



EDITOR'S CHOICE • OPEN ACCESS

Cosmological horizons radiate

To cite this article: Ulf Leonhardt 2021 *EPL* **135** 10002

View the [article online](#) for updates and enhancements.

You may also like

- [Finite features of quantum de Sitter space](#)
Dionysios Anninos, Damián A Galante and
Beatrix Mühlmann
- [Infrared effects and the Unruh state](#)
Paul R Anderson, Shohreh Gholizadeh
Siahmazgi and Zachary P Scofield
- [Schottky anomaly of deSitter black holes](#)
Jack Dinsmore, Patrick Draper, David
Kastor et al.

Cosmological horizons radiate

ULF LEONHARDT^(a)
Department of Physics of Complex Systems, Weizmann Institute of Science - Rehovot 7610001, Israel

received 12 April 2021; accepted in final form 25 June 2021

published online 6 September 2021

Abstract – Gibbons and Hawking (*Phys. Rev. D*, **15** (1977) 2738) have shown that the horizon of de Sitter space emits radiation in the same way as the event horizon of the black hole. But actual cosmological horizons are not event horizons, except in de Sitter space. Nevertheless, this paper proves Gibbons' and Hawking's radiation formula as an exact result for any flat space expanding with strictly positive Hubble parameter. The paper gives visual and intuitive insight into why this is the case. The paper also indicates how cosmological horizons are related to the dynamical Casimir effect, which makes experimental tests with laboratory analogues possible.




Copyright © 2021 The author(s)

Published by the EPLA under the terms of the [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/) (CC BY). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

Introduction. – Horizons are known [1] to turn quantum fluctuations into radiation. The event horizon [2] of a black hole radiates [3]. The causal horizon [2] of accelerated observers is predicted to radiate [4–7]. What about cosmological horizons [8], will they radiate as well? Gibbons and Hawking have shown [9] that exponentially expanding flat space —de Sitter space [10]— emits thermal radiation with temperature

$$k_B T = \frac{\hbar H}{2\pi}, \quad (1)$$

where k_B denotes Boltzmann's constant, \hbar the reduced Planck constant and H the Hubble parameter [11]

$$H = \frac{\dot{a}}{a} \quad (2)$$

that is constant if the expansion factor a grows exponentially with cosmological time t . This regime of exponential growth is believed to be the asymptotic limit of the expanding universe. The actual expansion [12] is non-exponential with varying Hubble parameter H [13,14]. Will Gibbons' and Hawking's result (1) remain valid?

This question appears far from being trivial if we consider what cosmological horizons are and what they are not [14]. Imagine an arbitrary point co-moving with the universe. According to Hubble's law [8] the rest of the universe appears to withdraw from this point with a velocity that grows as $H\ell$ with proper distance ℓ . At some

distance ℓ_H the expansion velocity $H\ell_H$ reaches the speed of light c . The sphere with radius ℓ_H around the point is called the Hubble sphere and its surface the cosmological horizon. One would expect that no light from beyond that sphere would ever enter it (and reach the point in its center) but this is not true [14]. According to the cosmological standard model¹ all galaxies with redshifts $z > 1.6$ lie beyond our horizon, and yet they are visible. Not to mention the Cosmic Microwave Background that originates from $z \sim 10^3$. Figure 1 shows how this is possible. The figure also illustrates that the asymptotic de Sitter horizon does prevent light from entering its Hubble sphere. The de Sitter horizon is an event horizon [2] which justifies Gibbons' and Hawking's theory [9], but the actual cosmological horizon is not. So will it radiate? And if it does what is its temperature?

The answer to this question is not entirely academic, despite the temperature of eq. (1) being in the range of 10^{-29} K for the present Hubble parameter [15,16]. It turns out [17] that the Gibbons-Hawking effect is essential for establishing the correct order of magnitude of the cosmological constant Λ from the Lifshitz theory of vacuum fluctuations [18,19]. Furthermore, Λ responds to changes in the inverse Gibbons-Hawking temperature with time, which may resolve [20] one of the major puzzles in contemporary astrophysics [21], the tension between

^(a)E-mail: ulf.leonhardt@weizmann.ac.il (corresponding author)

¹We use the standard Λ CDM model with parameters inferred from measurements of the Cosmic Microwave Background: $H^2 = H_0^2(\Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_\Lambda)$ with time in units of $1/H_0$ and $\Omega_R = 0.925 \times 10^{-4}$, $\Omega_M = 0.3153 \pm 0.0073$, $\Omega_\Lambda = 0.6847 \pm 0.0073$.

the present Hubble parameter inferred from the early [15] or the late [16] cosmic evolution. The literature disagrees whether eq. (1) holds in general with arguments in favor [2,17,22] or against [23,24]. Reference [17] uses a fluid-mechanical analogue [25–29] to establish eq. (1) for arbitrary cosmic expansion. Here I prove eq. (1) without analogues as an exact result for any $H(t) > 0$ (and conformally invariant quantum fields in a classical Friedmann-Lemaître-Robertson-Walker metric with zero spatial curvature).

Visualization. – For simplicity, and in agreement with astronomical observation [30], space is assumed to be flat with time-dependent length scale, the expansion factor $a(t) > 0$. Space-time is not flat; the space-time metric reads

$$ds^2 = c^2 dt^2 - a^2 d\mathbf{r}^2 \quad (3)$$

and has, for non-zero H , non-zero Riemann curvature². In conformal time

$$\tau = \int \frac{dt}{a} \quad (4)$$

the metric (3) reduces to $ds^2 = a^2(c^2 d\tau^2 - d\mathbf{r}^2)$. Light rays ($ds = 0$) thus propagate in conformal time τ and co-moving space \mathbf{r} like in flat Minkowski space —along the diagonals in a $(c\tau, \mathbf{r})$ space-time diagram (fig. 1).

Consider radially incident rays. They encounter the cosmological horizon at $r = c/(aH)$, because there the time derivative of the proper distance ar reaches c . The horizon separates the stream of incident light into light falling in and light staying outside, if the horizon is light-like itself, which is only the case for de Sitter space where $H = \text{const.}$ Only the de Sitter horizon (fig. 1) is an event horizon. Otherwise light is able to cross the cosmological horizon. For light reaching us at present time we find for the parameters of the cosmological standard model (see footnote ¹) that the crossing has occurred at redshift [13] $z = a - 1 \approx 1.6$ mentioned in the introduction. So the light from galaxies with larger redshifts and the Cosmic Microwave Background (with $z \sim 10^3$ [15]) reaches us from beyond the cosmological horizon.

Consider now the opposite situation: light-like particles emitted from a point taken as the coordinate origin. The surface they can reach is called the particle horizon (fig. 1). The particle horizon coincides with the cosmological horizon if H is proportional to a^{-2} . This is the equation of state of the radiation-dominated universe [13]. So, fittingly, in the radiation-dominated era [13] the particle horizon is also the cosmological horizon.

Figure 1 has illustrated the horizons in a space-time diagram of conformal time τ and co-moving radius r where light propagates as in Minkowski space. Alternatively, one can also compensate for the spatial expansion factor a in a

²From eq. (16) follows that the curvature scalar $R \neq 0$ unless $H = 0$ (empty space) or $\partial_t H^{-1} = 2$ (radiation-dominated universe). But even in the latter case the energy-momentum tensor and hence the Ricci tensor does not vanish.

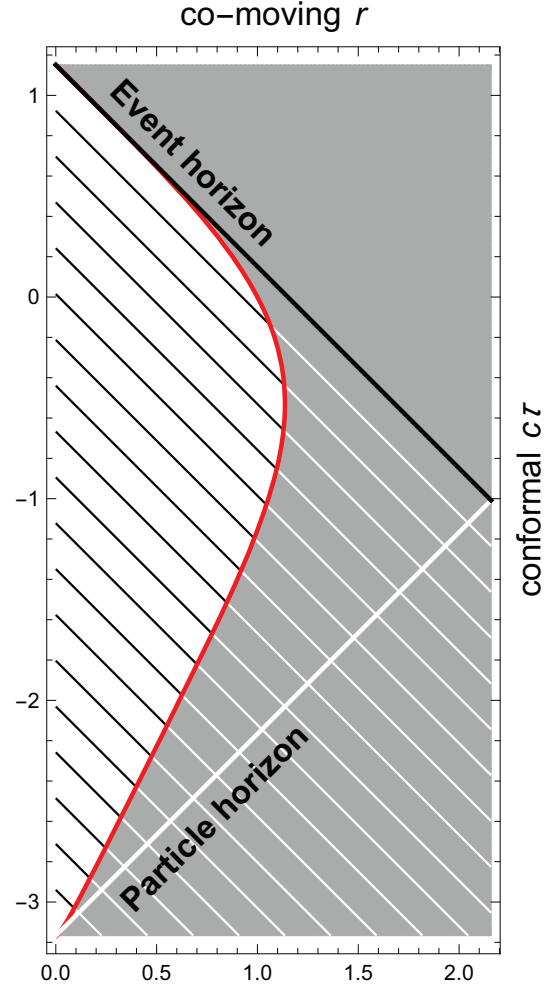


Fig. 1: Horizons. Radial space-time diagram showing the entire history of the universe (see footnote ¹) in (finite) conformal time τ and at co-moving radius r from a point placed at the coordinate origin (length unit: horizon at present time). The cosmological horizon (red) separates the inside (white) from the outside (gray) of the Hubble sphere. Light propagates along the diagonals of the space-time diagram, as in Minkowski space, and freely crosses the cosmological horizon, unless the horizon becomes light-like as in the final stage of cosmic evolution when the universe approaches de Sitter space. In the early stage the universe is radiation-dominated and the cosmological horizon coincides with the particle horizon.

diagram of cosmological time t and proper distance $\ell = ar$ (fig. 2). Consider the coordinates [17]

$$\mathbf{x} = a\mathbf{r}. \quad (5)$$

One obtains from the line element (3) the metric

$$ds^2 = c^2 dt^2 - (d\mathbf{x} - H\mathbf{x} dt)^2. \quad (6)$$

The null-geodesics in the $\{t, \mathbf{x}\}$ coordinates thus move with velocities

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \mathbf{c} + \mathbf{u}, \quad (7)$$

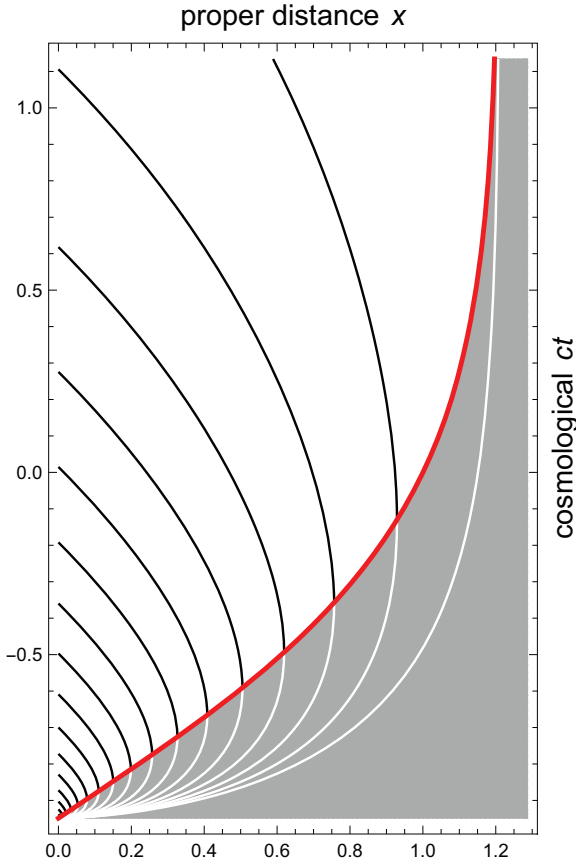


Fig. 2: Moving medium. Viewed in the radial space-time diagram of cosmological time t and proper distance x the universe appears to light like a fluid expanding with velocity Hx in agreement with Hubble's law (units as in fig. 1). At the cosmological horizon (red) the flow reaches the speed of light. The figure shows the same rays as in fig. 1. The rays are incident in the region (gray) outside the horizon, but dragged outward by the medium until the Hubble parameter H has fallen such that they may enter the Hubble sphere (white). At the cosmological horizon the rays are vertical in the space-time diagram: light stands still, although only for a fleeting moment. In de Sitter space, when $H = \text{const}$, this moment would last forever.

with $|c| = c$ and

$$\mathbf{u} = H\mathbf{x}. \quad (8)$$

Consequently, in $\{t, \mathbf{x}\}$ coordinates light propagates with c , but perceives the expanding universe as a moving medium³ with outward flow \mathbf{u} growing in speed with proper distance as $H\ell$ (Hubble's law) [17]. At the cosmological horizon the medium reaches the speed of light. Figure 2 shows how incident light rays get to a momentary halt at the horizon. For exponentially expanding space, the proper distance to the horizon, c/H , remains constant, and so light is trapped there forever. For the actual expansion of the universe, however, the proper distance to the horizon increases—the cosmological horizon appears

³Note that the addition theorem of velocities in eq. (7) is non-relativistic, which is typical for analogues of gravity [25–29].

to move outwards (fig. 2). Light trapped for a fleeting moment there gets released and moves on. Nevertheless, this moment of light standing still will suffice for Gibbons-Hawking radiation.

To see this, consider a specific but arbitrary moment in time labelled by the conformal time τ_0 with the expansion factor a_0 and Hubble parameter H_0 . Imagine a light wave of constant frequency ω' in frames co-moving with the Hubble flow (8). Such waves may exist as mathematical objects, regardless whether one subscribes to their fluid-mechanical interpretation, and we may regard all incident light waves as superpositions of them. The co-moving frequency ω' is related to ω , the frequency with respect to cosmological time, by the Doppler formula⁴

$$\omega' = \left(1 - \frac{H|\mathbf{x}|}{c}\right)\omega. \quad (9)$$

Expressed in terms of the co-moving radius r we thus require

$$\omega|_{\tau_0} = \frac{\omega'}{1 - a_0 H_0 r/c}, \quad \omega' = \text{const.} \quad (10)$$

The frequency ω is related to the phase φ as $\omega = -\partial_t \varphi$. As the wave propagates in conformal time and co-moving radius like in Minkowski space (fig. 1) the phase of an incident radial wave must depend on $r + c\tau$. We thus obtain by integration

$$\varphi = \frac{\omega'}{H_0} \ln \left| 1 + a_0 H_0 \left(\tau_0 - \tau - \frac{r}{c} \right) \right|. \quad (11)$$

Consider now the wave number $k = \partial_r \varphi$. Inside the region with $r/c + \tau - \tau_0 < (a_0 H_0)^{-1}$ that coincides with the Hubble sphere at $\tau = \tau_0$ the wave number k is negative, whereas outside this region $k > 0$. Purely incident waves must have entirely negative spatial Fourier components and hence negative wave numbers. Therefore we must require that the wave vanishes for $r/c + \tau - \tau_0 > (a_0 H_0)^{-1}$. This does not mean that the wave ends at the cosmological horizon throughout its history (fig. 3). The edge of the wave coincides with the horizon only at $\tau = \tau_0$. As we can decompose all radially incident waves inside the temporary horizon at τ_0 in terms of waves of constant but different co-moving frequencies ω' , all waves reaching the origin are superpositions of waves with an edge.

The quantum vacuum, however, is not confined by such edges, for the following simple reason [17]: The universe is homogeneous and isotropic on cosmological scales [31] and so must be the vacuum. One can move the coordinate origin to any spatial point, but the vacuum must remain invariant. This would be impossible if the quantum vacuum were confined by wave edges around a given point. From this follows that the radiation incident on any point must not be in the vacuum state. Cosmological horizons radiate. To work out the details takes a calculation.

⁴The Doppler formula (9) follows from the transformation $t = t'$, $r = x/a$ for waves with phase φ and $\omega = -\partial_t \varphi$, $k = \partial_r \varphi$, $\omega' = -\partial_{t'} \varphi$ with $k = \omega/c$.

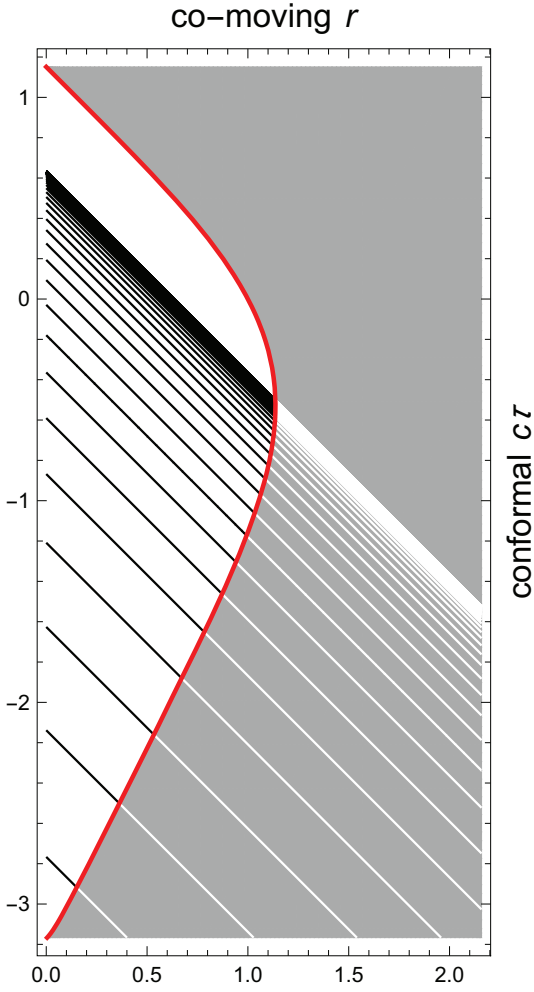


Fig. 3: Wave of constant co-moving frequency. Contour lines of the phase (eq. (11) for $\tau - \tau_0 + r/c < (a_0 H_0)^{-1}$) shown in the space-time diagram of conformal time and co-moving radius for $\tau_0 = -0.5$ (units as in fig. 1). The contour lines represent light rays incident from outside (gray) the Hubble sphere and crossing to the inside (white) at the cosmological horizon. For $\tau \geq \tau_0$ they are confined within the Hubble sphere. At $\tau = \tau_0 + (a_0 H_0)^{-1}$ the last ray reaches the origin. All incident radiation confined within the Hubble sphere at τ_0 can be represented as superposition of these waves with different constant co-moving frequencies.

Calculation. –

The model. Consider a simple model for massless bosons such as the photons of the electromagnetic field: a conformally invariant, massless scalar field [32] described by the mode decomposition

$$\hat{A} = \sum_{\mathbf{k}} \left(A_{\mathbf{k}} \hat{a}_{\mathbf{k}} + A_{\mathbf{k}}^* \hat{a}_{\mathbf{k}}^\dagger \right), \quad (12)$$

where the mode functions $A_{\mathbf{k}}$ obey the wave equation [32]

$$\frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu A + \frac{R}{6} A = 0 \quad (13)$$

in terms of the metric tensor $g_{\alpha\beta}$, its determinant g and inverse $g^{\mu\nu}$, while R denotes the curvature scalar [11]. The mode operators \hat{a} and \hat{a}^\dagger shall obey the Bose commutation relations. This simple model captures the essence of conformally invariant bosonic fields experiencing the Gibbons-Hawking effect while avoiding technicalities associated with their polarization and internal structure.

For the Bose commutation relation to hold for the mode operators, the mode functions are required [33] to be orthonormal with respect to the scalar product [32]

$$(A_1, A_2) = \frac{ic}{\hbar} \int (A_1^* \partial^0 A_2 - A_2 \partial^0 A_1^*) \sqrt{-g} dV \quad (14)$$

that, as a consequence of the wave equation (13), does not depend on time.

Specifically, the quantum field \hat{A} shall evolve in a space-time geometry with metric (3) expressed in spherical coordinates:

$$ds^2 = c^2 dt^2 - a^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \quad (15)$$

with curvature scalar [11]

$$R = -\frac{6}{c^2} (\dot{H} + 2H^2) \quad (16)$$

and $\sqrt{-g} = a^3 r^2 \sin \theta$. We express the mode functions in terms of the spherical harmonics Y_{lm} as

$$A = a(t) A_l(t, r) Y_{lm}(\theta, \phi) \quad (17)$$

and arrive from eqs. (13) and (15)–(17) at the equation of the partial waves A_l :

$$\partial_\tau^2 A_l = c^2 \left(\partial_r^2 + \frac{2}{r} \partial_r \right) A_l - \frac{c^2}{r^2} l(l+1) A_l \quad (18)$$

expressed in terms of conformal time τ defined in eq. (4). In flat Minkowski space one gets exactly the same wave equation. So not only do null geodesics propagate in conformal time and co-moving space as in Minkowski space, the full quantum field \hat{A} does it as well. This is a consequence of its conformal invariance [32]. For guidance and intuition, consider first the case $l = 0$ of purely radial propagation, before proceeding to the general $l \geq 0$.

Radial propagation. Put $l = 0$. In this case the general solution of the radial wave equation (18) takes the familiar d'Alembert form

$$A_0 = \frac{1}{r} [f_+(r + c\tau) + f_-(r - c\tau)] \quad (19)$$

with the functions f_\pm describing incoming (+) and outgoing (–) waves. Consider in particular

$$A_0 = \frac{\mathcal{A}}{r} (\eta \mp \rho)^{i\nu+1} \quad (20)$$

with constant \mathcal{A} , the dimensionless variables

$$\eta = 1 + H_0 a_0 (\tau_0 - \tau), \quad \rho = \frac{H_0 a_0}{c} r \quad (21)$$

and the dimensionless constant

$$\nu = \frac{\omega'}{H_0}. \quad (22)$$

The radial wave given by eqs. (20)–(22) with the minus sign in eq. (20) has the phase profile (11) of a wave with constant co-moving frequency discussed in the previous section and illustrated in fig. 3. There we argued that for being strictly ingoing the wave needs to be confined in the region $\rho < \eta$ (fig. 3).

After the incident wave focuses at the origin it is reflected there and leaves as an outgoing wave. We thus write for the propagation of the incident mode:

$$A_+ = \frac{\mathcal{A}}{r} \left(\Theta(\eta - \rho) (\eta - \rho)^{i\nu+1} - (\eta + \rho)^{i\nu+1} \right). \quad (23)$$

The minus sign between the incoming and the outgoing term ensures that A_+ does not diverge at the origin. Instead, it forms there a diffraction-limited spot [34]. There is one more subtlety to consider: eq. (23) holds only for $\eta \geq 0$. At $\eta = 0$ the incident wave runs out and vanishes for $\eta < 0$ (fig. 3). No further light can be reflected; the edge of the last reflected wave moves as $\rho = -\eta$ with falling η given by eq. (21). We indicate this by writing a $\Theta(\eta + \rho)$ in front of the reflected wave for $\eta < 0$:

$$A_+ = -\frac{\mathcal{A}}{r} \Theta(\eta + \rho) (\eta + \rho)^{i\nu+1}. \quad (24)$$

Now imagine a wave A_- of constant co-moving frequency that stays outside $\rho > \eta$. Here, according to eq. (10), the co-moving frequency ω' has the opposite sign of the frequency with respect to cosmological time, ω . For having a wave oscillating with positive frequencies ω the co-moving frequency needs to be negative, and so does the parameter ν in eq. (22). We thus complex-conjugate the solution of eq. (20) and define A_- as

$$A_- = \frac{\mathcal{A}}{r} \Theta(\rho - \eta) (\rho - \eta)^{-i\nu+1}. \quad (25)$$

Like eq. (23) this equation is only valid for $\eta > 0$ and hence $\tau - \tau_0 < (a_0 H_0)^{-1}$. At later times A_- reaches the origin and is reflected there:

$$A_- = \frac{\mathcal{A}}{r} \left((\rho - \eta)^{-i\nu+1} - \Theta(-\eta - \rho) (-\eta - \rho)^{-i\nu+1} \right) \quad (26)$$

for $\eta < 0$. The need for complex conjugation in the definition of A_- also becomes evident from normalizing A_- with respect to the scalar product defined in eq. (14). We recall that the scalar product does not depend on time and evaluate it at $\eta = 0$ where we get

$$(A_1, A_2) = \frac{8\pi \mathcal{A}^2 H_0 \nu}{\hbar} \int_0^\infty \rho^{i(\nu_1 - \nu_2) + 1} d\rho \quad (27)$$

for $\nu_1 \sim \nu_2 \sim \nu$ (the scalar product vanishes if $\nu_1 \neq \nu_2$ for otherwise it would be time-dependent). We substitute

$\exp(-\xi)$ for ρ and obtain the standard integral of the delta function:

$$(A_1, A_2) = \frac{(4\pi)^2 \mathcal{A}^2 H_0 \nu}{\hbar} \delta(\nu_1 - \nu_2). \quad (28)$$

The norm is positive, as required, and it would be negative without the complex conjugation in the definition of A_- . For A_+ we also evaluate the scalar product at $\eta = 0$ and obtain exactly the same norm.

We have thus established two sets of modes with parameter ν , one (A_+) describing radiation incident from inside $\rho < \eta$ and the other (A_-) incident radiation staying outside. Similar to the closely related Rindler modes [1] these modes form a complete orthonormal set for incident radiation.

Next we follow Damour's and Ruffini's elegant argument [35] and note that the quantum vacuum occupies modes that are analytic in $\rho - \eta$ (or $\rho + \eta$). In particular, they must not vanish for $\rho < \eta$ or $\rho > \eta$. It is easy to construct analytic $A_{\text{vac}\pm}$ from the A_\pm of eqs. (23) and (25):

$$A_{\text{vac}\pm} = A_\pm \cosh \zeta + A_\mp^* \sinh \zeta \quad (29)$$

with

$$\tanh \zeta = e^{-\pi\nu}. \quad (30)$$

The Bogoliubov transformation (29) preserves the orthonormality of the mode functions and the Bose commutation relations of the associated mode operators [33]

$$\hat{a}_{\text{vac}\pm} = \hat{a}_\pm \cosh \zeta - \hat{a}_\mp^\dagger \sinh \zeta. \quad (31)$$

Defining the vacuum state $|0\rangle$ as the eigenstate of the $\hat{a}_{\text{vac}\pm}$ with eigenvalue zero gives [33]

$$|0\rangle = \frac{1}{\sqrt{Z}} \sum_{n=0}^\infty e^{-n\pi\nu} |n\rangle_+ |n\rangle_- \quad (32)$$

in terms of the particle number states $|n\rangle_\pm$ of the modes A_\pm and with $Z^{-1} = 1 - e^{-2\pi\nu}$. The state described in eq. (32) is an Einstein-Podolski-Rosen state (a two-mode squeezed vacuum) [33], the strongest entangled state for a given mean energy [36]. Indeed, we see from eq. (32) that the A_\pm modes are strongly correlated: a particle of frequency ω' detected at τ_0 at the origin is accompanied by a partner particle in mode A_- . The partner may also appear at the origin after the A_- mode comes in when the conformal time $\tau_0 + (a_0 H_0)^{-1}$ has elapsed, which may take a rather long cosmological time.

The reduced quantum state for the A_+ mode is described by the density operator

$$\hat{\rho} = \text{tr}\{|0\rangle\langle 0|\}_- = \frac{1}{Z} \sum_{n=0}^\infty e^{-n(2\pi\nu)} |n\rangle_+ \langle n|_+. \quad (33)$$

This is a thermal state [33], because we can understand $e^{-n(2\pi\nu)}$ as the Boltzmann factor $e^{-E_n/(k_B T)}$ for the energies $E_n = \hbar n \omega$ and the temperature T given by

$$2\pi\nu = \frac{\hbar\omega}{k_B T}. \quad (34)$$

When reduced to one of the two entangled modes, all Einstein-Podolski-Rosen states appear as thermal states. What makes the state created by horizons special is the fact that the temperature T is the same throughout the spectrum [1]. In our case, this is because ν is proportional to ω according to definition (22) and eq. (10) at $r = 0$. The proportionality factor gives Gibbons' and Hawking's formula (1) for any H_0 and hence for any $H(t) > 0$.

General propagation. Radial propagation is sufficient for describing the field captured by a single point (with coordinate origin set to this point). Yet the field energy at this point depends on derivatives of the field amplitudes and hence on multipole moments, and so do other correlation functions. It is therefore necessary to consider the general case of arbitrary angular momentum l . Inspired by the radial case of $l = 0$, we seek the solution of the wave equation (18) as

$$A_l = \frac{\mathcal{A}_l}{r} (\eta \mp \rho)^{i\nu+1} p_l(\pm z) \quad (35)$$

with

$$z = -\frac{\eta}{\rho} \quad (36)$$

and find

$$p_l(z) = \frac{(i\nu)!}{(i\nu - l)!} {}_2F_1 \left(-l, l+1, -i\nu, \frac{1-z}{2} \right) \quad (37)$$

in terms of Gauss' hypergeometric function [37]. The prefactor is chosen for later convenience. From the hypergeometric series follows that the $p_l(z)$ are polynomials of order l in z , with the first two given by

$$p_0(z) = 1, \quad p_1(z) = 1 + i\nu - z. \quad (38)$$

We use eq. 3.4.(6) of ref. [37] to express eq. (37) in terms of Legendre functions and deduce from eq. 3.8.(12) of ref. [37] the recurrence relation

$$p_{l+2}(z) + (2l+3)z p_{l+1}(z) + (l-i\nu)(l+2+i\nu) p_l(z). \quad (39)$$

Applying this relation with the initial values of eq. (38) we can easily compute the $p_l(z)$. In particular, we obtain for the constant term in the polynomials

$$p_l(0) = \prod_{m=1}^l (2m - l + i\nu), \quad \text{for } l > 0 \quad (40)$$

and $p_0 = 1$. These are all the mathematical preparations needed for constructing and normalizing the mode functions of the incident radiation.

In analogy to eqs. (23) and (24) we write for the mode incident inside $\rho < \eta$ the compact expression

$$A_+ = \frac{\mathcal{A}_l}{r} \left(\Theta(\eta - \rho) (\eta - \rho)^{i\nu+1} p_l(z) - (-1)^l \Theta(\eta + \rho) (\eta + \rho)^{i\nu+1} p_l(-z) \right). \quad (41)$$

The incoming wave is reflected with coefficient $(-1)^l$ for the following reason. The highest singularity in $p_l(z)$ for

$r \rightarrow 0$ is given by the highest term in $z = -\eta/\rho$, which is proportional to z^l in the l -th order polynomial $p_l(z)$. Subtracting the ingoing and outgoing term with the difference $(-1)^l$ thus removes the leading singularity. It also removes all other singularities, as there must exist a regular solution as linear combination of the two fundamental solutions (35). The so-constructed mode function describes light that reaches the origin at $\eta > 0$ when $\tau - \tau_0 < (a_0 H_0)^{-1}$.

The light outside the cosmological horizon at τ_0 propagates inwards as well, but reaches the origin at $\tau - \tau_0 > (a_0 H_0)^{-1}$ when $\eta < 0$. We describe the corresponding mode function as

$$A_- = \frac{\mathcal{A}_l}{r} \left(\Theta(-\eta + \rho) (\eta + \rho)^{-i\nu+1} p_l(z) - (-1)^l \Theta(-\eta - \rho) (-\eta - \rho)^{-i\nu+1} p_l(-z) \right) \quad (42)$$

where we take the complex conjugates of the fundamental solutions (35) for having a positive norm. The normalization is best done at the time when $\eta = 0$ and hence $z = 0$. We obtain for both A_+ and A_-

$$(A_1, A_2) = \frac{(4\pi)^2 \mathcal{A}_l^2 |p_l(0)|^2 H_0 \nu}{\hbar} \delta(\nu_1 - \nu_2) \quad (43)$$

with the $p_l(0)$ given by eq. (40). We can thus proceed exactly as in the previous subsection (without the need to consider gray-body factors [1] as for the Hawking radiation of black holes). We obtain also for general wave propagation the Einstein-Podolski-Rosen state of eq. (32) as the vacuum state seen by the observer modes A_{\pm} . An observer co-moving with the universe would thus perceive the vacuum as a thermal state with Gibbons-Hawking temperature (1).

Conclusions. – Cosmological horizons radiate with Gibbons-Hawking temperature (1) in a spatially flat expanding universe with $H(t) > 0$. This paper has proven this statement as an exact result for a Friedmann-Lemaître-Robertson-Walker metric with zero spatial curvature [13]. We may also imagine a contracting universe with $H(t) < 0$ where the Hubble flow of eq. (8) points inwards and grows in magnitude with growing proper distance. Also in this case a cosmological horizon is established when the Hubble flow reaches the speed of light. We can run the entire argument of the paper with H replaced by $|H|$ and arrive at eq. (1) with $|H|$. An interesting —and different— scenario occurs when H changes sign, in particular when H is oscillating (as in anti de Sitter space [38]). In this case light may cross the cosmological horizon multiple times. Similar to black-hole lasing [39–41] each interaction with the horizon may create radiation, but it depends on the phase acquired between the interactions whether the radiation is amplified or de-amplified.

While the “oscillating universe” is a purely theoretical case in astrophysics, it is in fact the most realistic case for laboratory analogues of cosmological horizons and their

radiation, complementing and generalizing the analogue of de Sitter space with a Bose-Einstein condensate [42]. The reason is that one can create periodic modulations of the refractive index [43–46] that act like a periodically modulated expansion factor in the space-time metric (3). The Hubble constant is proportional to the modulation frequency ω_0 . When ω_0 is comparable with the radiation frequency ω the effective temperature (1) becomes significant and detectable radiation is generated. This process is closely related to the dynamical Casimir effect [44–46] where a boundary is modulated (or the optical length to a boundary). The radiation of de Sitter space has been mapped to the radiation produced by an accelerated mirror [47]. For these reasons, the Gibbons-Hawking radiation of cosmological horizons may be regarded as a pure and intriguing case within the wider area of the dynamical Casimir effect [48–51].

* * *

This paper is dedicated to the memory of RENAUD PARENTANI. I am most grateful to him for introducing me to the quantum physics of horizons. I also thank D. BERMUDEZ, D. BERECHYA, and N. EBEL for discussions directly related to this paper. The paper has been supported by the Israel Science Foundation and the Murray B. Koffler Professorial Chair.

Data availability statement: All data that support the findings of this study are included within the article (and any supplementary files).

REFERENCES

- [1] BROUT R., MASSAR S., PARENTANI R. and SPINDEL P., *Phys. Rep.*, **260** (1995) 329.
- [2] JACOBSON T. and PARENTANI R., *Found. Phys.*, **33** (2003) 323.
- [3] HAWKING S. W., *Nature*, **248** (1974) 30.
- [4] FULLING S. A., *Phys. Rev. D*, **7** (1973) 2850.
- [5] DAVIES P. C. W., *J. Phys. A*, **8** (1975) 609.
- [6] UNRUH W. G., *Phys. Rev. D*, **14** (1976) 870.
- [7] TAKAGI S., *Prog. Theor. Phys. Suppl.*, **88** (1986) 1.
- [8] HARRISON E., *Cosmology: The Science of the Universe* (Cambridge University Press, Cambridge) 2000.
- [9] GIBBONS G. W. and HAWKING S. W., *Phys. Rev. D*, **15** (1977) 2738.
- [10] DE SITTER W., *Mon. Not. R. Astron. Soc.*, **78** (1917) 3.
- [11] LANDAU L. D. and LIFSHITZ E. M., *The Classical Theory of Fields* (Butterworth-Heinemann, Amsterdam) 2003.
- [12] RIESS A. *et al.*, *Astrophys. J.*, **659** (2007) 98.
- [13] WEINBERG S., *Cosmology* (Oxford University Press, Oxford) 2008.
- [14] DAVIS T. M. and LINEWEAVER C. H., *Publ. Astron. Soc. Aust.*, **21** (2004) 97.
- [15] PLANCK COLLABORATION, <http://pla.esac.esa.int/pla/#home>.
- [16] RIESS A. G. *et al.*, *Astrophys. J.*, **876** (2019) 85.
- [17] LEONHARDT U., *Ann. Phys. (N.Y.)*, **411** (2019) 167973.
- [18] RODRIGUEZ A. W., CAPASSO F. and JOHNSON S. G., *Nat. Photon.*, **5** (2011) 211.
- [19] SIMPSON W. M. R. and LEONHARDT U. (Editors), *Forces of the Quantum Vacuum* (World Scientific, Singapore) 2015.
- [20] BERECHYA D. and LEONHARDT U., arXiv:2008.04789.
- [21] VERDE L., TREU T. and RIESS A. G., *Nat. Astron.*, **3** (2019) 891.
- [22] DAVIES P. C. W. and DAVIES T. M., *Found. Phys.*, **32** (2002) 1877.
- [23] CAI R.-G. and KIM S. P., *JHEP*, **02** (2005) 050.
- [24] HU Y.-P., *Phys. Lett. B*, **701** (2011) 269.
- [25] UNRUH W. G., *Phys. Rev. Lett.*, **46** (1981) 1351.
- [26] VOLOVIK G., *The Universe in a Helium Droplet* (Oxford University Press, Oxford) 2003.
- [27] BARCELO C., LIBERATI S. and VISSER M., *Living Rev. Relativ.*, **8** (2005) 12.
- [28] UNRUH W. G. and SCHÜTZHOLD R. (Editors), *Quantum Analogues: From Phase Transitions to Black Holes and Cosmology* (Springer, Berlin) 2007.
- [29] FACCIO D., BELGIORNO F., CACCIATORI S., GORINI V., LIBERATI S. and MOSCHELLA U. (Editors), *Analogue Gravity Phenomenology: Analogue Spacetimes and Horizons, from Theory to Experiment*, *Lect. Notes Phys.*, Vol. **870** (Springer, Cham) 2013.
- [30] PLANCK COLLABORATION, *Astron. Astrophys.*, **641** (2020) A6.
- [31] GOTT J. R. III *et al.*, *Astrophys. J.*, **624** (2005) 463.
- [32] BIRRELL N. D. and DAVIES P. C. W., *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge) 1984.
- [33] LEONHARDT U., *Essential Quantum Optics: From Quantum Measurements to Black Holes* (Cambridge University Press, Cambridge) 2010.
- [34] LEROSEY G. *et al.*, *Science*, **315** (2007) 1120.
- [35] DAMOUR T. and RUFFINI R., *Phys. Rev. D*, **14** (1976) 332.
- [36] BARNETT S. M. and PHOENIX S. J. D., *Phys. Rev. A*, **40** (1989) 2404.
- [37] ERDÉLYI A., MAGNUS W., OBERHETTINGER F. and TRICOMI F. G., *Higher Transcendental Functions* (McGraw-Hill, New York) 1981.
- [38] KOLOMEISKY E. B., *Phys. Rev. B*, **100** (2019) 140301(R).
- [39] CORLEY S. and JACOBSON T., *Phys. Rev. D*, **59** (1999) 124011.
- [40] LEONHARDT U. and PHILBIN T. G., *Lect. Notes Phys.*, Vol. **718** (Springer) 2017, p. 229.
- [41] TETTAMANTI M., CARUSOTTO I. and PAROLA A., *EPL*, **133** (2021) 20002.
- [42] FEDICHEV P. O. and FISCHER U. R., *Phys. Rev. Lett.*, **91** (2003) 240407.
- [43] DEZAEL F. X. and LAMBRECHT A., *EPL*, **89** (2010) 14001.
- [44] WILSON C. M. *et al.*, *Nature*, **479** (2011) 376.
- [45] LÄHTEENMÄKI P. *et al.*, *Proc. Natl. Acad. Sci. U.S.A.*, **110** (2013) 4234.
- [46] VEZZOLI S. *et al.*, *Commun. Phys.*, **2** (2019) 84.
- [47] GOOD M. R. R., ZHAKENULY A. and LINDER E. V., *Phys. Rev. D*, **102** (2020) 045020.
- [48] SCHWINGER J., *Proc. Natl. Acad. Sci. U.S.A.*, **89** (1992) 4091.
- [49] MENDONÇA J. T., *Theory of Photon Acceleration* (CRC Press, Bristol) 2000.
- [50] MENDONÇA J. T. and GUERREIRO A., *Phys. Rev. A*, **72** (2005) 063805.
- [51] DODONOV V. V., *Phys. Scr.*, **82** (2010) 038105.