimes. The purpose of the present paper is to obtain τ in the presence of magnetic field.

Magneto-induced backscattering. – The self-functions of the Hamiltonian (1) corresponding to the energies $\epsilon_\pm = \pm vp$ at $B = 0$ are

$$\psi_+ = \frac{1}{\sqrt{L}}\begin{pmatrix}1\\0\end{pmatrix} e^{ipx + i\int \mathrm{d}x V(x)/v},$$
$$\psi_- = \frac{1}{\sqrt{L}}\begin{pmatrix}0\\1\end{pmatrix} e^{ipx - i\int \mathrm{d}x V(x)/v}.$$

The backscattering amplitude is determined by the first-order perturbation on the non-diagonal part of the Hamiltonian $H_{nd} \propto \Delta_1$, *i.e.*, the matrix element between the $|\psi_{-p,-\sigma}\rangle$ and $|\psi_{p,\sigma}\rangle$ states of the same energy $\epsilon_{p,\sigma}$:

$$\begin{aligned} A &= \langle\psi_{-p,-}|H_{nd}|\psi_{p,+}\rangle \\ &= \frac{\Delta_1}{L}\int_{-L}^{L}\exp\left(2ipx + \frac{2i}{v}\int V(x)\,\mathrm{d}x\right)\mathrm{d}x. \end{aligned}$$

Weak impurity potential. The perturbation theory. – In the first order with regard to potential $V(x)$ we find:

$$\begin{aligned} A &= 2i\langle\psi_{-p,-}|H_{nd}|\psi_{p,+}\rangle = \frac{2i\Delta_1}{Lpv}\tilde{V}(2p), \\ \tilde{V}(p) &= \int_{-\infty}^{\infty} V(x)e^{ipx}\mathrm{d}x. \end{aligned}$$

Consider the electron scattering at the impurity potential $V(\mathbf{r}) = \sum_n u(\mathbf{r} - \mathbf{r}_n)$, where r_n is the n-th impurity position and $u(\mathbf{r} - \mathbf{r}_n)$ is its potential. The probability of backscattering has to be averaged over the random impurity positions in the 2D layer, so that

$$1/\tau = \frac{L}{v}\langle|A|^2\rangle. \tag{4}$$

The 2D Fourier transform of the Coulomb impurity is

$$u(q) = \frac{2\pi e^2}{\kappa q}.$$

Therefore,

$$\langle|V(q_x)|^2\rangle = n_i\int\frac{\mathrm{d}q_y}{2\pi}u^2(q) = 2\pi^2 n_i\frac{e^4}{q_x\kappa^2}, \quad q^2 = q_x^2 + q_y^2.$$

Here n_i is the impurity density. For the backscattering probability we find

$$1/\tau_0 = \frac{8\pi^2 e^4 n_i|\Delta_1|^2}{\hbar\kappa^2(vp_F)^3} = \frac{8e^4 n_i|\Delta_1|^2}{\hbar^4\kappa^2 v^3\pi n_e^3}, \tag{5}$$

where n_e is the 1D electron density.

Let us estimate the value of τ_0 for out-of-plane magnetic field $B_z = 1$ T. Utilizing parameters $v = 4.2\cdot 10^7$ cm/s [11] $n_e = 10^6$ cm^{-1}, $n_i = 10^{11}$ cm^{-2}, and $\kappa = 20$, we obtain $\tau_0 = 0.2\cdot 10^{-9}$ s. However, $\tau_0 = 36.5\cdot 10^{-9}$ for the in-plane magnetic field $B_z = B_y = 0$, and $B_x = 1$ T.

The probability (5) essentially increases at a small electron density. Therefore, screening of the Coulomb potential by the gate has to be taken into account. Then

$$u(q) = \frac{2\pi e^2}{\kappa q}(1 - e^{-2qd}). \tag{6}$$

Here d is the distance to the gate. The last factor in eq. (6) limits the impurity matrix elements to small q values. Therefore, for the $1/\tau$ value, we obtain:

$$\frac{1}{\tau} = \frac{8\pi^2 e^4 n_i|\Delta_1|^2}{\hbar\kappa^2(vp_F)^3}\phi(p_F d), \tag{7}$$

where

$$\phi(y) = \frac{2}{\pi}\int_0^\infty\frac{(1 - e^{-4y\cosh t})^2}{\cosh t}\mathrm{d}t,$$

$\phi(y) \approx 16y\ln 2/\pi$ at $y \ll 1$ and $\phi(y) \to 1$ at $y \to \infty$. Thus, screening reduces the divergence of the $1/\tau$ value at a small electron density: $\tau \propto n_e^2$, instead of $\tau \propto n_e^3$ for $n_e \to 0$.

It should be noted that the expressions obtained in this section hold only for small impurity potentials. This confirms the applicability of the perturbation theory for this potential as well as for the gap value.

Strong impurity potential. – The conductivity of an electron gas with a quadratic spectrum is too complicated if it is considered beyond the frames of the perturbation theory. This is not the case for the 1D electron gas with

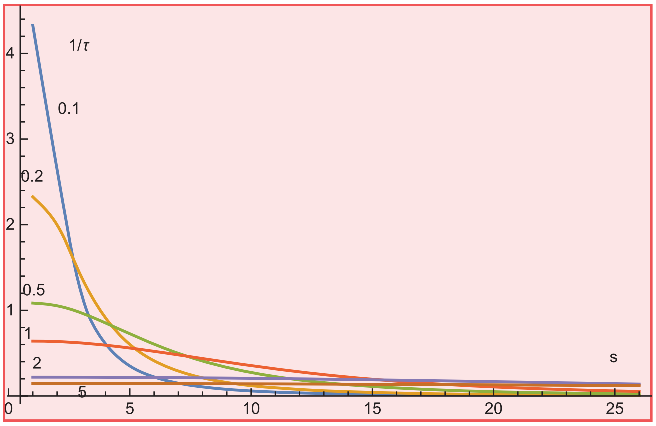

Fig. 1: $1/\tau$ (in units of $1/\tau^* = 2(g\mu_B B)^2 d/(v\hbar^2)$) *vs.* s and β (shown at the curves).

a linear spectrum because this problem has an exact solution [12]. This allows us to obtain the result at $n_e \to 0$.

Consider the problem of a small gap and arbitrary impurity potential. Assume the random impurity potential obeys the Gaussian distribution. This is correct if $|u(\boldsymbol{r} - r_n| \ll vp_F$. Then, in the Gaussian approximation from eq. (4), we obtain

$$\langle |A|^2\rangle = L^{-2}|\Delta_1|^2 \int\int \exp\Big[2ip(x-x') - \frac{4}{v^2}\int\int_{x'}^{x} \mathrm{d}x_1\mathrm{d}x_2 W(x_1-x_2)\Big]\mathrm{d}x\mathrm{d}x',$$

where

$$W(x-x') \equiv \int_{-\infty}^{\infty} \mathrm{d}k(2\pi)^{-1}e^{-ik(x-x')}\tilde{W}(k) = \langle V(x)V(x')\rangle$$

is the potential correlation function. Its Fourier transform is

$$\tilde{W}(k) = \frac{4\pi^2 e^4}{\kappa^2} n_i \frac{\phi(kd/2)}{k}.$$

Then using eq. (4) we obtain

$$\frac{1}{\tau} = 2(d/v)|\Delta_1|^2 \times \int_{-\infty}^{\infty} \mathrm{d}z \cos(sz)\exp\left(-\beta\int\frac{\mathrm{d}y}{y^3}\phi(y/2)\sin^2(yz)\right), \tag{8}$$

$$s = 4p_F d/\hbar,\ \beta = \frac{2\pi e^4}{\kappa^2 v^2\hbar^2} n_i d^2.$$

The dependence of $1/\tau$ *vs.* s is presented in fig. 1. At $B_z = 1\,\mathrm{T}$ and $d = 10^{-6}\,\mathrm{cm}$ $\tau^* \approx 0.2\cdot 10^{-11}\,\mathrm{s}$. We have restored $\hbar$ in the last expression. The $1/\tau$ quantity has linear dependence on β at small β values (or at a small impurity density, in agreement with eqs. (5), (7)). At large β this dependence has an exponential decrease. This decrease results from the random potential, which increases the local Fermi momentum and, consequently, reduces the scattering.

Electron transport on the edge states. – In the classical approach the edge conductivity of an infinite sample at a low temperature is expressed via the backscattering time τ by the Drude expression

$$\sigma = \frac{e^2\tau v}{\pi}. \tag{9}$$

The classical approach requires the absence of the phase coherence at a finite temperature (similar problem was considered in connection with the TI strip [7]). The phase coherence violates the applicability of the classical approach. The dephasing occurs due to non-elastic processes (*e.g.*, electron-phonon interaction) when an electron, traveling at the distance $v\tau$ forward and backward, randomly changes the phase to an opposite one. Dephasing originates from the the phonon field fluctuations in the absence of transitions between the electron branches and may not include backscattering. Hence, at a small magnetic field and finite temperature, the backscattering can be weaker than the dephasing. That restores the classical approach.

This means that the conductivity obeys the Drude expression (9). At zero temperature, however, the dephasing is switched out. In this case the edge states become localized. Therefore, we need to consider the conductance G of a finite sample with a length L. Without backscattering the zero temperature conductance of the edge state G is equal to the conductance quantum $G_0 = e^2/h$ independently of the edge length.

The presence of backscattering leads to the localization of the edge states and, therefore, to the exponential drop of the conductance, if the edge length exceeds the backscattering length: $\ln G/G_0 \propto -L/v\tau$; or $\ln G/G_0 \propto -B^2$ in agreement with the obtained τ behavior. Then the τ dependence on the magnetic field results in the magnetoconductivity of an infinite sample as well as magnetoconductance of a finite sample. They are characterized by negative quadratic dependencies on a low magnetic field.

Discussions. – Thus, we found that the magnetic field results in the elastic backscattering of the edge states electrons with the linear spectrum. At zero temperature this scattering leads to the localization in an infinite system. In the system of finite size L the zero-temperature conductance G exponentially decays with the ratio of L to the back scattering length $v\tau$. The dependence $\tau \propto B^2$ yields a similar dependence of $\log G$.

At non-zero temperature the finite conductivity of infinite system is established due to dephasing processes. At low B the dephasing time is shorter than τ. In this case the conductivity is proportional to B^{-2}.

It is shown that the probability of backscattering increases with the impurity concentration at a low concentration followed by a decrease at a high impurity concentration.

Note that our results are correct, if only the Fermi level is far apart from the gap $|E_F| \gg |\Delta_1|$. This is necessary to justify the expansion over the magnetic field.

We have found that the out-of-plane magnetic field is much more effective for magnetoresistance than the in-plane one. However, this result has been obtained in the Bernevig-Hueghes-Zhang model with the interface-induced mixing of the electron and heavy hole states. It can be revised in a more general model.

Another note concerns the non-magnetic backscattering mechanisms. It is shown in [13] that in a TI with a smooth edge the overlapping of linear topology-protected edge states with Dirac gapped branches leads to elastic backscattering. In this case, the magneto-induced backscattering complements the latter in the energy domains where these kinds of scattering coexist.

* * *

The work is supported by the RFBR Grant No. 20-02-00622.

REFERENCES

[1] Hasan M. Z. and Kane C. L., *Rev. Mod. Phys.*, **80** (2010) 3046.

[2] Qi X.-L. and Zhang Sh.-Ch., *Rev. Mod. Phys.*, **83** (2011) 1057; Fruchart M. and Carpentier D., *C. R. Phys.*, **14** (2013) 779.

[3] Shen S.-Q., *Topological Insulators: Dirac Equation in Condensed Matters*, *Springer Series in Solid-State Sciences*, Vol. **174** (Springer-Verlag, Berlin, Heidelberg) 2012.

[4] Bernevig B. Andrei, *Topological Insulators and Topological Superconductors* (Princeton University Press) 2013.

[5] Väyrynen J. I., Goldstein M. and Glazman L. I., *Phys. Rev. Lett.*, **110** (2013) 216402.

[6] Väyrynen J. I., Goldstein M., Gefen Y. and Glazman L. I., *Phys. Rev. B*, **90** (2014) 115309.

[7] Magarill L. I. and Entin M. V., *JETP Lett.*, **100** (2014) 541.

[8] Piatrusha S. U., Tikhonov E. S., Kvon Z. D., Mikhailov N. N., Dvoretsky S. A. and Khrapai V. S., *Phys. Rev. Lett.*, **123** (2019) 056801.

[9] Durnev M. V. and Tarasenko S. A., *Phys. Rev. B*, **93** (2016) 075434.

[10] König M., Buhmann H., Molenkamp L. W., Hughes T., Liu C.-X., Qi X.-L. and Zhang S.-C., *J. Phys. Soc. Jpn.*, **77** (2008) 031007.

[11] Qi X.-L. and Zhang S.-C., *Rev. Mod. Phys.*, **83** (2011) 1057.

[12] Entin M. V. and Braginsky L., *EPL*, **120** (2017) 17003.

[13] Mahmoodian M. M. and Entin M. V., arXiv:2101.05156v1 [cond-mat.mes-hall], submitted to *EPL* (2021).