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Investigating bounds on the extended uncertainty principle metric through astrophysical tests

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Abstract – In this paper, we consider the gravitational tests for the extended uncertainty principle (EUP) metric, which is a large-scale quantum correction to Schwarzschild metric. We calculate gravitational redshift, geodetic precession, Shapiro time delay, precession of Mercury and S2 star's orbits. Using the results of experiments and observations, we obtain the lower bounds for the EUP fundamental length scale L_*. We obtain the smallest bound $L_* \sim 9 \times 10^{-2}$ m for gravitational redshift, and the largest bound $L_* \sim 4 \times 10^{10}$ m for the precession of S2's orbit.

Introduction. – Modifications of Heisenberg uncertainty principle (HUP) play a vital role in gravitational physics. There are two kinds of modifications. The first kind of modification takes into account the quantum gravity effects near the Planck scale, and is called the generalized uncertainty principle (GUP). The simplest form of GUP is given by [1]

$$\Delta x \Delta p \leq 1 + \beta L_{Pl}^2 \Delta p^2, \tag{1}$$

where β is a dimensionless GUP parameter[1]. Apart from GUP, a second kind of modification takes into account a long scale correction, and is called the extended uncertainty principle (EUP). The simplest form of EUP is given by [2]

$$\Delta x \Delta p \leq 1 + \frac{\alpha}{L_*^2} \Delta x^2, \tag{2}$$

where L_* is a fundamental length scale and α is a dimensionless EUP parameter[2].

Since GUP includes quantum gravity effects, it has been intensively studied in the literature. Various GUP models were proposed [1,3–6]. GUP may totally prevent the black hole evaporation. Therefore, black hole thermodynamics can be considered in the context of GUP [7–10]. Investigations of GUP can be extended to different applications of cosmology [11–14], deformed quantum and statistical mechanics [6,15–17], etc.[3].

On the other hand, EUP affects large scale gravitational physics since it includes quantum effects at large distance. Recently, much attention has been focused on EUP. In ref. [19], Bambi and Urban derived EUP from a gedanken experiment in de Sitter spacetime. Another derivation, which is based on the modified commutation relation from a non-Euclidean space, can be found in ref. [20]. A new type of EUP was proposed in ref. [21]. The author studied the deformations of classical mechanics, calculus, and quantum mechanics for the new type of EUP. Just like GUP, EUP also gives some interesting results for the modification of black hole thermodynamics and Friedmann equations. In ref. [22], Dabrowski and Wagner obtained EUP relations for Rindler and Friedmann horizons. They studied black hole temperature and entropy for both relations. They showed that temperature decreases while entropy increases. In ref. [23], Moradpour *et al.* interestingly showed that the EUP correction to black hole entropy is similar to Rnyi entropy. Considering the Bohr-like approach, they also studied the stable-unstable phase transition for an excited black hole. In another paper [24], Chung and Hassanabadi studied Schwarzschild black hole thermodynamics and Unruh effect for EUP. Unlike GUP case, they found a lower bound for the black hole temperature. They also showed that Unruh temperature increases for the EUP correction. In ref. [25], Giné and Luciano obtained the modified inertia for two EUP relations. They showed that EUP may provide a natural explanation for

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[1] We use the units $\hbar = c = 1$ through the paper. We only restore the physical constants for the numerical calculations.

[2] Taking into account both momentum and position uncertainty corrections to HUP, a third kind of modification is also possible. It is called generalized extended uncertainty principle (GEUP), and is given by $\Delta x \Delta p \leq 1 + \beta L_{Pl}^2 \Delta p^2 + \frac{\alpha}{L_*^2} \Delta x^2$.

[3] The literature on GUP is comprehensive. The interested reader may refer to the review in ref. [18].

MoND. EUP can also be considered thermodynamics of the Friedmann-Robertson-Walker (FRW) universe. For example, Zhu *et al.* studied Friedmann equations for GUP and EUP [26]. They found corrected entropy of apparent horizon for GUP and EUP. They obtained modified Friedmann equations from modified entropy and first law of thermodynamics at apparent horizon. It is also possible to consider EUP corrections for conventional thermodynamics systems. In ref. [27], EUP modified number of microstates was obtained to investigate the thermodynamics of monatomic and interacting gas models.

Besides the above mentioned studies, EUP may modify the black hole solutions. EUP black hole solution was proposed by Mureika [2]. He obtained the modified black hole characteristics such as horizon radius, ISCO, and photosphere. It was shown that if L_* is 10^{12}–10^{14} m, EUP will become relevant for the black holes in the range 10^9–$10^{11} M_\odot$. Finally, he calculated the Hawking temperature of an EUP black hole, and found that an EUP black hole temperature has a smaller than standard temperature. Recently, EUP black holes were considered for gravitational lensing [28], shadow and weak deflection angle [29,30].

In this paper, we would like to find lower bounds of the new fundamental length scale L_*. Therefore, we will study some astrophysical tests such as gravitational redshift, geodetic precession, Shapiro time delay, precession of Mercury and S2 star's orbits for EUP metric[4]. Getting constraints on L_* may provide us with a better understanding of large scale EUP effects. Besides, finding bounds on EUP from experiments and observations is sparse in the literature. In the GUP case, the studies on this direction are not new. There are a lot of studies aimed to obtain upper bounds from various experiments and observations [4,35–54]. As for the EUP case, constraints on EUP were studied in refs. [28,54–56]. In ref. [28], Lu and Xie obtained constraints on L_* from gravitational lensing. In ref. [54], Aghababaei *et al.* set bounds on GUP and EUP from Hubble tension. In ref. [55], Nozari and Dehghani found bounds on EUP for both Newtonian and relativistic cosmologies based on Verlinde's entropic gravity. In ref. [56], assuming equality between EUP and gravity sector of Standart Model Extension modified Hawking temperatures, Illuminati et. al. found bounds on EUP dimensionless parameters.

The rest of the paper is arranged as follows. In the next section, we briefly review the EUP metric and derive effective potential of a particle around orbit in EUP metric. In the third section, we use the EUP metric to compute gravitational redshift, geodetic precession, Shapiro time delay, precession of Mercury and S2 star's orbits. Finally, we discuss our results.

The extended uncertainty principle metric. – In this section, we review the EUP metric proposed in ref. [2]. Considering the confinement of N gravitons to Schwarzschild radius $\Delta X \sim r_S = 2G_N M$, each graviton momentum uncertainty Δp_g is given by [2]

$$\Delta p_g \sim \frac{1}{2G_N M}\left(1+\frac{4\alpha G_N^2 M^2}{L_*^2}\right), \tag{3}$$

where M is the black hole mass. If the total mass of N gravitons is considered, then we have $\frac{N}{2G_N M}$. Therefore, total momentum uncertainty ΔP is given by

$$\Delta P \sim M\left(1+\frac{4\alpha G_N^2 M^2}{L_*^2}\right). \tag{4}$$

One may interpret eq. (4) as EUP corrected mass

$$M_{EUP} = M\left(1+\frac{4\alpha G_N^2 M^2}{L_*^2}\right), \tag{5}$$

and assume that EUP correction corresponds to the stress-energy tensor,

$$M_{EUP} = \int \mathrm{d}^3x\sqrt{g}(T^0_{0GR}+T^0_{0EUP}). \tag{6}$$

Replacing M with M_{EUP} leads to EUP corrected Schwarzschild metric, *i.e.*,

$$F(r) = 1-\frac{2G_N M}{r}\left(1+\frac{4\alpha G_N^2 M^2}{L_*^2}\right), \tag{7}$$

and the event horizon of EUP metric is given by

$$r_H = 2G_N M\left(1+\frac{4\alpha G_N^2 M^2}{L_*^2}\right). \tag{8}$$

At this point, we give some comments on dimensionless EUP parameter α. It is assumed that α is taken to be of the order of unity. So, we only get bounds on the new fundamental length scale L_*. Choosing $\alpha = -1$ seems problematic. If α is negative, there is a maximum mass for $r_H = 0$. Another problem arises as repulsive potential for sufficiently large masses. (Please see ref. [2] for more details.) Therefore, we exclude the negativity of α, and consider $\alpha = 1$.

Particle motion in the EUP metric. We begin to consider a particle in the equatorial plane $\theta = \pi/2$. We give the Lagrangian of the particle [57],

$$\mathcal{L} = \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \frac{1}{2}\left[-F(r)\dot{t}^2+\frac{\dot{r}^2}{F(r)}+r^2\dot{\phi}^2\right], \tag{9}$$

where $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\lambda$, and λ is the affine parameter. Following the standard procedure, constants of motion can be obtained,

$$p_t = \frac{\partial\mathcal{L}}{\partial\dot{t}} = -F(r)\dot{t} = -e \quad\Longrightarrow\quad \dot{t} = \frac{e}{F(r)}, \tag{10}$$

$$p_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = r^2\dot{\phi} = \ell \quad\Longrightarrow\quad \dot{\phi} = \frac{\ell}{r^2}, \tag{11}$$

where e and ℓ denote the energy and angular momentum of the particle, respectively. Employing the above expressions in $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -k$ ($k = 0$ for masseles particle

[4]Astrophysical tests may provide constraints for various modifed theories of gravity. The reader may refer to refs. [31–34].

and $k = 1$ for massive particle), we find

$$-\frac{e^2}{F(r)} + \frac{\dot{r}^2}{F(r)} + \frac{\ell^2}{r^2} = -k. \tag{12}$$

Using eq. (7), the above expression can be rearranged as

$$\frac{e^2 - k}{2} = \frac{1}{2}\dot{r}^2 + V_{\mathit{eff}}, \tag{13}$$

where the effective potential V_{eff} is given by

$$V_{\mathit{eff}} = -k\frac{G_N M}{r} + \frac{\ell^2}{2r^2} - \frac{G_N M \ell^2}{r^3} - \alpha\frac{4G_N^3 M^3}{L_*^2 r}\left(k + \frac{\ell^2}{r^2}\right). \tag{14}$$

Astrophysical tests of the EUP metric. – In this section, we focus on gravitational tests of EUP metric. Comparing our results with observations and experiments, we find bounds for the fundamental length scale L_*.

Gravitational redshift. Let us first consider the gravitational redshift of electromagnetic signal. If the electromagnetic signal travels from point A to point B in a gravitational field, then gravitational redshift is defined by [57]

$$\frac{\nu_B}{\nu_A} = \sqrt{\frac{F(r_A)}{F(r_B)}}. \tag{15}$$

For EUP metric in eq. (7), the above expression is given by

$$\frac{\nu_B}{\nu_A} = \sqrt{\frac{1 - \frac{2G_N M}{r_A}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)}{1 - \frac{2G_N M}{r_B}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)}}. \tag{16}$$

Expanding eq. (15), the frequency shift is given by

$$\begin{aligned}\frac{\Delta\nu}{\nu_A} = \frac{G_N M(r_A - r_B)}{r_A r_B}\Bigg[1 &+ \frac{G_N M(3r_A + r_B)}{2r_A r_B} \\ &+ \frac{4\alpha G_N^2 M^2}{L_*^2}\left(1 + \frac{G_N(3r_A + r_B)}{r_A r_B}\right)\Bigg],\end{aligned} \tag{17}$$

where $\Delta\nu = \nu_B - \nu_A$.

In order to get a bound for L_*, we refer to the Pound-Snider experiment [58] which was carried out in a tower with height $h = 22.86$ m. The relative deviation of frequency is

$$\frac{\frac{\Delta\nu}{\nu_A} - \left(\frac{\Delta\nu}{\nu_A}\right)^{GR}}{\left(\frac{\Delta\nu}{\nu_A}\right)^{GR}} < 0.01. \tag{18}$$

Using eq. (17) in eq. (18) yields

$$\begin{aligned}\frac{\alpha}{L_*^2} < \frac{c^4}{4G_N^2 M^2}&\left(\frac{1}{100} - \frac{G_N M(3r_A + r_B)}{2r_A r_B c^2}\right) \\ &\times\left(1 + \frac{G_N M(3r_A + r_B)}{c^2 r_A r_B}\right)^{-1},\end{aligned} \tag{19}$$

where $M = M_\oplus = 5.972 \times 10^{24}\,\mathrm{kg}$, $R_A = R_\oplus = 6378\,\mathrm{km}$, and $R_B = R_\oplus + h$. The lower bound of L_* is given by

$$9 \times 10^{-2}\,\mathrm{m} \lesssim L_*. \tag{20}$$

Geodetic precession. Let us consider a gyroscope rotating in an orbit around a spherical massive body. General relativity predicts that the spin direction of gyroscope changes. This phenomenon is called geodetic precession. A gyroscope with a spin four-vector $\boldsymbol{s}$ is characterized by [59]

$$\frac{\mathrm{d}s^\alpha}{\mathrm{d}\tau} + \Gamma^\alpha_{\mu\nu} s^\mu u^\nu = 0, \tag{21}$$

where $\Gamma^\alpha_{\mu\nu}$ is the Christoffel symbol. We call eq. (21) gyroscope equation. It determines the components of spin vector. The spin four-vector $\boldsymbol{s}$ and velocity four-vector $\boldsymbol{u}$ satisfy the following conditions:

$$\boldsymbol{s}\cdot\boldsymbol{u} = g_{\mu\nu}s^\mu u^\nu = 0, \qquad \boldsymbol{s}\cdot\boldsymbol{s} = g_{\mu\nu}s^\mu s^\nu = s_*^2, \tag{22}$$

where s_* is the magnitude of spin. Choosing equatorial plane ($\theta = \pi/2$) and circular orbit ($\dot{r} = 0 = \dot{\theta}$) obviously simplifies the problem. The components of the velocity four-vector are given by

$$\boldsymbol{u} = u^t(1, 0, 0, \Omega), \tag{23}$$

where $\Omega = \mathrm{d}\phi/\mathrm{d}t$ is the orbital angular velocity. Since $\dot{r}$ vanishes for the stable circular orbits, eq. (13) yields

$$\frac{e^2 - 1}{2} = V_{\mathit{eff}}, \tag{24}$$

and circular orbit radius R is found from

$$\frac{\mathrm{d}V_{\mathit{eff}}}{\mathrm{d}r} = 0. \tag{25}$$

From eqs. (24) and (25), one gets

$$\begin{aligned}e^2 &= \left[1 - \frac{2G_N M}{R}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\right]^2 \\ &\quad\times\left[1 - \frac{3G_N M}{R}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\right]^{-1},\end{aligned} \tag{26}$$

$$\begin{aligned}\ell^2 &= G_N M R\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\Bigg[1 - \frac{3G_N M}{R} \\ &\quad\times\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\Bigg]^{-1},\end{aligned} \tag{27}$$

$$\Omega = \frac{\mathrm{d}\phi}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{F(R)}{R^2}\frac{\ell}{e} = \sqrt{\frac{G_N M}{R^3}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)}. \tag{28}$$

Now, let us begin to solve the gyroscope equations. We suppose that $\boldsymbol{s}$ is radially directed at the beginning, *i.e.*, only $s^r(0) \neq 0$. From the orthogonality condition in eq. (22), the relation between components s^t and s^ϕ is given by

$$s^t = \Omega R^2\left[1 - \frac{2G_N M}{R}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\right]^{-1} s^\phi. \tag{29}$$

From eqs. (23) and (29), the gyroscope equations are given by

$$\frac{\mathrm{d}s^r}{\mathrm{d}\tau}+\Omega\left[3G_NM\left(1+\frac{4\alpha G_N^2M^2}{L_*^2}\right)-R\right]s^\phi u^t=0, \quad (30)$$

$$\frac{\mathrm{d}s^\theta}{\mathrm{d}\tau}=0, \quad (31)$$

$$\frac{\mathrm{d}s^\phi}{\mathrm{d}\tau}+\frac{\Omega}{R}s^r u^t=0. \quad (32)$$

It is clearly seen that s^θ remains zero due to $s^\theta(0)=0$. Since $u^t=\mathrm{d}t/\mathrm{d}\tau$, eqs. (30) and (32) can be rearranged as

$$\frac{\mathrm{d}s^r}{\mathrm{d}t}+\left[3G_NM\left(1+\frac{4\alpha G_N^2M^2}{L_*^2}\right)-R\right]\Omega s^\phi=0, \quad (33)$$

$$\frac{\mathrm{d}s^\phi}{\mathrm{d}t}+\frac{\Omega}{R}s^r=0, \quad (34)$$

respectively. Substituting eq. (34) into eq. (33) leads to a second-order differential equation,

$$\frac{\mathrm{d}^2s^\phi}{\mathrm{d}t^2}+\tilde{\Omega}^2s^\phi=0, \quad (35)$$

where $\tilde{\Omega}$ is defined by

$$\tilde{\Omega}=\sqrt{1-\frac{3G_NM}{R}\left(1+\frac{4\alpha G_N^2M^2}{L_*^2}\right)}\Omega. \quad (36)$$

One can solve eqs. (33) and (35) which give the results

$$s^r = s_*\sqrt{1-\frac{2G_NM}{R}\left(1+\frac{4\alpha G_N^2M^2}{L_*^2}\right)}\cos\left(\tilde{\Omega}t\right), \quad (37)$$

$$s^\phi = -s_*\frac{\Omega}{\tilde{\Omega}R}\sqrt{1-\frac{2G_NM}{R}\left(1+\frac{4\alpha G_N^2M^2}{L_*^2}\right)}\times\sin(\tilde{\Omega}t), \quad (38)$$

where we employ the conditions $\boldsymbol{s}\cdot\boldsymbol{s}=s_*^2$ and $s^t(0)=s^\phi(0)=0$.

The spin initially starts along a unit vector $\boldsymbol{e}_{\hat{r}}$. After one complete rotation in a time $P=2\pi/\Omega$, the change of spin direction is given by

$$\left[\frac{\boldsymbol{s}}{s_*}\cdot\boldsymbol{e}_{\hat{r}}\right]_{t=P}=\cos\left(\frac{2\pi\tilde{\Omega}}{\Omega}\right). \quad (39)$$

Therefore, the geodetic precession angle is given by

$$\Delta\Phi_{geodetic}=2\pi-2\pi\sqrt{1-\frac{3G_NM}{R}\left(1+\frac{4\alpha G_N^2M^2}{L_*^2}\right)}, \quad (40)$$

which can approximately be written as

$$\Delta\Phi_{geodetic}\approx\Delta\Phi_{GR}\left(1+\frac{4\alpha G_N^2M^2}{c^4L_*^2}\right), \quad (41)$$

where $\Delta\Phi_{GR}=\frac{3\pi G_NM}{Rc^2}$ is predicted by general relativity.

In order to get a bound for L_*, we refer to measurements of Gravity Probe B (GPB) [60], which was a satellite in a orbit around the Earth. Considering GPB was located at 642 km altitude and had 97.65 min orbital period, the general relativity predicts $\Delta\Phi_{GR}=6606.1$ mas/year. The measurement of GPB is given by

$$\Delta\Phi_{geodetic}=(6601.8\pm18.3)\,\text{mas/year}, \quad (42)$$

which gives 6620.1 mas/year and 6583.5 mas/year. Since later value imposes $\alpha=-1$, we consider the maximum value, *i.e.*, 6620.1 mas/year. Therefore, we found

$$2\times10^{-1}\,\text{m}\lesssim L_*. \quad (43)$$

Up to now, we have considered Earth-based experiments to constrain L_*. In the rest of the paper, we consider gravitational tests for Solar system and beyond.

Shapiro time delay. If an electromagnetic signal travels in a gravitational field, the travel time of signal takes longer than the travel time of the same signal in flat spacetime. This effect is called Shapiro time delay [61]. In this section, we follow the arguments of ref. [57].

Let us consider that the electromagnetic signal travels from a point A to point B in the Solar system. Without loss of generality, we again consider the equatorial plane, *i.e.*, $\theta=\pi/2$. Employing

$$\frac{\mathrm{d}r}{\mathrm{d}\lambda}=\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\lambda}=\frac{\mathrm{d}r}{\mathrm{d}t}\frac{e}{F(r)}, \quad (44)$$

Equation (12) can be rearranged as

$$\frac{e^2}{F(r)^3}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2+\frac{\ell^2}{r^2}-\frac{e^2}{F(r)}=0, \quad (45)$$

for massless particles. For $r=r_O$ (the closest distance to the Sun), one gets

$$\ell^2=\frac{er_O^2}{F(r_O)}. \quad (46)$$

Employing eq. (46) in eq. (45), we find

$$\mathrm{d}t=\pm\frac{\mathrm{d}r}{\sqrt{F(r)^2\left(1-\frac{F(r)r_O^2}{F(r_O)r^2}\right)}}. \quad (47)$$

Expanding in r_S/r and r_S/r_O, eq. (47) can ben given in the integral form as follows:

$$t=\int\frac{\mathrm{d}r}{\sqrt{F(r)^2\left(1-\frac{F(r)r_O^2}{F(r_O)r^2}\right)}}\approx\int\frac{r\mathrm{d}r}{\sqrt{r^2-r_O^2}}+\int\left(1+\frac{r_S^2}{L_*^2}\alpha\right)\times\left(\frac{r^2r_S}{(r^2-r_O^2)^{3/2}}+\frac{rr_Or_S}{2(r^2-r_O^2)^{3/2}}-\frac{3r_O^2r_S}{2(r^2-r_O^2)^{3/2}}\right)\mathrm{d}r. \quad (48)$$

So, we find the travel times from point A to point O and point O to point B,

$$t_{AO}=\sqrt{r_A^2-r_O^2}+\left(1+\frac{r_S^2}{L_*^2}\alpha\right)\times\left(\frac{r_S}{2}\sqrt{\frac{r_A-r_O}{r_A+r_O}}+r_S\ln\left(\frac{r_A+\sqrt{r_A^2-r_O^2}}{r_O}\right)\right), \quad (49)$$

$$t_{BO} = \sqrt{r_B^2 - r_O^2} + \left(1 + \frac{r_S^2}{L_*^2}\alpha\right) \times \left(\frac{r_S}{2}\sqrt{\frac{r_B - r_O}{r_B + r_O}} + r_S \ln\left(\frac{r_B + \sqrt{r_B^2 - r_O^2}}{r_O}\right)\right), \quad (50)$$

respectively. The total travel time of signal is given by

$$t_{tot} = 2(t_{AO} + t_{BO}). \quad (51)$$

For flat spacetime, it is given by

$$\tilde{t}_{tot} = 2\left(\sqrt{r_A^2 - r_O^2} + \sqrt{r_B^2 - r_O^2}\right). \quad (52)$$

Considering $r_O \ll r_A, r_B$, the time delay is given by

$$\delta t = t_{tot} - \tilde{t}_{tot} = 4G_N M\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right) \times \left(1 + \ln\left(\frac{4r_A r_B}{r_O^2}\right)\right). \quad (53)$$

In order to get a bound on L_*, we compare eq. (53) with the time delay which is defined in parameterized post-Newtonian (PPN) formalism [62],

$$\delta t_{PPN} = 4G_N M\left(1 + \left(\frac{1+\gamma}{2}\right)\ln\left(\frac{4r_A r_B}{r_O^2}\right)\right), \quad (54)$$

where γ is a dimensionless PPN parameter. We refer to measurements of Cassini spacecraft [63]. The constraint on γ is $|\gamma - 1| < 2.3\times 10^{-5}$. Comparing eqs. (53) with (54), we get

$$\frac{8\alpha G_N^2 M^2}{c^4 L_*^2}\left(1 + \frac{1}{\ln\left(\frac{4r_A r_B}{r_O^2}\right)}\right) = |\gamma - 1| < 2.3\times 10^{-5}. \quad (55)$$

Finally, taking $r_A = 1\text{AB}$, $r_B = 8.46\text{AB}$ and $r_O = 1.6R_\odot$, one gets

$$9\times 10^5\,\text{m} \lesssim L_*. \quad (56)$$

Precession of Mercury and S2 star's orbits. Now let us turn our attention to the perihelion shift of Mercury and precession of S2's orbit. In this section, we follow the arguments of ref. [64]. For a massive particle ($k = 1$), eq. (12) can be rearranged as

$$\dot{r} = \pm\sqrt{e^2 - F(r)\left(1 + \frac{\ell^2}{r^2}\right)}. \quad (57)$$

Dividing eq. (11) by eq. (57), we have

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = \pm\frac{\ell}{r^2}\left[e^2 - F(r)\left(1 + \frac{\ell^2}{r^2}\right)\right]^{-1/2}. \quad (58)$$

From eq. (58), one may write the orbital precession as

$$\psi_{prec} = 2\int_{r_-}^{r_+}\frac{\ell}{r^2}\left[e^2 - F(r)\left(1 + \frac{\ell^2}{r^2}\right)\right]^{-1/2}\mathrm{d}r - 2\pi, \quad (59)$$

where r_+ and r_- are the maximum and minimum points, respectively. Since $\mathrm{d}r/\mathrm{d}\phi$ vanishes for $r = r_\pm$, eq. (58) gives

$$\frac{1}{r_\pm^2} + \frac{1}{\ell^2} = \frac{e^2}{\ell^2 F(r_\pm)}. \quad (60)$$

Solving these equations yields

$$e^2 = \frac{F(r_+)F(r_-)(r_+^2 - r_-^2)}{r_+^2 F(r_-) - r_-^2 F(r_+)}, \quad (61)$$

$$\ell^2 = \frac{r_+^2 r_-^2 (F(r_-) - F(r_+))}{r_-^2 F(r_+) - r_+^2 F(r_-)}. \quad (62)$$

Substituting eqs. (61) and (62) into the integral in eq. (59), we have

$$\psi_{prec} = 2\int_{r_-}^{r_+}\zeta^{-1/2}\frac{\mathrm{d}r}{\sqrt{F(r)}r^2} - 2\pi, \quad (63)$$

where ζ is defined by

$$\zeta = \frac{r_-^2\left(\frac{1}{F(r)} - \frac{1}{F(r_-)}\right) - r_+^2\left(\frac{1}{F(r)} - \frac{1}{F(r_+)}\right)}{r_+^2 r_-^2\left(\frac{1}{F(r_+)} - \frac{1}{F(r_-)}\right)} - \frac{1}{r^2} = C\left(\frac{1}{r_-} - \frac{1}{r}\right)\left(\frac{1}{r} - \frac{1}{r_+}\right). \quad (64)$$

Since ζ vanishes for $r = r_\pm$, it can be expressed with the second line in the above equation and the constant C can be obtained in the limit $r \to \infty$. It is given by

$$C = \frac{r_-^2 F(r_+)(F(r_-) - 1) - r_+^2 F(r_-)(F(r_+) - 1)}{r_+ r_-(F(r_+) - F(r_-))} = 1 - 2G_N M\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\left(\frac{1}{r_-} + \frac{1}{r_+}\right), \quad (65)$$

or we can approximately write

$$C^{-1/2} \approx 1 + G_N M\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\left(\frac{1}{r_-} + \frac{1}{r_+}\right). \quad (66)$$

Therefore, total precession is given by

$$\psi_{prec} = 2C^{-1/2}\int_{r_-}^{r_+}\left(\frac{1}{r_-} - \frac{1}{r}\right)^{-1/2} \times \left(\frac{1}{r} - \frac{1}{r_+}\right)^{-1/2}\frac{\mathrm{d}r}{r^2\sqrt{F(r)}} - 2\pi. \quad (67)$$

This integral can be solved by choosing a suitable change of variable. So, we introduce

$$\frac{1}{r} = \frac{1}{2}\left(\frac{1}{r_+} + \frac{1}{r_-}\right) + \frac{1}{2}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)\sin\rho. \quad (68)$$

For eq. (68), the integral in eq. (67) is given by

$$\psi_{prec} = 2\left[1 + G_N M\left(\frac{1}{r_-} + \frac{1}{r_+}\right)\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\right] \times \int_{-\pi/2}^{\pi/2} 1 + \frac{G_N M}{2}\left[\left(\frac{1}{r_-} + \frac{1}{r_+}\right) + \left(\frac{1}{r_+} - \frac{1}{r_-}\right)\sin\rho\right] \times \left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\mathrm{d}\rho - 2\pi. \qquad (69)$$

Finally, total precession is

$$\psi_{prec} = 3\pi G_N M\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right)\left(\frac{1}{r_+} + \frac{1}{r_-}\right), \qquad (70)$$

or

$$\psi_{prec} = \frac{6\pi G_N M}{L}\left(1 + \frac{4\alpha G_N^2 M^2}{L_*^2}\right), \qquad (71)$$

where we use the semilatus rectum L which is defined by

$$\frac{1}{L} = \frac{1}{2}\left(\frac{1}{r_+} + \frac{1}{r_-}\right). \qquad (72)$$

In order to find a bound on L_*, we consider total precession in PPN formalism, which is given by [62]

$$\psi_{prec}^{PPN} = \frac{6\pi G_N M}{L}\left(1 + \frac{2\gamma - \widetilde{\beta} - 1}{3}\right), \qquad (73)$$

where $\widetilde{\beta}$ and γ are Eddington parameters. For the perihelion shift of Mercury, the constraint on PPN parameters provided by the Messenger spacecraft [65] is given by $|2\gamma - \widetilde{\beta} - 1| < 7.8 \times 10^{-5}$. Therefore, we obtain

$$\frac{12 G_N^2 M^2}{c^4 L_*^2} = |2\gamma - \widetilde{\beta} - 1| < 7.8 \times 10^{-5}, \qquad (74)$$

which approximately gives

$$5.8 \times 10^5\,\mathrm{m} \lesssim L_*. \qquad (75)$$

On the other hand, the S2 star orbiting around Sagittarius A^* gives a laboratory to test general relativity in the strong gravitational field. In our case, it can provide a much larger lower bound for L_*. Recently, the GRAVITY Collaboration [66] measured the precession of S2's orbit $(2 + 2\gamma - \widetilde{\beta})/3 = 1.10 \pm 0.19$ which gives 1.29 and 0.91. Since $\alpha = -1$ for minimum value 0.91, we only consider maximum value 1.29. So, we get

$$\frac{4 G_N^2 M^2}{c^4 L_*^2} < 0.29. \qquad (76)$$

Taking $M = 4.25 \times 10^6 M_\odot$, the lower bound on L_* is given by

$$4 \times 10^{10}\,\mathrm{m} \lesssim L_*. \qquad (77)$$

Discussions and conclusions. – EUP takes into account position uncertainty correction to standard uncertainty principle, and makes quantum effects available at the large distance scale. In this paper, we investigated the observational constraints for the EUP metric. We studied gravitational redshift, geodetic precession, Shapiro time delay, perihelion shift of Mercury and orbit precession of S2 star. Using the results of Solar system and S2 star orbiting around Sgr A^*, we obtained the lower bounds of new fundamental length scale L_*. In table 1, we summarized the lower bounds of L_* from various observations.

Table 1: Lower bounds of new fundamental length scale L_*.

Test	L_*
Light deflection [28]	9.1×10^5 m
Strong lensing (Sgr A*) [28]	2×10^{10} m
Strong lensing (M87) [28]	3×10^{13} m
Gravitational redshift	9×10^{-2} m
Geodetic precession	2×10^{-1} m
Shapiro time delay	9×10^5 m
Perihelion shift of Mercury's orbit	5.8×10^5 m
Precession of S2's orbit	4×10^{10} m

As can be seen in table 1, the bounds from Earth-based experiments such as gravitational redshift and geodetic precession are the smallest bounds, 10^{-2}–10^{-1} m. Solar scale observations give much bigger bounds, 10^5–10^6 m. Beyond the Solar system, the bound 10^{10} m from the precession of S2 star's orbit is the biggest bound in this work. Comparing our bounds with ref. [28], the lower bound 10^{13} m from strong gravitational lensing is the biggest bound for the supermassive black hole in M87.

Before finishing the paper, we give some comments on the nature of L_*. One may ask whether L_* is universal just like its counterpart Planck length L_{Pl} or depends on a particular gravitational system. Although L_* does not have a well-defined value, one may expect that L_* has one value. In order to affect the physics of supermassive black holes, the value of L_* must be sufficiently large in this case ($L_* \sim 10^{10}$ m or beyond). In the second case, one may consider L_* depending on the mass of a particular gravitational system. In this case, L_* varies between 10^{-2} and 10^{13} m according to this work and ref. [28]. However, the second case may not be favourable, because it is well known that the Solar system tests are not sensitive tools to set precise bounds on the large scale structures [67].

The observational constraints for EUP may open a new window to understand the quantum features at large distance scale. Since the new fundamental length scale L_* may play a key role in the properties of supermassive black hole, more research is needed in the future.

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