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Heterogeneous investments induced by emotions promote cooperation in public goods games

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Abstract – Emotion plays an important role in heterogeneous investments and has some direct effects on the cooperation behaviour of a player in a public goods game (PGG). How this irrational factor affects the heterogeneous investments and what level of cooperation is present in players with emotions are still unknown to us. Here, the heterogeneous investments induced by emotions into a PGG were introduced. The emotional index was firstly quantified by considering a memory-cumulative effect, and then an investment formula was proposed based on this emotional index. At last, the effect of emotions on the cooperation behaviour in a PGG was investigated. Results show that the heterogeneous investments induced by emotions can improve cooperation significantly in a PGG, and that an increase of the memory length, the emotional increment, or the memory discounting factor can improve the cooperation level.

Introduction. – Cooperation is a common phenomenon and plays an important role in evolutions in a human society, animal world and biological systems [1,2]. It has become a hot topic to explain the cooperation behaviour in diverse systems. Public goods game (PGG) is one of the powerful tools for explaining the cooperation behaviour and is widely used in solving all kinds of problems in the energy development, in the environmental protection issue, and in the supply chains among companies, etc. [3–5]. In the early stages, many researchers investigated the cooperation behaviour in a PGG with a homogeneous investment, which has some shortcomings in explaining many social cooperation phenomena, such as voluntary service, charitable giving, trust and reciprocity. In real life, heterogeneous investments are more common than homogeneous investments because players investment is affected by many factors, such as network structures, risk appetites, prospective earnings, emotions, etc.

Heterogeneous investments have been widely investigated for many years. Cao *et al.* [6], Lei *et al.* [7] and Wang *et al.* [8] proposed an investment model by considering the degree of a focal player, and they introduced a tunable parameter to change the investment distribution to the groups with different nodes. Vukov *et al.* [9], Zhang *et al.* [10] suggested to contribute an investment to a group according to a benefit obtained from this group in the previous simulation step. It is no doubt that this investment can obviously improve a cooperation level because a cooperator contributes more investments to the groups who are more beneficial to him. Meloni *et al.* [11] found that a Pareto distribution exists in the payoff when contributing an investment according to a payoff in his previous round and they claimed that some formal cooperators act as defectors if they invest little in the previous round. Yuan *et al.* [12], Gao *et al.* [13] suggested to give a larger investment into a group with a higher fraction of cooperators. This greatly enhances a collective cooperation behaviour in PGGs. Szolnoki *et al.* [14] proposed a selecting model for a cooperator who can contribute external investments exclusively to his most successful neighbor. This investment model is novel and interesting because it has no need for recording the historical incomes and no need for knowing the topology of interaction graph. Lee *et al.* [15] further investigated the effects of a small fraction of selective cooperators on the cooperation level in PGGs and proved that a full cooperator state can still be reached even under a small fraction of selecting cooperators.

The above investigations were conducted under a hypothesis of a rational man. In fact, people have emotions in different games, which also affect the cooperation level [16–19]. The effects of emotions on the cooperation behaviour in different games can be grouped along the following two aspects. Firstly, emotions could be imitated

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and spread in a network. A pioneering work about the effect of emotions on the cooperation behaviour was reported by Szolnoki *et al.* [20,21]. Szolnoki *et al.* claimed that a player has a sympathy emotion and an envy emotion when behaving towards less and more successful neighbors. And he found that imitating these emotions instead of strategies greatly promotes cooperation and leads to a high level of social welfare. Wang *et al.* [22], Ji *et al.* [23] and Xie *et al.* [24] built on Szolnoks work to study the Prisoners Dilemma game with some more emotional parameters and also found that the diversity of emotions can emerge spontaneously and that the cooperation level is greatly improved. Secondly, emotions affect the players decision making in updating their strategies. Wang *et al.* [25] defined an emotional decision model based on a prosocial preference of a player when facing his neighbors with some different behaviour. He examined the effects of emotional decisions on the evolution of the cooperation in a PGG, and found that emotions significantly improve a high cooperation level in a PGG. Chen *et al.* [26] quantified the emotion patterns by considering an emotional cumulative length and proposed a probability function for a player learning from his neighbors. He found that this emotional-updating strategy decreases the cooperation level for competitive individuals at a large memory length.

Except for these two aspects, emotions also affect the heterogeneous investments, which are usually ignored by our researchers. A common phenomenon can be seen in real life that people usually do some investments according to their emotions without a rational analysis. If they feel happy, they invest more. If they feel angry, they invest less or even nothing. Moreover, emotions have a memory-cumulative effect [27–29]. If one takes a defection strategy with a high frequency, he will be more disgusted by others and may get less in the future investments from his neighbors. Even so, some defectors still survive in reality. How does this phenomenon happen? How does this emotion affect the investments? What is the cooperation level for a heterogeneous investment caused by these emotions? There problems are still mysterious to us.

Motivated by these discussions, we introduced the effect of heterogeneous investments caused by players' emotions on the evolution of cooperation in a PGG. Firstly, the emotional index in a PGG was quantified by considering a memory-cumulative effect, and then an investment formula was proposed according to this emotional index. At last, the effect of emotions on the cooperation behaviour in a PGG was investigated. The results shows that heterogeneous investments induced by emotions can significantly improve cooperation level, comparing with the results obtained in a traditional PGG with homogeneous investments [30].

This paper is organized as follows. The next section introduces the model of heterogeneous investments induced by emotions. The third section presents the results and discussion. The last section draws conclusions.

Model. – A two-dimensional square lattice with a 100×100 nodes network structure and with periodic boundary conditions is considered. Each player occupies a node with 4 linked neighbors, and participates in a PGG with its neighbors. There are two strategies for each player, cooperation by contributing some investments into a public pool, or defection by contributing nothing. The payoff $u_{i,j}(t)$ of player i obtained from a group centered on player j is given by

$$u_{i,j}(t) = \frac{r}{5} \sum_{k \in \Omega_j} I_{k,j}(t) \cdot s_k - I_{i,j}(t) \cdot s_i, \tag{1}$$

where Ω_{j} is a group centered on player j, s_i the strategy value which is equal to 1 for a cooperator and 0 for a defector, r the synergy factor, which is usually larger than 1 and produces a multiplication effect on the total investment, $I_{i,j}$ the investment of player i contributing to the group centered on player j.

Traditional PGG supposes that all players are rational and contribute equally to all their neighbors. But in real games, a cooperator has two different preferences when contributing to a cooperator or a defector. A cooperator will be happy when contributing to a cooperative neighbor, and will be regretful when contributing to a defector. A defector will be rational to all of his neighbors because he contributes nothing to his neighbors.

To quantify the emotion of happiness or regret, an emotional index is defined. Player i increases his emotional index by one unit δ on his neighbor (player j) at a time step (t), if he feels happy when contributing to player j. Player i decreases his emotional index by one unit δ on player j if he feels regretful to player j. Player i keeps a constant emotional index on player j if he feels rational. Thus, the variation of emotional index of player i on player j could be given by

$$\Delta_{i,j}(t) = \begin{cases} \delta, & \text{happiness}, \\ 0, & \text{rationality}, \\ -\delta, & \text{regret}. \end{cases} \tag{2}$$

Usually, all players show a rational attitude at the initial time $(t = 1)$, and their emotional index increases over time and shows some different values when contributing to different neighbors. The emotional index of player i on player j at step t has a memory-cumulative effect, and it could be expressed as

$$E_{i,j}(t+1) = \begin{cases} \sum\limits_{l=1}^{t} \varepsilon^{l-1} \Delta_{i,j}(t-l+1), & t < m, \\ \sum\limits_{l=1}^{m} \varepsilon^{l-1} \Delta_{i,j}(t-l+1), & t \geq m, \end{cases} \tag{3}$$

where $E_{i,j}$ is the emotional index of player i on player j. If $E_{i,j}$ exceeds 0, player i feels happy with player j. If $E_{i,j}$ is less than 0, player i feels regretful with player j. If $E_{i,j}$ is 0, player i keeps rational to player j. m is the memory length, ε the memory discounting factor, $0 < \varepsilon \leq 1$. If $0 < \varepsilon < 1$, it means that the more time passes, the less impression it gets, and it also indicates that the current emotion is the most impressive one. If $\varepsilon = 1$, it means the memory has no decay, which also happens in real life, such as a computer memory, an artificial memory for a short memory length.

Suppose that the total investment of a cooperator is fixed and equals 1. The investment of player i contributed to player j depends on his emotions. Thus, $I_{i,j}(t+1)$ is determined by the emotional index,

$$I_{i,j}(t+1) = \frac{e^{\alpha E_{i,j}(t)}}{\sum\limits_{k\in\Omega_i} e^{\alpha E_{i,k}(t)}}, \tag{4}$$

where $I_{i,j}(t+1)$ is the investment of player i to player j at time $t+1$. At the initial time, the investment of all cooperators to their neighbors is 0.2, as said by [31]. α is an adjustable parameter, and $\alpha > 0$.

The total payoff of player i at time t can be calculated from his groups, as follows:

$$U_i(t) = \sum_{j\in\Omega_i} u_{i,j}(t). \tag{5}$$

All players update their strategies synchronously by means of the Fermi rule [31]. Player i randomly chooses one of his neighbors j, and adopts the strategy of player j with a probability

$$W(S_i \leftarrow S_j) = \frac{1}{1 + e^{(U_i - U_j)/K}}, \tag{6}$$

where K is the intensity of noise and is usually set to be 0.1 as reported by Zhang *et al.* [31].

Initially, each player keeps a rational attitude, and adopts a cooperation strategy (C) or a defection strategy (D) with a randomly equal probability. All cooperators contribute an investment of 0.2 to their neighbors at the initial time. Subsequently, every payoff value is updated and an emotional index is calculated: A cooperator increases his emotional index towards a cooperator neighbor and decreases his emotional index towards a defector, according to eqs. (2),(3). Such an emotional model allows a player to possess a different emotion towards different neighbors at the same time. Importantly, a new investment plan for each player to his neighbors in the next round will be given out according to his emotional indexes. It should be noted that the investments of a player to his neighbors can also be different at the same time. Finally, all players update their strategies synchronously according to the Fermi rule in eq. (6). Each player randomly chooses one neighbor and adopts this neighboring strategy with a probability shown in eq. (6). To measure the cooperation level, the Monte Carlo simulation is calculated with a sufficiently long time (1×10^4) and the cooperation results are obtained by averaging over 20 independent trials with random initial conditions.

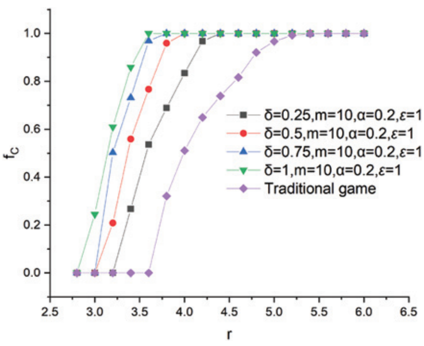


Fig. 1: Fraction of cooperators f_c *vs.* the synergy factor r.

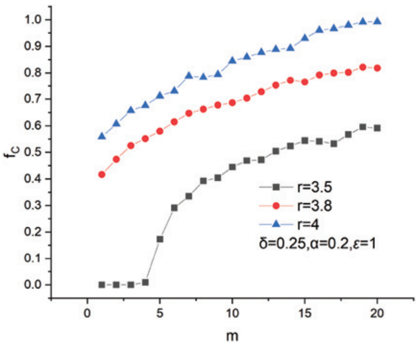
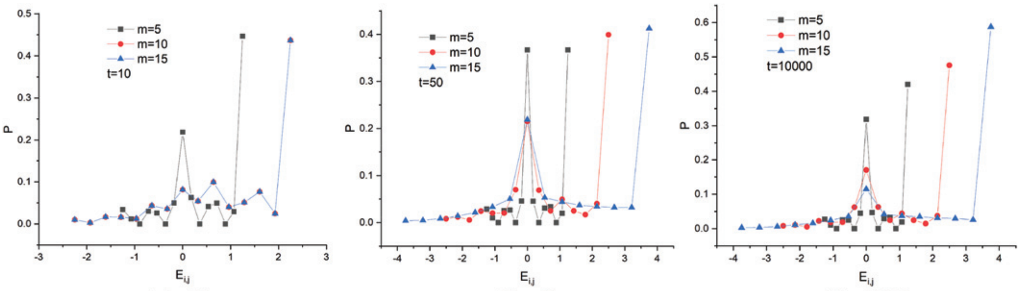


Fig. 2: Fraction of cooperators f_c *vs.* the memory length m.

Results and discussions. – The synergy factor has a great effect on the cooperation level. A small synergy factor produces a small payoff, resulting in a deadlock that no one chooses to be an investor. When the synergy factor exceeds a critical value, the fraction of cooperators begins to increase until a full cooperator state with an increase of the synergy factor, as shown in fig. 1. This means that a proper synergy factor could change this deadlock because a mixed Nash equilibrium exists in this game. When the synergy factor is large enough, all players choose to be a cooperator because the total payoff is very large even under a small investment.

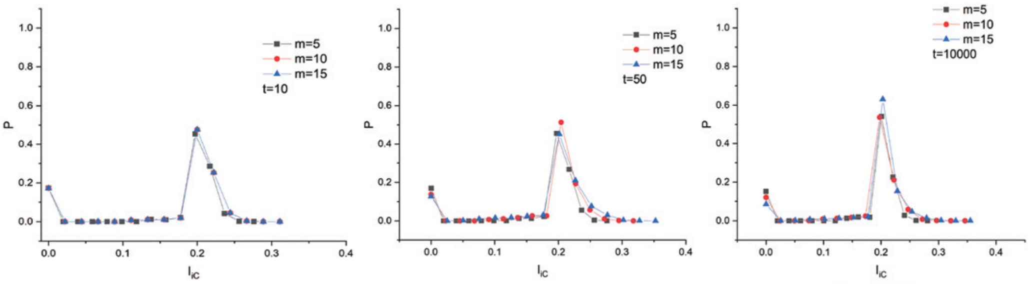
Compared with the result of the traditional game, the curves of the cooperator fraction in this new game obviously move to the left when considering the emotional effect. On the one hand, the emotion could break up the deadlock under a small synergy factor condition. On the other hand, the cooperation level in this new model is also higher than that in the traditional game.

Usually, a player has a memory capacity for remembering the historical behaviour of his neighbors. This memory length directly affects the cooperation level, as shown in fig. 2. An increase of the memory length promotes the cooperation level. In reality games, a cooperator usually has a regretful emotion on his neighbor who takes a defection strategy. And this attitude becomes stronger if this defection behaviour occurs repeatedly. Thus, a longer memory length results in a poorer investment to a player who takes a defection strategy with multi-times. Even though



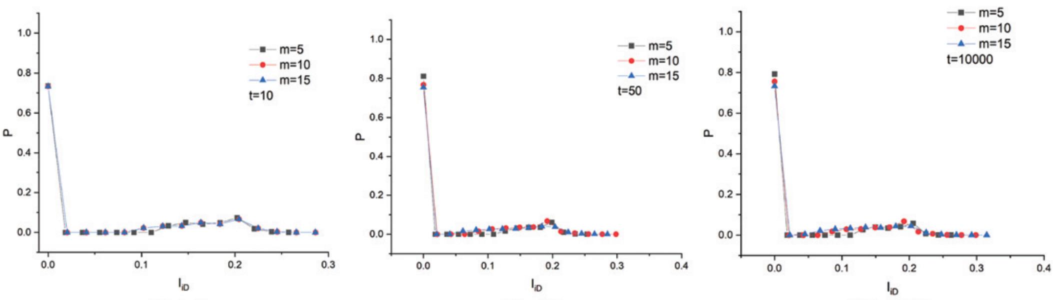
(*a*) *t*=10 (b) *t*=50 (c) *t*=10000

Fig. 3: The probability distribution of the emotional index at several representative times, $\delta = 0.25$, $\varepsilon = 1$.



(*a*) *t*=10 (b) *t*=50 (c) *t*=10000

Fig. 4: The probability distribution of the investments to cooperators at several representative times, $\delta = 0.25$, $\varepsilon = 1$.



(*a*) *t*=10 (b) *t*=50 (c) *t*=10000

Fig. 5: The probability distribution of the investments to defectors at several representative times, $\delta = 0.25$, $\varepsilon = 1$.

a defector could obtain some benefits from his cooperator neighbors, the average payoff in his group decreases. Thus, the other players would reject to adopt the strategies in the group centered on this defector. That is to say, a large memory length results in isolation from this defector.

To understand the evolution of the emotional index, a probability distribution of the emotional index at several representative times is shown in fig. 3. A long memory length has a large distribution of the emotional index and produces a wide range of investments. It could be found that the probability of rational players decreases while the probability of happy players increases with an increase of the memory length. This means that the memory length could increase the enthusiasm of players to contribute the investment.

The distribution of the emotional indexes brings in a complex investment. A probability distribution of the investments to cooperators and defectors is given out in figs. 4, 5. In figs. 4, 5, I_{iC} represents the investment of players to cooperators, and I_{iD} represents the investment of players to defectors. In terms of the investments to cooperators, their probability curves are characterized by a bimodal distribution, as shown in fig. 4. The first crest appears at $I_{iC} = 0$, which explains the phenomenon that some defectors scatter around cooperators. The second crest appears at $I_{iC} = 0.2$, which is an average investment as that in the traditional PGG [30].

In the early stage, an overlong memory length has no effect on its probability distribution curve when the evolutionary time is shorter than the memory length. Thus, the two probability curves at $m = 10$, 15 overlap perfectly as shown in fig. 4(a), fig. 5(a), because these memory lengths have already exceeded the evolutionary time.

In the developing stage, it could be found that a large memory length reduces the probability of the first crest, as shown in fig. 4(b). This means that the number of

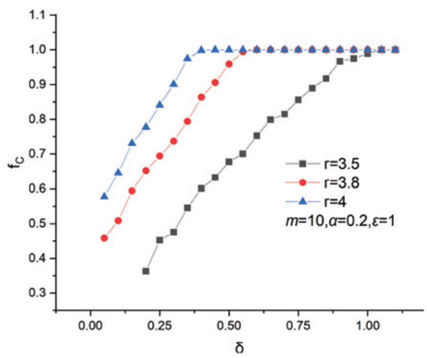


Fig. 6: Fraction of cooperators *vs.* the emotional increment.

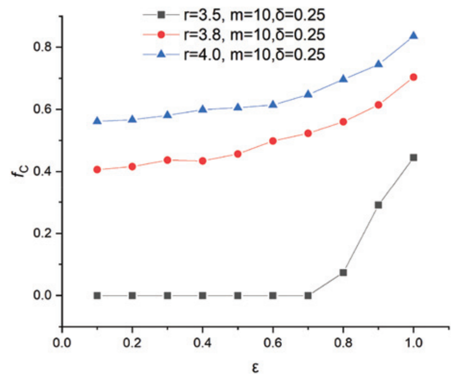


Fig. 7: Fraction of cooperators *vs.* the memory factor.

defectors around these cooperators decreases when increasing the memory length. It could be also found that the probabilities of high investments ($I_{i\mathrm{C}} > 0.25$) increase with an increase of a memory length. In terms of the investment to defectors, its peak value decreases when increasing the memory length, as shown in fig. 5(b). This means that the number of defectors also decreases at a large memory length.

In the developed stage, it could be found that its probability curve, as shown in fig. 4(c), is similar to that in fig. 4(b). But the probability of the second crest ($I_{iC} = 0.2$) increases with an increase of the memory length. This indicates that most of players take a cooperation strategy at a large memory length in the developed stage.

It should be noted that the probability distribution of the investments to cooperators or to defectors has some changes. The basic shapes and peak values of these probability distribution curves have little change with time, but their standard deviations changes dramatically. This indicates that a player constantly adjusts the investments to his neighbors. By comparing with the investments to cooperators and the investments to defectors, it can be found that players contribute much more investments to cooperators than to defectors. This further proves that the emotion has an isolation effect on defectors and this effect is enhanced when increasing the memory length.

The emotional increment could reflect the emotional variation of players when facing with cooperator neighbors or defector neighbors. Figure 6 shows the fraction of cooperators *vs.* the emotional increment. It could be found that a large increment improves the cooperation level. Once the emotional increment is big enough, all players become full cooperators. As we know, the difference between the investment to a cooperator and that to a defector becomes more and more obvious when increasing this emotional increment. That is to say, a player should be careful when choosing a defection strategy, because the defection behaviour for the short-term interest rapidly produces an isolating effect by neighbors in the subsequent game. The emotional increment may have a more obvious effect in improving the cooperation level than that caused by a large memory length, because one has some allowable space for the defection behaviour under a small emotional increment and a large memory length condition.

The memory discounting factor directly affects the emotional index. A small factor means a player has a bad memory. Once the discounting factor decreases, effects of the history defection behaviour will be weakened, which also means that the infrequent defection behaviour would not greatly reduce the neighboring investment to him. That is also the reason why cooperation level decreases when decreasing the discounting factor, as shown in fig. 7.

Finally, the robustness of the results is checked in Erdös-Rényi (ER) random networks with 1000 nodes and an average degree 4 [32]. The heterogeneous investments induced by emotions are applied in this heterogeneous network. The cooperation fraction *vs.* the synergy factor is shown in fig. 8. The cooperation level of the heterogeneous investments induced by emotions is also higher than that of a homogeneous investment in ER networks. The effects of m, δ, ε on cooperation are similar to the results obtained on the homogeneous network.

Conclusions. – A heterogeneous investment induced by historical emotions in a PGG is investigated. Players' emotions are quantified, and the investment is defined by the emotional index. The effects of the memory length, the emotional increment and the memory discounting factor on the evolution of cooperation are studied. It could be found that the heterogeneous investments induced by emotions can significantly improve cooperation level. Comparing with these three parameters, the memory length, the emotional increment, and the memory discounting factor in this new model, it could be found that the emotional increment greatly affects the current investment. One had better not to choose a defection strategy at a large emotional increment because it instantly brings an isolating effect to himself. A memory with a long length is just like a credit database, which records the historical behaviour of players. One will get more investments if he always keeps cooperative, and will get fewer investments if he always takes a defection strategy. But some allowable space for choosing a defection strategy with a low frequency may be forgivable under a long memory length. For the memory discounting factor, it has a weakened effect on the memory

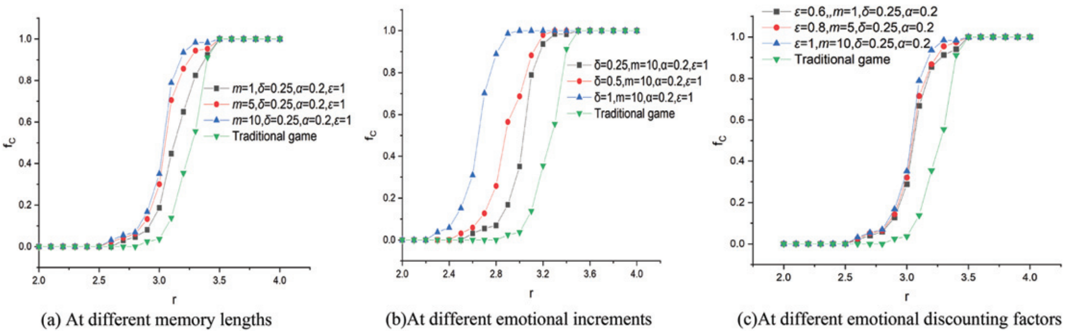


Fig. 8: Fraction of cooperators *vs.* the synergy factor in an ER network with 1000 nodes and an average degree 4.

length. A small memory discounting factor means a bad memory, and players in the networks gradually forgive a historical defector once he changes his mind to be a new cooperator in the last few steps. The emotional effect on the evolution of cooperation is consistent with the real-world phenomena, and we hope our findings can provide more perspectives on understanding the emergence and the maintenance of cooperation in the real world.

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