A LETTERS JOURNAL EXPLORING
THE FRONTIERS OF PHYSICS

PERSPECTIVE

The exotic hadron states in quenched and unquenched quark models

To cite this article: H. X. Huang *et al* 2021 *EPL* **135** 31001

A Letters Journal Exploring the Frontiers of Physics
August 2021
EPL, **135** (2021) 31001
www.epljournal.org
doi: 10.1209/0295-5075/ac25aa

Perspective

The exotic hadron states in quenched and unquenched quark models

H. X. Huang, X. Jin, Y. Tan and J. L. Ping[(a)]

Department of Physics and Jiangsu Key Laboratory for Numerical Simulation of Large Scale Complex Systems, Nanjing Normal University - Nanjing 210023, PRC

received 30 July 2021; accepted in final form 10 September 2021
published online 24 September 2021

Abstract – In the last two decades, a large number of exotic hadron states have been observed in experiments, which arouses great attention in the hadron physics community. In this short review, we briefly summarize progresses of our group on several exotic hadron states. Two approaches, quenched and unquenched quark models, are adopted in the calculation. The channel coupling effects, the multiquark states couple to open channels and the quark-antiquark states couple to meson-meson states, are emphasized. $X(3872)$ is showed to be a mixture state of $c\bar{c}$ and $D\bar{D}^*$ in the unquenched quark model. $X(2900)$ can be explained as a resonance state $\bar{D}^*K^*$ with the quantum numbers $IJ^P = 00^+$ in both the quark delocalization color screening model and the chiral quark model. The reported state $X(6900)$ can be explained as a compact resonance state with $IJ^P = 00^+$ in both two quark models, and several fully heavy tetraquark states are predicated. The possible hidden-charm pentaquarks are systematically investigated in QDCSM, and seven resonance states are obtained in the corresponding baryon-meson scattering process, among which the $\Sigma_c D$ with $J^P = \frac{1}{2}^-$, $\Sigma_c D^*$ with $J^P = \frac{3}{2}^-$ and $J^P = \frac{1}{2}^-$ are consistent with the experimental report of $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, respectively. More experimental data on exotic hadron states are vital for understanding exotic hadron states in quark models.

perspective

Introduction. – It is well known that in the conventional quark model a meson is composed of a pair of quark and antiquark ($q\bar{q}$) and a baryon is composed of three quarks (qqq). However, the quantum chromodynamics (QCD) does not deny the existence of other hadrons besides $q\bar{q}$ mesons and qqq baryons. Many charmonia or charmonium-like states reported in the last 20 years are difficult to fit into the conventional quark model and they are considered as exotic hadron states. Although none of the exotic hadron states is now definitely confirmed by experiment, more and more theoretical and experimental work has been devoted to investigate the exotic hadron states.

In 2003, the observation of $D_{s0}(2317)$ in the inclusive $D_s^+\pi$ invariant mass distribution by the BaBar Collaboration [1] and $X(3872)$ in the $J/\psi\pi^+\pi^-$ invariant mass spectrum of $B \to KJ/\psi\pi^+\pi^-$ by the Belle Collaboration [2] stimulated the investigation of exotic hadron states. Afterwards, dozens of so-called XYZ particles and other exotic states were announced by experiments. Especially, the discovery of charged charmonium-like states, *e.g.*, $Z_c(3900)$ [3], is regarded as the smoking gun of multiquark states. Nowadays, over twenty exotic states are collected in the collection of Particle Data Group [4]. Recently, a new narrow structure $X(6900)$ was observed by the LHCb Collaboration in the J/ψ-pair mass spectrum [5], which aroused interest in full-heavy exotic hadron states. The two new exotic structures $X_0(2900)$ and $X_1(2900)$ discovered by the LHCb Collaboration in the D^-K^+ invariant mass distributions of the decay process $B^+ \to D^+D^-K^+$ [6] renew our interest in four different flavor hadron states.

In regard to pentaquarks, the most noteworthy states in recent years are the hidden-charm pentaquarks. In 2015, the LHCb Collaboration observed two hidden-charm pentaquark states in the $J/\psi p$ invariant mass spectrum of $\Lambda_b^0 \to J/\psi K^- p$ [7], $P_c(4380)$ and $P_c(4450)$. Four years later, the LHCb Collaboration updated their results, a new state $P_c(4312)$ was proposed, and $P_c(4450)$ is split to two states $P_c(4440)$, and $P_c(4457)$ [8]. Actually, before LHCb's discovery of these two P_c states, the possible hidden-charm pentaquarks were studied

(a) E-mail: jlping@njnu.edu.cn (corresponding author)

extensively in the framework of both the coupled-channel unitary approach [9] and the one-boson exchange model [10], where the existence of hidden-charm pentaquarks was predicted. More detailed information about tetraquarks and pentaquarks can be found in review articles [11–13].

QCD is believed to be the fundamental theory of the strong interaction. However, the low-energy physics of QCD, such as hadron structure, hadron-hadron interactions, and multiquark systems, is much harder to calculate directly from QCD. Lattice QCD has provided numerical results describing quark confinement between two static colorful quarks, a preliminary picture of the QCD vacuum and the internal structure of hadrons in addition to a phase transition of strongly interacting matter, but it has still a long way to obtain reliable results for multiquark systems. Besides lattice QCD, many theoretical frameworks with some kind of QCD spirit were proposed, such as the QCD sum rule [14], the Dyson-Schwinger equation [15], constituent quark model [16] and so on.

Among them, the constituent quark model is a successful model in understanding hadron spectroscopy and hadron-hadron interactions even though we have not yet derived the constituent quark model directly from QCD. De Rújula *et al.* [17] first put forward a quark-gluon coupling model based on constituent quark and gluon effective degrees of freedom. Isgur and Karl obtained a good description of hadron spectroscopy based on this model [18]. However, extension of the model to baryon-baryon interactions does not reproduce the nucleon-nucleon (NN) intermediate attraction. To remedy this shortcoming, two approaches are adopted. One modification is the inclusion of scalar meson exchange and Goldstone bosons exchange on the quark level [19–22]. A typical approach is the chiral quark model (ChQM) [23,24], in which the constituent quarks interact with each other through colorless Goldstone bosons exchange in addition to the colorful one-gluon exchange and confinement. The chiral partner σ meson exchange is introduced to obtain the immediate-range attraction of NN interaction. An alternative approach is the quark delocalization color screening model (QDCSM), which was developed in the 1990s with the aim of explaining the similarities between nuclear and molecular forces [25]. These two models have been successfully applied to the study of the dibaryon systems, such as the properties of deuteron [26], the baryon-baryon scattering phase shifts [27,28], the dibaryon candidates d^* [27] and the $N\Omega$ systems [29].

In describing the multiquark system, the channel coupling effect has to be taken into account, especially for the resonances, where the coupling to the open channels will shift the mass of the resonance and give the decay width to the resonance, or destroy the resonance. Another factor affecting the results in some calculation is the finite coordinate space used. In quark model calculation, all the eigen-energies are discrete due to the finite space. A stabilization method, for example real-scaling method [30] has been invented to pick up the genuine resonance states from these states with discrete energies.

Another way to study the exotic hadron states is to invoke the unquenched quark model, which was initiated by Beveren *et al.* in the 1980s [31]. In fact, in the quantum field theory, the number of particles is not a conserved quantity. A meson with the baryon number $B = 0$ and a baryon with $B = 1$ can be the expansion of the following Fock states:

$$\begin{aligned}|Meson\rangle &= C_0|q\bar{q}\rangle + C_1|q\bar{q}q\bar{q}\rangle + C_2|q\bar{q}g\rangle \\ &\quad + C_3|q\bar{q}q\bar{q}q\bar{q}\rangle + \cdots, \\ |Baryon\rangle &= C_0|qqq\rangle + C_1|qqqq\bar{q}\rangle + C_2|qqqg\rangle \\ &\quad + C_3|q^3q\bar{q}q\bar{q}\rangle + \cdots. \end{aligned} \tag{1}$$

The lattice QCD has evolved from the early quenching approximation, in which the quark-antiquark excitation is not considered, to the unquenched lattice QCD. The quark model also needs to move forward to unquenched quark model by introducing multi-quark components, and even hybrid components to the hadron states. In recent years, the unquenched quark model has been developed as an extension of the constituent quark model that includes the effects of sea quarks via a $^3P_0$ quark-antiquark pair creation mechanism, and it has been employed to the study of baryons [32], mesons [33], and some exotic states [34,35]. In this paper, we will give a concise review on the research progress of some exotic hadron states in quenched and unquenched quark models.

Quenched and unquenched quark models. – In this section, we give a brief introduction to two quenched quark models: ChQM and QDCSM, and a modified unquenched quark model is also presented.

Quenched quark model. Two quenched quark models are employed. One is the Salamanca version of ChQM, another is QDCSM. These two models have been successfully applied to describe hadron spectra, nucleon-nucleon interaction, and multiquark states. The model details can be found in refs. [24,25]. Here we only describe the Hamiltonian briefly.

$$H = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i} V_{ij},$$

$$V_{ij} = V_{ij}^{C1} + V_{ij}^{G} + V_{ij}^{\chi} + V_{ij}^{\sigma}, \qquad \text{for ChQM}, \tag{2}$$

$$V_{ij} = V_{ij}^{C2} + V_{ij}^{G} + V_{ij}^{\chi}, \qquad \text{for QDCSM}, \tag{3}$$

$$\chi = \pi, K, \eta,$$

$$V_{ij}^{C1} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (r_{ij}^2 + V_0), \tag{4}$$

$$V_{ij}^{C2} = \begin{cases} -\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (a_c r_{ij}^2 + V_0), & i, j \text{ in the same cluster}, \\ -\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c a_c \frac{1-e^{-\mu_{ij} r_{ij}^2}}{\mu_{ij}}, & \text{otherwise}, \end{cases} \tag{5}$$

where T_c is the kinetic energy of the center of mass. There are two differences between two models, one is about the Hamiltonian, the ordinary quadratic confinement, and scalar meson (σ) exchange is used in ChQM, whereas the screened confinement is used for two interacting quarks resident in different clusters, and the scalar meson (σ) exchange is missing in QDCSM. The details of each potential can be found in refs. [24,25]. Another is about the model space. Generally the left and right centered single Gaussian functions are adopted as the single particle orbital wave functions in ChQM, whereas the quark delocalization in QDCSM is realized by writing the single particle orbital wave function as a linear combination of the left and right Gaussians, and the combination coefficients are determined by the dynamics of the quark system. In this way, the system can choose its most favorable configuration through its own dynamics in a larger Hilbert space.

Model parameters are determined by fitting the hadron properties, deuteron properties and hadron-hadron scattering phase shifts, respectively.

The modified unquenched quark model. Here, we take meson as an example. In the unquenched quark model, the mass of a meson is obtained by solving the Schrödinger equation,

$$H\Psi_{IM_I}^{JM_J} = E\Psi_{IM_I}^{JM_J}, \tag{6}$$

where $\Psi_{IM_I}^{JM_J}$ is the unquenched wave function of the system which contain two- and four-quark components (as a preliminary work). It can be written as

$$\Psi_{IM_I}^{JM_J} = c_2\Psi_{IM_I}^{JM_J}(2q) + \sum_{i=1}^{N} c_{4i}\Psi_{i,IM_I}^{JM_J}(4q), \tag{7}$$

where $\Psi(2q)$ and $\Psi(4q)$ are the wave functions with two- and four-quark components, respectively (the simplified symbols are used to save space), and the N is the total number of four-quark channels.

In the nonrelativistic quark model, the number of particles is conserved. So there is no rigorous way to write down the Hamiltonian of the unquenched quark model. Here we only give a prescription of the Hamiltonian H as follows:

$$H = H_{2q} + H_{4q} + T_{24}, \tag{8}$$

where H_{2q} is stipulated to act on the wave function of the quark-antiquark component, $\Psi(2q)$, and H_{4q} only acts on the wave function of the four-quark component, $\Psi(4q)$. T_{24} is the transition operator, which undertakes the coupling of the two- and four-quark components. Here, the modified version of the transition operator in the $^3P_0$ model (quark pair creation model) [36–38] is developed (in coordinate space) [33,39],

$$\begin{aligned} T_{24} = & -3\gamma\sum_m \langle 1m1-m|00\rangle \int \mathrm{d}\mathbf{r}_3\mathrm{d}\mathbf{r}_4 \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} ir2^{-\frac{5}{2}}f^{-5} \\ & \times Y_{1m}(\hat{\mathbf{r}})e^{-\frac{r^2}{4f^2}}e^{-\frac{R_{AV}^2}{f_0^2}}\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}b_3^\dagger(\mathbf{r}_3)d_4^\dagger(\mathbf{r}_4). \end{aligned} \tag{9}$$

Here, $\mathbf{R}_{AV} = \mathbf{R}_A - \mathbf{R}_V$ is the relative coordinate between the valence quark-antiquark pair "A" and the created quark-antiquark pair in the vacuum. The damping factor $e^{-r^2/(4f^2)}$ suppresses the creation of quark-antiquark pair with high energy and the factor $e^{-R_{AV}{}^2/R_0{}^2}$ takes into account the fact that the created quark-antiquark pair should not be far away from the valence particles. The introduction of two damping factors can justify the validity of the quenched quark model as a good zeroth approximation of hadrons.

The calculation method. – In our work, two ways are introduced to consider the channel coupling effect, especially for the search of the resonances. It is well known that the energies of multiquark systems are stable if the sub-clusters are colorful (hidden-color states) without coupling to the color-singlet hadron-hadron channels (open channels). In most cases, these energies are higher than that of the color-singlet states. It is an interesting problem that the hidden-color states survive as resonances with the coupling to the color-singlet states.

The resonating group method (RGM). The resonating group method (RGM) [40], a well-established method for studying a bound-state problem or a scattering one, is used to calculate the hadron-hadron scattering phase shifts and cross-sections, and investigate the resonance states. Resonances are unstable particles usually observed as bell-shaped structures in scattering cross-sections of their open channels. For a simple narrow resonance, its fundamental properties correspond to the visible cross-section features: mass is at the peak position, and decay width is the half-width of the bell shape. More details can be found in ref. [40].

A stabilization method. A stabilization method, also named as a real scaling method, which has proven to be a valuable tool for estimating the energies of the metastable states of electron-atom, electron-molecule, and atom-diatom complexes [30], is employed to identify the genuine resonances. In this method, with the scaling of the distance between two clusters, the continuum state will fall off towards its threshold, while a resonance state will tend to be stable if it does not couple to the open channels, or acts as an avoid-crossing structure if it couples to the open channels and the structure will appear repeatedly with the increment of the scaling.

Results of several exotic hadron states. –

$X_0(2900)$. The observation of $X_0(2900)$ and $X_1(2900)$ indicates the first evidence of an open-charm tetraquark state with the quark content $ud\bar{s}\bar{c}$. In fact, this open charm state has been predicted in the color-magnetic interaction model [41] and the coupled channel unitary approach [42]. Recently, QCD sum rules, quark model, one-boson exchange model, effective field theory and other approaches all suggest the explanation of $X_0(2900)$ as an S-wave $\bar{D}^*K^*$ ($D^*\bar{K}^*$) molecule state.

Table 1: Parameters of two models. $m_\pi = 0.7\ \text{fm}^{-1}$, $m_K = 2.51\ \text{fm}^{-1}$, $m_\eta = 2.77\ \text{fm}^{-1}$, $\Lambda_\pi = 4.2\ \text{fm}^{-1}$, $\Lambda_K = 5.2\ \text{fm}^{-1}$, $\Lambda_\eta = 5.2\ \text{fm}^{-1}$.

	b (fm)	m_u (MeV)	m_s (MeV)	m_c (MeV)	V_0 (fm^2)
ChQM	–	313	536	1728	−0.775
QDCSM	0.518	313	470	1270	−0.733
	a_c (MeV/fm^2)	α_s^{uu}	α_s^{us}	α_s^{uc}	α_s^{sc}
ChQM	101.0	0.57	0.54	0.48	0.44
QDCSM	58.03	1.50	1.46	1.45	1.44

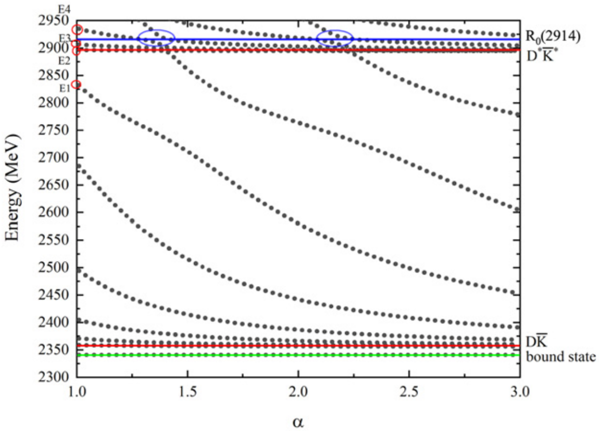

Fig. 1: Energy spectrum of $^1S_0$ $cs\bar{q}\bar{q}$ system.

We systematically investigate the S-wave tetraquarks composed of $ud\bar{s}\bar{c}$ in both the ChQM [43] and QDCSM [44]. Two structures, meson-meson and diquark-antidiquark, are considered. The parameters used are listed in table 1. The Gaussian expansion method is employed to do the calculation in the ChQM. In our calculations, many discrete energy levels are obtained because the model space is finite. To identify the nature of these states with discrete energy, the real-scaling method is invoked. The results are shown in fig. 1. To be specific, the discussion is limited to the states with energies around 2900 MeV. From fig. 1, one can see that there are four energy levels around 2900 MeV, $E_1(2836)$, $E_2(2896)$, $E_3(2906)$ and $E_4(2936)$. The behavior of state $E_1(2836)$ rapidly falls to the lowest threshold of $D\bar{K}$ with the increasing of scaling factor α (spaces getting bigger), the state E_{2896} is just the threshold of $D^*\bar{K}^*$, and both $E_3(2906)$ and $E_4(2936)$ fall to $D^*\bar{K}^*$ threshold. However, the $E_4(2936)$ undergoes an avoid-crossing structure and the structure repeats at $\alpha = 2.2$, which indicates the existence of a resonance state $R_0(2914)$. The mechanism behind the avoid-crossing structure is as follows. The energies of scattering state fall to its threshold rapidly with the increasing of α, and the energy of resonance state is stable if it stands alone. The scattering state will interact with the resonance state weakly when the energy difference is large, and the interaction will become strong when the energy difference gets small, and it will induce an avoid-crossing structure when the energy of the scattering state approaches that of resonance state. The structure will repeat with the increasing of α. The decay width can be estimated from the avoid-crossing structure, the result for $R_0(2914)$ is about 41.8 MeV [43]. Comparing with the experiment, the $R_0(2914)$ is possible to be a good candidate of $X_0(2900)$. Besides, there is a bound state with mass 2341 MeV obtained, which is denoted by a horizontal line below the $D\bar{K}$ threshold [43].

In QDCSM, a dynamic bound-state calculation is carried out by using the RGM [40]. The single channel dynamical calculation indicates that there is a bound state $\bar{D}^*K^*$ with energy 2820.7 MeV and quantum numbers $IJ^P = 00^+$, which is possible to explain the reported $X_0(2900)$. However, the state can decay to the $\bar{D}K$ channel. To confirm whether the states of $\bar{D}^*K^*$ can survive as a resonance state after coupling to the scattering state, further study of the scattering process of $\bar{D}K$ is needed, which is going on. The energy of the state would be pushed up by coupling to the open channel, much closer to the mass of the $X_0(2900)$. Besides, two bound states $\bar{D}K$ and $\bar{D}^*K$ are obtained by the channel coupling calculation. Their energies are 2341.2 MeV and 2489.7 MeV, with quantum numbers $IJ^P = 00^+$ and $IJ^P = 01^+$, respectively. This indicates that the channel coupling may induce bound states or resonances.

Fully heavy tetraquark states. The fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}, Q = c, b$) attracted extensive attention for the last few years, since such states with very large energy can be accessed experimentally and easily distinguished from other states. The experimental data and theoretical importance of $cc\bar{c}\bar{c}$ tetraquarks were reviewed in the articles [45,46].

The low-lying fully heavy systems $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ are systematically investigated in both ChQM and QDCSM with three different sets of parameters [47]. Two structures, meson-meson and diquark-antidiquark, are considered. The dynamic bound-state calculation is carried out to search for any bound state in the fully heavy systems. To explore the effect of the multi-channel coupling, both the single channel and the channel coupling calculation are performed. The resonance states can be found in figs. 8–10 of ref. [47]. The numerical results show that:

1) For the fully charm $cc\bar{c}\bar{c}$ systems, there are four possible resonance states with $IJ^P = 00^+$, and the energy ranges and decay width are 6205–6270 MeV and 13.2–22.3 MeV, 6825–6975 MeV and 54.5–61.3 MeV, 7140–7170 MeV and 27.6–40.7 MeV, and 7210–7260 MeV and 64.7–87.1 MeV, respectively. The possible decay channels are $\eta_c\eta_c$ and $J/\psi J/\psi$. The reported state $X(6900)$ can be explained as a compact resonance state with $IJ^P = 00^+$ in the present calculation. Besides, we also obtain two resonance states with $IJ^P = 01^+$. The energies and decay width are 6740–7150 MeV and 257.3–285.5 MeV,

Table 2: The mass and decay width (in MeV) of the resonance states in the S-wave scattering channels.

	$IJ^P=\frac{1}{2}\frac{1}{2}^-$					
	$\Sigma_c D$		$\Sigma_c D^*$		$\Sigma_c^* D^*$	
	$M^{'}$	Γ_i	$M^{'}$	Γ_i	$M^{'}$	Γ_i
$N\eta_c$	4311.3	4.5	4448.8	1.0	4525.8	4.0
NJ/ψ	4307.9	1.2	4459.7	3.9	nr	–
$\Lambda_c D$	4306.7	0.02	4461.6	1.0	nr	–
$\Lambda_c D^*$	4307.7	1.4	4449.0	0.3	nr	–
Γ_{total}	7.12		6.20		4.00	
	$IJ^P=\frac{1}{2}\frac{3}{2}^-$					
	$\Sigma_c D^*$		$\Sigma_c^* D$		$\Sigma_c^* D^*$	
	$M^{'}$	Γ_i	$M^{'}$	Γ_i	$M^{'}$	Γ_i
NJ/ψ	4445.7	1.5	4376.4	1.5	nr	–
$\Lambda_c D^*$	4444.0	0.3	4374.4	0.9	4523.0	1.0
Γ_{total}	1.8		2.4		1.0	

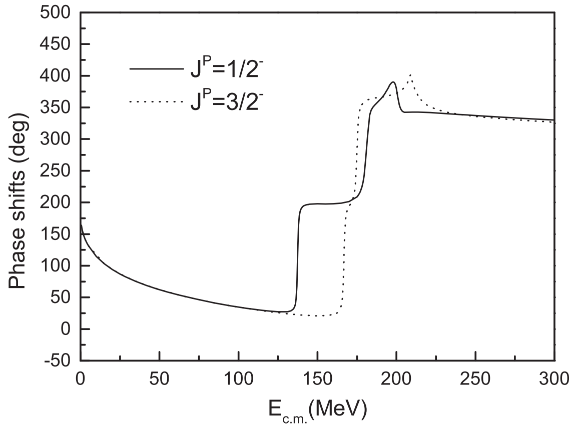

Fig. 2: The phase shifts of the open channel NJ/ψ with channel coupling of the bound-state channels for the $IJ^P=\frac{1}{2}\frac{1}{2}^-$ and $IJ^P=\frac{1}{2}\frac{3}{2}^-$ systems.

7250–7280 MeV and 55.3–87.8 MeV, respectively. The possible decay channel is $\eta_c J/\psi$. Moreover, a resonance state with $IJ^P=02^+$ is obtained. The energy and decay width are 6725–7050 MeV and 202.1–296.2 MeV. The possible decay channel is $\eta_c\eta_c$. All these fully heavy resonance states are worth searching in the future experiments. We suggest the experiment to look for more fully heavy tetraquark states in possible decay channels.

2) For the fully beauty $bb\bar{b}\bar{b}$ systems, there is not any bound state or resonance state in two structures in ChQM, while in QDCSM, the wide resonances are possible in the diquark-antidiquark structure, and the resonance energies are 19122–19344 MeV for $IJ^P=00^+$, 19184–19354 MeV for $IJ^P=01^+$, and 19236–19374 MeV for $IJ^P=02^+$, respectively.

3) Comparing the results of two quark models, the energy for the meson-meson configuration is almost the same, because the σ meson exchange of the ChQM is inoperative and the quark delocalization of QDCSM is close to 0 in the systems of fully heavy quarks. In the diquark-antidiquark configuration, the energy of QDCSM is smaller than that of ChQM, due to the color octet symmetry of the diquark and antidiquark, which makes the quark delocalization work in QDCSM, and leads to lower energy in this model. Nevertheless, the conclusions are consistent in these two quark models.

Hidden-charm pentaquarks. The observations of the hidden-charm pentaquarks bring great interest in pentaquarks. Excellent review on the hidden-charm pentaquarks can be found in refs. [11–13]. The main interpretation with $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecular configurations are provided by QCD sum rules, potential models, effective field theory, and other approaches.

We have investigated the possible hidden-charm molecular pentaquarks with $Y=1$, $I=\frac{1}{2}$, $J^P=\frac{1}{2}^{\pm}$, $\frac{3}{2}^{\pm}$, and $\frac{5}{2}^{\pm}$ in QDCSM [48]. The channel coupling effects are also taken into account. Besides, to provide more information for experiments, the corresponding baryon-meson scattering processes are also investigated [49]. The resonance mass and decay width of resonance states are listed in table 2. To save space, we take the phase shifts of the open channel NJ/ψ as an example, which are shown in fig. 2. Our results indicate that:

1) For the negative parity states, there are several bound states and resonance states. For the $J^P=\frac{1}{2}^-$ system, there is a bound state of 3881–3884 MeV by seven channels coupling, and the main channel is $N\eta_c$, the single channel calculation of $N\eta_c$ cannot find bound state. Three molecular resonance states are obtained. To show the behavior of resonances, the channel coupling calculations of the resonance with the open channel are performed. The $\Sigma_c D$ appears as a resonance state in the open channels $N\eta_c$, NJ/ψ, $\Lambda_c D$ and $\Lambda_c D^*$, with the resonance mass of 4307–4311 MeV, and decay width of 7 MeV (the first rising of the solid line in fig. 2. To save space, only the results of the coupling among resonances and open channel NJ/ψ are shown), which is consistent with the subsequent experimental results of $P_c(4312)$. The $\Sigma_c D^*$ also behaves as a resonance state in the above four open channels with the resonance mass of 4449–4462 MeV, and decay width of 6.2 MeV (the second rising of the solid line in fig. 2), which can be used to explain the reported $P_c(4457)$. Besides, $\Sigma_c^* D^*$ also appears as a resonance state in the $N\eta_c$ phase shifts, with the resonance mass of 4526 MeV, and decay width of 4 MeV. However, there is only a cusp around the threshold of the state $\Sigma_c^* D^*$ in the phase shifts of the open channel NJ/ψ. The reason is that the channel coupling pushes the higher state above the threshold.

2) For the $J^P=\frac{3}{2}^-$ system, the main channel of the bound state is NJ/ψ with the mass of 3997–3998 MeV, which is a resonance state in the D-wave $N\eta_c$ scattering process. Besides, three resonance states are obtained. $\Sigma_c D^*$ is a resonance state in the open channels NJ/ψ and

$\Lambda_c D^*$, with the resonance mass of 4444.0–4445.7 MeV, and decay width of 1.8 MeV, which can be used to explain the reported $P_c(4440)$. $\Sigma_c^* D$ is also a resonance state in the open channels NJ/ψ and $\Lambda_c D^*$. Although the resonance mass 4374.4–4376.4 MeV is close to the $P_c(4380)$, the decay width is only 2.4 MeV, much smaller than the experimental data. Additionally, the $\Sigma_c^* D^*$ also behaves as a resonance state in the $\Lambda_c D^*$ scattering process, and the resonance mass and decay width are 4523.0 MeV and 1.0 MeV, respectively.

3) For the $J^P = \frac{5}{2}^-$ system, there is a resonance state $\Sigma_c^* D^*$ with the mass of 4512–4517 MeV, which can also decay to several D-wave open channels.

4) All of these heavy pentaquarks are worth investigating in future experiments. Besides the open channel NJ/ψ, other channels, like $N\eta_c$, NJ/ψ, $\Lambda_c D$ and $\Lambda_c D^*$ with $J^P = \frac{1}{2}^-$, and $\Lambda_c D^*$ with $J^P = \frac{3}{2}^-$, are possible decay channels for these heavy pentaquark resonances mentioned above. In addition, the theoretical study shows that the $\Sigma_c^* D^*$ with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$, and $J^P = \frac{5}{2}^-$ are all resonance states, but none of them has been verified by experiments. So we suggest the experiment to search more heavy pentaquarks in more decay channels.

5) The calculation is also extended to the hidden-bottom pentaquarks. The results are similar to the case of the hidden-charm molecular pentaquarks. Some narrow pentaquark resonances with hidden-bottom above 11 GeV are found from corresponding scattering processes.

$X(3872)$. The quantum numbers of state $X(3872)$ can be reproduced in the quark-antiquark picture, but it has some strange properties: its mass is very close to the threshold of $D\bar{D}^*$, and its decay width is very narrow (less than 1.2 MeV). Different theoretical explanations for $X(3872)$ were proposed, which include the meson picture, the molecular state, the diquark-antidiquark state, the tetraquark state, the hybrid charmonium, the 1^{++} cusp, the S-wave threshold effect due to the $D^0\bar{D}^{0*}$ threshold, and the mixture state of $c\bar{c}$ and $D\bar{D}^*$.

We invoked unquenched quark model to investigate the nature of $X(3872)$ [39]. The multichannel couplings of the quark-antiquark state with all the possible meson-meson states are considered. To justify the approach, the masses of η_c and J/ψ are fitted by adjusting the parameters which related to charm quark, other parameters are taken from ref. [33]. We found that the dominant components of η_c and J/ψ are $c\bar{c}$, over 90%, which means that the valence quark model is a good one for the low-lying states. Meanwhile, the calculated unquenched mass of $\chi_{c1}(2P)$ is 3871.7 MeV, which is almost the experimental value of $X(3872)$, and the component of $c\bar{c}$ is about 70%, the fraction of $D\bar{D}^* + D^*\bar{D}$ is around 22.5%, $D^*\bar{D}^*$, 5.6% and $D_s^{(*)}\bar{D}_s^{(*)}$, 2.4%. The results infer that the states above the open channels will have large meson-meson components generally.

Besides, the decay width of $X(3872)$ as a mixing of $c\bar{c}$ and $D\bar{D}^*$ can be estimated. From the electromagnetic decay width $\sim$ 300 keV [50] of $c\bar{c}$ 2^3P_1 state, and 8.7–49.5 keV [51] of $D\bar{D}^*$ molecular state, we can estimate the electromagnetic decay width of $X(3872)$ in our model is around 220 keV. The strong decay width of $X(3872)$ depends on the mass of the state strongly. If the mass of $X(3872)$ is below the threshold of $D\bar{D}^*$, the width is 0, and the mass is 3872.0 MeV, the strong decay width is about 1 MeV [52], So the results of $X(3872)$ is our calculation is consistent with the experimental data.

Summary. – Over the past decades, dozens of exotic hadron states have been reported in experiments worldwide. These states provide us with an ideal platform to deepen our understanding of the non-perturbative QCD. With the development of the experiments, in this short review, we briefly summarize the progresses of our group on several exotic hadron states. The channel coupling effects are emphasized. By giving this review, we also learn some valuable lessons and gain great inspirations.

1) As we know, quark model plays an important role in the development of hadron physics. The discovery of Ω^- is based on the prediction of the quark concept of Gell-Mann–Zweig. The naïve quark model of Glashow-Isgur gave a remarkable description of the properties of ground-state hadrons. Applying it to the excited states of hadron, hadron-hadron interaction and multiquark systems, extensions of the naïve quark model have to be made. Based on the different extension of the naïve quark model, a proliferation of bound states or resonances are predicted. The prediction of dibaryons d^* and $N\Omega$ was supported by experiments [53,54]. Three hidden-charm resonance states, which are $\Sigma_c D$ with $J^P = \frac{1}{2}^-$, $\Sigma_c D^*$ with $J^P = \frac{3}{2}^-$ and $J^P = \frac{1}{2}^-$, were obtained in QDCSM before the experiment. These progresses of quark model are a great encouragement.

2) For a multiquark system, there are several structures by arranging quarks/antiquarks into sub-clusters, and there are also many channels due to color, spin and flavor coupling for each structure. It is easy to obtain stable states for the structure with colorful sub-clusters. However, the observable of these states depends on its coupling to the open channels (the states with color-singlet sub-clusters). The coupling can shift the energy of the resonance, give the width to the resonance and even destroy the resonance.

3) The unquenched quark model may be another critical development of the quark model, where the valence quarks and real/virtual quark pair are treated equally. Caramés and Valcarce have studied the possible multiquark contributions to the charm baryon spectrum by considering higher-order Fork space components [55]. The employment of the unquenched quark model to the study of $X(3872)$ indicates that $X(3872)$ is a mixture state of $c\bar{c}$ and $D\bar{D}^*$. By incorporating new ingredients, the phenomenological quark model is expected to give a unified description of ordinary and exotic hadrons.

4) Up to now, none of the exotic hadron states is definitely confirmed by experiment. To each experimental observation, there always exist many different theoretical interpretations. To provide the necessary information for experiments to search for exotic hadron states, mass spectrum calculations alone are not enough. The coupling calculation between the bound channels and open channels is indispensable. The study of the scattering process and the real-scaling method are two effective ways to look for the genuine resonances. However, to distinguish the various explanations and confirm the existence of the exotic hadron states is still very difficult, and requires the joint efforts of both theorists and experimentalists.

* * *

This work is supported partly by the National Natural Science Foundation of China under Contract Nos. 11675080, 11775118 and 11535005.

Data availability statement: No new data were created or analysed in this study.

REFERENCES

[1] BaBar Collaboration (Aubert B. *et al.*), *Phys. Rev. Lett.*, **90** (2003) 242001.
[2] Belle Collaboration (Choi S. K. *et al.*), *Phys. Rev. Lett.*, **91** (2003) 262001.
[3] BESIII Collaboration (Ablikim M. *et al.*), *Phys. Rev. Lett.*, **110** (2013) 252001.
[4] Particle Data Group (Tanabashi M. *et al.*), *Phys. Rev. D*, **98** (2018) 030001.
[5] LHCb Collaboration (Aaij R. *et al.*), *Sci. Bull.*, **65** (2020) 1983.
[6] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **125** (2020) 242001.
[7] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **115** (2015) 072001.
[8] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **122** (2019) 222001.
[9] Wu J. J. *et al.*, *Phys. Rev. Lett.*, **105** (2010) 232001.
[10] Yang Z. C. *et al.*, *Chin. Phys. C*, **36** (2012) 6.
[11] Chen H. X. *et al.*, *Phys. Rep.*, **639** (2016) 1.
[12] Liu Y. R. *et al.*, *Prog. Part. Nucl. Phys.*, **107** (2019) 237.
[13] Yang G. *et al.*, *Symmetry*, **12** (2020) 1869.
[14] Albuquerque R. M. *et al.*, *J. Phys. G*, **46** (2019) 093002.
[15] Maris P. and Roberts C. D., *Int. J. Mod. Phys. E*, **12** (2003) 297.
[16] Chakrabarty S. and Deoghuria S., *Prog. Part. Nucl. Phys.*, **33** (1994) 577.
[17] De Rújula A., Georgi H. and Glashow S. L., *Phys. Rev. D*, **12** (1975) 147.
[18] Isgur N. and Karl G., *Phys. Rev. D*, **18** (1978) 4187; **19** (1979) 2653; **20** (1979) 1191.
[19] Faessler A. and Fernandez F., *Phys. Lett.*, **124** (1983) 145.
[20] Obukhovsky I. T. and Kusainov A. M., *Phys. Lett. B*, **238** (1990) 142.
[21] Fernandez F. *et al.*, *J. Phys. G*, **19** (1993) 2013.
[22] Fujiwara Y. *et al.*, *Phys. Rev. Lett.*, **76** (1996) 2242; Fujiwara Y. *et al. Phys. Rev. C.* **65** (2001) 014002.
[23] Yu Y. W. *et al.*, *Phys. Rev. C*, **52** (1995) 3393.
[24] Valcarce A. *et al.*, *Rep. Prog. Phys.*, **68** (2005) 965 and references therein.
[25] Wang F. *et al.*, *Phys. Rev. Lett.*, **69** (1992) 2901.
[26] Chen L. Z. *et al.*, *Phys. Rev. C*, **76** (2007) 014001.
[27] Ping J. L. *et al.*, *Phys. Rev. C*, **79** (2009) 024001.
[28] Chen M. *et al.*, *Phys. Rev. C*, **83** (2011) 015202.
[29] Huang H. X. *et al.*, *Phys. Rev. C* **92** (2015) 065202.
[30] Simon J., *J. Chem, Phys.*, **75** (1981) 2465.
[31] Beveren E. *et al.*, *Phys. Rev. D*, **21** (1980) 772.
[32] Bijker R., Gmez S., Lopez-Ruiz M. *et al.*, *AIP Conf. Proc.*, **1753** (2016) 050003.
[33] Chen X. Y. *et al.*, *Phys. Rev. D*, **97** (2018) 094016.
[34] Kalashnikova Y. S., *Phys. Rev. D* **72** (2005) 034010.
[35] Ortega P. G., Segovia J., Entem D. R. *et al.*, *Phys. Rev. D*, **81** (2010) 054023.
[36] Micu L., *Nucl. Phys. B*, **10** (1969) 521.
[37] Yaouanc A. L. *et al.*, *Phys. Rev. D*, **8** (1973) 2223.
[38] Roberts W. and Silvestre-Brac B., *Few-Body Syst.*, **11** (1992) 171.
[39] Tan Y. and Ping J. L., *Phys. Rev. D*, **100** (2019) 034022.
[40] Kamimura M., *Prog. Theor. Phys. Suppl.*, **62** (1977) 236.
[41] Cheng J. *et al.*, *Phys. Rev. D*, **101** (2020) 114017.
[42] Molina R. *et al.*, *Phys. Rev. D*, **82** (2010) 014010.
[43] Tan Y. and Ping J., *Chin. Phys. C*, **45** (2021) 093104.
[44] Xue Y. *et al.*, *Phys. Rev. D*, **103** (2021) 054010.
[45] Richard J.-M., arXiv:2008.01962 [hep-ph].
[46] Chao K. and Zhu S., *Sci. Bull.*, **65** (2020) 1952.
[47] Jin X. *et al.*, *Eur. Phys. J. C*, **80** (2020) 1083.
[48] Huang H. X. *et al.*, *Eur. Phys. J. C*, **76** (2016) 624.
[49] Huang H. X. and Ping J. L., *Phys. Rev. D*, **99** (2019) 014010.
[50] Barnes T. *et al.*, *Phys. Rev. D*, **72** (2005) 054026.
[51] Wang Z. Y. *et al.*, *Phys. Rev. D* **97** (2018) 016015.
[52] Wang H. *et al.*, *Eur. Phys. J. A*, **50** (2014) 76.
[53] WASA-at-COSY Collaboration (Adlarson P. *et al.*), *Phys. Rev. Lett.*, **112** (2014) 202301.
[54] STAR Collaboration (Adam J. *et al.*), *Phys. Lett. B*, **790** (2019) 490.
[55] Caramés T. F. and Valcarce A., *Phys. Rev. D*, **90** (2014) 014042.