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Transverse proton gluon anisotropy points behind the Standard Model

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Abstract – The present work focuses on transverse gluon densities in the proton and derives exemplar distributions showing azimuthal anisotropies. Such anomalies relative to the Standard Model may be visible in scattering experiments involving protons. I describe baryons as mass eigenstates of a Hamiltonian structure on an intrinsic $U(3)$ configuration space. This has yielded the neutral flavour baryon spectrum and given a rather accurate value for the neutron mass 939.9(5) MeV from first principles. Quark and gluon fields are shaped by the momentum form of the intrinsic wave function. This has led to parton distribution functions for the u and d valence quarks for the proton and to a proton spin structure function agreeing with experiments over four orders of magnitude in the parton momentum fraction.

Introduction. – The Standard Model covers a vast field of experimental results in the quantum phenomenology of particle physics within a few standard deviations from expectations [1]. Nevertheless there must be a more fundamental theory behind it. In search of such a background, the present work suggests to look for anisotropies in transverse gluon densities in the proton interior. This is a field of current interest with analyses of charged particle multiplicities in scattering experiments [2–4] and theoretical predictions of shear stress transverse distributions from form factors based on QCD [5] and chiral quark soliton models [6]. Confer [7] for a future prospect and [8] for an exemplar model assuming specific internal proton structure. Also hot spot models look promising for data analysis [9]. The present work predicts anisotropies in the transverse plane of gluon densities in the proton interior. If confirmed this will be a sign of an intrinsic proton structure and would support the idea of going behind the Standard Model to better understand its origin. The main difference lies in the configuration spaces. The Standard Model starts out with quantum field configurations in an infinite spacetime whereas intrinsic quantum mechanics starts from a compact configuration space.

The Standard Model has no explanation of its fermion mass spectrum, its quark and lepton mixing matrices, its electroweak mixing angle, its Higgs particle mass —just to mention the most obvious shortcomings. Why has Nature chosen the three gauge groups, $SU(3)$, $SU(2)$ and $U(1)$? How does it confine quarks and gluons? And where do the quantum fields come from?

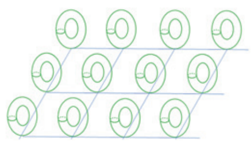

Fig. 1: The intrinsic baryon configuration space $U(3)$ is compact with a toroidal structure, indicated as $2D$ tori. It can be excited at any spacetime point in scattering experiments by kinematical generators from laboratory space, indicated as the floor tiling. Figure from [10] inspired by Maldacena [11].

Replies from work relating to the present are interpretations like these: When elementary particles undergo scattering experiments they excite intrinsic degrees of freedom, cf. fig. 1. The generators for these excitations are nine kinematic operators in laboratory space, namely three momentum operators, three angular momentum operators and three Laplace-Runge-Lenz operators. These nine operators generate an intrinsic configuration space $U(3)$ for baryons which are strongly interacting particles. The configurations subsequently undergo spontaneous symmetry breaking where the gauge groups of the electroweak sector

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come out as a subspace $U(2)$ that by the Higgs mechanism factorises into $SU(2)\times U(1)$ [12,13]. This idea is supported by a wide list of results like concrete calculations of the baryon mass spectrum [14], the Higgs mass [15,16], the Cabibbo and Weinberg angles [17], predictions of Higgs to gauge boson couplings [18] and suggestions for the origin of the u and d quark masses [14]. Local gauge transformations mimic translations in the intrinsic coordinate fields. Quarks and gluons are confined per construction as the configuration space is compact. The fields are generated by the momentum form of the intrinsic wavefunction acting on the configuration space generators, the nine kinematic generators from laboratory space. This has led to u and d valence quark parton distributions and a spin structure function for the proton [14].

The present work focuses on gluons in the proton and is structured in three steps. The first step is to present a Hamiltonian for baryon mass eigenstates. From that we can later get quark and gluon fields for the Lagrangian of quantum chromodynamics, QCD. Hamiltonians are well suited for mass spectra calculations whereas Lagrangians are natural for quantum field scattering [19].

The second step is to explain how quark and gluon fields derive from the intrinsic baryon states and to show that these fields transform under $SU(3)$ gauge group transformations as they should. The final step is to map the intrinsic structure onto the laboratory space to describe the interior distribution of quarks and gluons when scattering on the proton. The reader not familiar with coordinate forms on differentiable manifolds may refer to [20] and appendix E ("Vector fields, derivations and forms on smooth manifolds") in [21], and may skim the third section for a first reading.

Intrinsic baryon mass. – We shall use the following Hamiltonian equation[1] for baryon mass eigenstates $mc^2 = \mathcal{E}$ [24]:

$$\frac{\hbar c}{a}\left[-\frac{1}{2}\Delta + \frac{1}{2}\mathrm{Tr}\ \chi^2\right]\Psi(u) = \mathcal{E}\Psi(u), \quad u = e^{i\chi} \in U(3). \tag{1}$$

The configuration variable u has no physical dimension. So, the configuration space is *not* a version of string theory with, *e.g.*, six compactified spatial dimensions out of nine. On the contrary we shall make do all through with three spatial dimensions. We map laboratory space coordinates x_j into the three toroidal angles θ_j by a length scale a

$$\theta_j = x_j/a, \qquad j = 1,2,3. \tag{2}$$

Here $e^{i\theta_j}$ are the three eigenvalues of u. The θ_j's are dynamical variables conjugate to toroidal generators iT_j

$$iT_j = \frac{\partial}{\partial\theta_j}, \qquad [iT_j, \theta_i] = \delta_{ij}, \qquad p_j = \frac{\hbar}{a}T_j, \tag{3}$$

where δ_{ij} is the Kronecker delta. In order to use the quantisation inherent in (3) on all of the compact configuration space $U(3)$ we need to generalise the commutation relation to a global expression using left invariant coordinate fields $\partial_j = uiT_j$ and corresponding coordinate forms $\mathrm{d}\theta_j$. Thus the commutation relations of first quantisation generalises to the conjugacy of coordinate fields and coordinate forms

$$[iT_j, \theta_i] = \delta_{ij} \rightarrow \ \mathrm{d}\theta_i(\partial_j) = \delta_{ij}. \tag{4}$$

I interpret the three toroidal degrees of freedom as colour dimensions. Spin and hypercharge generators are contained in the Laplacian which can be expressed in a polar decomposition [25]

$$\Delta = \sum_{j=1}^{3} \frac{1}{J^2}\frac{\partial}{\partial\theta_j}J^2\frac{\partial}{\partial\theta_j} - \sum_{\substack{1=i<j,\\ k\neq i,j}}^{3} \frac{(S_k^2 + M_k^2)/\hbar^2}{8\sin^2\frac{1}{2}(\theta_i - \theta_j)}. \tag{5}$$

Here the van de Monde determinant [26] (the "Jacobian") is antisymmetric in the three colour degrees of freedom

$$J = \prod_{1=i<j}^{3} 2\sin\left(\frac{1}{2}(\theta_i - \theta_j)\right). \tag{6}$$

The off-toroidal generators S_k, M_k in coordinate representation and matrix representation [27] with λ_k being off-diagonal Gell-Mann matrices are, *e.g.*,

$$\begin{aligned} S_1 &= a\theta_2 p_3 - a\theta_3 p_2 = \hbar\lambda_7, \\ M_3/\hbar &= \theta_1\theta_2 + \frac{a^2}{\hbar^2}p_1 p_2 = \lambda_1. \end{aligned} \tag{7}$$

We can now express the configuration generator χ in (1) in an operational form

$$\begin{aligned} \chi &= (a\theta_j p_j + \alpha_j S_j + \beta_j M_j)/\hbar, \\ p_j &= \frac{\hbar}{a}T_j, \qquad \theta_j, \alpha_j, \beta_j \in \mathbb{R}, \end{aligned} \tag{8}$$

where the relation to kinematical operators from laboratory space becomes explicit in the coordinate representations of p_j, S_j, M_j in (3) and (7).

The S_j's equate angular momentum operators and the M_j's mix spin and flavour. Their commutation relations are

$$[M_i, M_j] = [S_i, S_j] = -i\hbar\varepsilon_{ijk}S_k. \tag{9}$$

They yield the quantum numbers $m2$ for the positive definite $\mathbf{M}^2$ [24]

$$m2 = \frac{4}{3}\left(n + \frac{3}{2}\right)^2 - s(s+1) - 3 - \frac{1}{3}y^2 - 4i_3^2, \tag{10}$$

where the intrinsic spin eigenvalues are the well-known half odd-integers $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$, following from the commutation relations [28], $n \geq 1$ is a positive integer, i_3 is the isospin three component and y is the hypercharge. Note the minus sign for the spin commutators in (9) in analogy

[1]The Hamiltonian is a radical reinterpretation of a Kogut-Susskind Hamiltonian [22] from lattice gauge theory with Manton's action [23] used now as intrinsic potential. Note that we distinguish between roman u (quark flavour) and italic u (baryonic unitary configuration variable).

with body fixed coordinates for intrinsic spin in nuclear physics [29]. One may interpret the $U(3)$ configuration variable u as a generalised spin variable.

The Laplacian (5) matches the potential $\frac{1}{2}\mathrm{Tr}\ \chi^2$ in (1) which only depends on the three toroidal angles because the trace is invariant under equivalence transformations $u' = v^{-1}uv$, in particular those that diagonalise u. Equation (1) can be solved quite accurately and has the neutral charge ground-state eigenvalue [14],

$$\mathrm{E} \equiv \mathcal{E}/(\hbar c/a) = 4.382(2), \tag{11}$$

which yields the neutron mass

$$m_n c^2 = \mathrm{E}\frac{\hbar c}{a}. \tag{12}$$

One might choose to fit the length scale a to match the experimental neutron mass and from the fit get the rest of the baryon spectrum as the tower of eigenvalues of (1). There is, however, a particular choice of a with more profound implications, namely [24],

$$\pi a = r_e, \tag{13}$$

where r_e is the classical electron radius defined by

$$m_e c^2 = \frac{e^2}{4\pi\varepsilon_0 r_e} = \alpha\frac{\hbar c}{r_e}. \tag{14}$$

Iterating in the fine structure coupling to $\alpha^{-1}(m_n) = 133.6$ leads to

$$m_n c^2 = \mathrm{E}\frac{\pi}{\alpha(m_n)} m_e c^2 = 939.9(5)\,\mathrm{MeV}. \tag{15}$$

This result is found from first principles directly from the intrinsic baryon configuration and agrees with the experimental value $m_n c^2 = 939.56542052(54)\,\mathrm{MeV}$ [1] within uncertainties. The theoretical accuracy is limited by the accuracy with which the fine structure coupling can be found at the neutronic energy scale and it is limited by the size of the set of functions on which the neutronic wavefunction can be expanded in the solution of (1). The theoretical uncertainty is comparable to Standard Model calculations of the neutron to proton mass difference by Borsanyi *et al.* in June 2014 [30] but from a radically different point of view. The present method was published [24] a year prior to the Borsanyi *et al.* result with a calculation of the relative neutron to proton mass difference publically available from 2011 [21]. The advantage of calculations based on (1) is the insight one gains in what may lie behind the Standard Model and the tremendously reduced needs in computing power. The result (15) is based on approximately two thousand base functions in the expansion set but takes less than two hours of diagonalisation time. This efficiency is due to the fact that —provided one uses a well-suited set of functions on which to expand the solution— the matrix elements of the Hamiltonian in (1) can all be solved analytically. Solving these integrals, on the other hand, has taken some years of consideration to "stay analytical". But once they are solved and the results programmed into your favourite matrix handling programme, it takes only those couple of hours to diagonalise the Hamiltonian —and you have the whole spectrum of neutral charge, neutral flavour baryons of appropiate isospin and hypercharge with the single choice of the length scale a in (1) motivated by (13). A thorough presentation of the method can be found in [14].

Quark and gluon fields. – The length scale set in (13) implies a projection from the intrinsic torus angles on tori of radius a to the classical electron radius in laboratory space[2]. The projection implied from the intrinsic configuration space to laboratory space acquires a formal setting by use of the momentum form of the wave function.

To solve (1) we introduce a measure-scaled wave function $\Phi = J\Psi$ with a uniform probability interpretation on the actual configuration space $U(3)$ rather than on the angular space $(\theta_j, \alpha_j, \beta_j)$. This wave function can be factorised to suit the polar decomposition of the Laplacian. Thus we have in total

$$\Phi(u) = R(\theta_1, \theta_2, \theta_3)\Upsilon(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3). \tag{16}$$

Colour quark (conjugate) fields ψ_j are generated by the momentum form of the measure-scaled toroidal wavefunction R by its acting on the toroidal generators

$$\psi_j(u) = \frac{-i\hbar}{a}\mathrm{d}R(uiT_j). \tag{17}$$

The momentum form is also called the exterior derivative and it becomes operational by the following derivation along a one-parameter curve at u ([20] and appendix E in [21]),

$$\mathrm{d}R_u(iT_j) = \frac{\mathrm{d}}{\mathrm{d}t}R(u\exp(tiT_j))\mid_{t=0}. \tag{18}$$

Note that the momentum form can also be identified as an exterior derivative along the direction given by the vector field induced by the generator iT_j,

$$\mathrm{d}R_u(iT_j) = \partial_j|_u[R] = u\partial_j|_e[R] = uiT_j[R], \tag{19}$$

where $e = \mathrm{diag}(1,1,1)$ is the *origo* of $U(3)$. From the left invariance of the coordinate fields in (19) we readily see that ψ_j transforms under $SU(3)$ gauge transformations as it should. We namely have

$$\psi_j(u) = \frac{-i\hbar}{a}\partial_j|_u[R] = \frac{-i\hbar}{a}u\partial_j|_e[R] = u\psi_j(e). \tag{20}$$

We define gluon field components in analogy with (17) by using the eight Gell-Mann matrices λ_k in the generators for directional derivatives on the full measure-scaled

[2] Heuristically one may understand the appearance of the electron on the scene as a "peel off" from the neutron inherent in its decay to a proton, an electron and an anti-electron neutrino [24].

intrinsic wave function Φ

$$G^{(k)}(u)=\frac{-i\hbar}{a}\mathrm{d}\Phi_u(\partial_k),\quad \partial_k=ui\lambda_k/2,\quad k=1,2,\ldots,8. \tag{21}$$

We define, with a suggestive coupling constant g_s,

$$\mathcal{G}(u)\equiv g_s\sum_{k=1}^{8}G^{(k)}(u)\ it_k,\quad t_k=\lambda_k/2. \tag{22}$$

This construct is expressed in a fixed $SU(3)$ representation base $\{it_k\}$ for QCD in laboratory space [1]. One can think of it as a read-off of *gluonic* degrees of freedom from the full measure-scaled intrinsic wave function Φ by the momentum form in (21) at a (random) intrinsic configuration variable. The read-off is related to a similar read-off at the origo e in the intrinsic configuration space. We find

$$\begin{aligned}\mathcal{G}(u)=ig_s\sum_k G^{(k)}(u)t_k=ig_s\sum_k d\Phi_u(\partial_k)t_k=\\ ig_s\sum_k\partial_k|_u[\Phi]t_k=ig_s\sum_k u\partial_k|_e[\Phi]t_k=\\ ig_su\sum_k d\Phi_e(it_k)t_k=ig_su\sum_k G^{(k)}(e)t_k=\\ u\mathcal{G}(e).\end{aligned} \tag{23}$$

Now, t_k belongs to an adjoint representation of $SU(3)$ in laboratory space and under a global gauge transformation with $g(x)=g$ in (20) it shifts to the basis

$$t_k\to gt_kg^{-1}. \tag{24}$$

Likewise when representing u in laboratory space as a 3×3 matrix U it transforms as

$$U\to gUg^{-1}. \tag{25}$$

Thus

$$\begin{aligned}&G^{(k)}(u)t_k=UG^{(k)}(e)t_k\to\\ &gUg^{-1}\ G^{(k)}(e)\ gt_kg^{-1}=\\ &g\ UG^{(k)}(e)t_k\ g^{-1}=g\ G^{(k)}(u)t_k\ g^{-1}.\end{aligned} \tag{26}$$

We therefore have for global gauge transformations

$$\mathcal{G}(u)\to g\ U\mathcal{G}(e)\ g^{-1}=g\ \mathcal{G}(u)\ g^{-1}. \tag{27}$$

We want to admit the gauge transformation to be local in spacetime. For that we need to generate gluon field components in spacetime. We therefore consider the time dependent edition of our baryon configurations (1)

$$\frac{\hbar c}{a}\left[-\frac{1}{2}\Delta+\frac{1}{2}\mathrm{Tr}\ \chi^2\right]\Psi(\tilde u)=i\hbar\frac{\partial}{\partial t}\Psi(\tilde u),\quad \tilde u=e^{i\theta_0T_0}u, \tag{28}$$

with $\theta_0=ct/a$ and

$$\frac{\partial}{\partial\theta_0}=\frac{a}{c}\frac{\partial}{\partial t}\equiv iT_0. \tag{29}$$

We can generalize this to suit the left invariance used in (19) such that we have a left invariant coordinate field also for θ_0

$$\partial_0|_{\tilde u}=\frac{\partial}{\partial\theta}(\tilde u\exp(\theta iT_0))|_{\theta=0}=\tilde u iT_0. \tag{30}$$

The coordinate field ∂_0 is conjugate to the corresponding coordinate form $\mathrm{d}\theta_0$. This implies a measure-scaled wave function (θ_0 is not a dynamical variable)

$$J\Psi(\tilde u)\equiv\tilde\Phi(\tilde u)=e^{-i\mathcal{E}t/\hbar}\Phi(u). \tag{31}$$

The spacetime basis in laboratory space is

$$e_\mu(x)=\frac{\partial}{\partial x^\mu}\equiv\partial_\mu,\quad x=(x^0,x^1,x^2,x^3),\quad \mu=0,1,2,3. \tag{32}$$

Here $x^0=ct$, $(x^1,x^2,x^3)=\boldsymbol{x}$ and we follow the metric sign convention $(1,-1,-1,-1)$ of Aitchison and Hey [31].

We expand the gluon field components

$$\tilde G^{(k)}(\tilde u)=\frac{-i\hbar}{a}\mathrm{d}\tilde\Phi_{\tilde u}(\partial_k) \tag{33}$$

on the spacetime coordinate fields ∂_μ to get components with spacetime indices μ

$$\tilde G^{(k)}_\mu(x)=\mathrm{d}\tilde G^{(k)}_{\tilde u}(\partial_\mu). \tag{34}$$

Accordingly

$$\mathcal{G}_\mu(x)=g_s\sum_{k=1}^{8}\tilde G^{(k)}_\mu(x)it_k. \tag{35}$$

Gluon fields and gauge fixing. In the Standard Model the gluon fields are interpreted as gauge fields with transformation properties given by [12]

$$A'_\mu=g(x)A_\mu g(x)^{-1}+(\partial_\mu g(x))g(x)^{-1}, \tag{36}$$

and used in gauge covariant derivatives [32]

$$D_\mu=\partial_\mu-A_\mu \tag{37}$$

to have invariance of the kinematic term $\frac{1}{2}(D_\mu\psi)^2$ in a Lagrangian density under local gauge transformations

$$\psi\to\psi'=g(x)\psi,\quad g(x)\in SU(3). \tag{38}$$

The gauge fields fulfilling (36) must be constrained before quantisation, *e.g.*, by using gauge fixing to operate in axial gauges $A_3^{(k)}=0$ where neither $A_0^{(k)}$ are independent variables [32].

In the intrinsic model (1), the gluon field spacetime components are defined from (35) based on (21). This means that their degrees of freedom are already quantised from (3) underlying (1) and constrained by the structure of the baryonic wavefunctions determined as solutions of (1).

The structure implied by the intrinsic wave function should be detectable for instance in scattering experiments through observations of gluon densities in a plane transverse to the beam axis as described in the fourth section.

Gauges at neighbouring points. We identify $u=g(x)\in SU(3)$ where $g(x)$ is an adjoint representation of

$SU(3)$ in laboratory space. We further exploit the fact that with the *introtangling* in (2) from laboratory space to the smooth Lie group intrinsic space, the representation of the local gauge transformation $g(x)$ in laboratory space —before second quantisation— remains a smooth function of the laboratory spacetime location x. For infinitesimal neighbours we thus suggest

$$u = g(x), \quad u' = g(x + \mathrm{d}x) \approx g(x) + \partial_\mu g(x)\mathrm{d}x^\mu. \quad (39)$$

In analogy with (27) we consider a local gauge transformation at $x + \mathrm{d}x$

$$g(x + \mathrm{d}x)\mathcal{G}_\mu(x + \mathrm{d}x)g(x + \mathrm{d}x)^{-1}. \quad (40)$$

To first order in $\mathrm{d}x$ we have

$$\begin{aligned} g(x + \mathrm{d}x)^{-1} &= (g(x) + \partial_\mu g(x)\mathrm{d}x^\mu)^{-1} \\ &= g(x)^{-1} + \partial_\mu g(x)^{-1}\mathrm{d}x^\mu. \end{aligned} \quad (41)$$

Since $g(x)g(x)^{-1} = \mathbf{1}$, we have [12]

$$0 = \partial_\mu(g(x)g(x)^{-1}) = (\partial_\mu g(x))g(x)^{-1} + g(x)\partial_\mu g(x)^{-1}. \quad (42)$$

This yields the useful expression

$$g(x)\partial_\mu g(x)^{-1} = -(\partial_\mu g(x))g(x)^{-1}. \quad (43)$$

After some algebra, using (41) and (43), we get for (40)

$$\begin{aligned} &g(x + \mathrm{d}x)\mathcal{G}_\mu(x + \mathrm{d}x)g(x + \mathrm{d}x)^{-1} = \\ &g(x)\mathcal{G}_\mu(x)g(x)^{-1} + \left[\partial_\nu(g(x)\mathcal{G}_\mu(x)g(x)^{-1})\right]\mathrm{d}x^\nu + O(\mathrm{d}x^2). \end{aligned} \quad (44)$$

Note that (44) is equivalent to (36) but not identical. Equation (36) relates A' and A at x and actually *defines* A' whereas (44) relates $\mathcal{G}_\mu(x + \mathrm{d}x)$ to $\mathcal{G}_\mu(x)$ at the neighbouring x. It does *not* constrain $\mathcal{G}_\mu(x + \mathrm{d}x)$ relative to $\mathcal{G}_\mu(x)$ more than what is implied by their common origin in the wave function Φ as of (21) and by the variation of the gauge transformation from $g(x)$ to $g(x + \mathrm{d}x)$.

The gluon densities in the proton can be tested experimentally. Figures 2 and 3 indicate the idea.

Gluon densities in a protonic state. – The gluons represent off-toroidal (off-diagonal) degrees of freedom and as such contribute to shear stresses in the energy-momentum tensor of the proton interior [33]. The off-diagonal generators E_{ij} are related to the off-toroidal gluonic generators S_k, M_k from (7) thus

$$iE_{ij} = \frac{1}{2}(-S_k + iM_k)/\hbar, \quad i, j, k \text{ cyclic} \quad (45)$$

e.g.,

$$E_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

From these we generate off-diagonal gluon fields in the E_{ij} basis as [33]

$$\pi_{ij}(u) = \frac{-i\hbar}{a}\mathrm{d}\Phi_u(iE_{ij}). \quad (46)$$

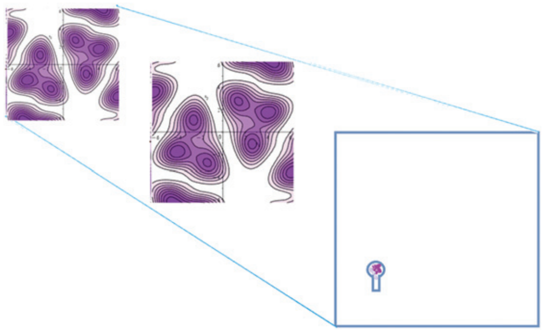

Fig. 2: Transverse gluon densities (squared) in the proton as seen in toroidal angular space. The density is probed through a "keyhole" by scattering at a specific impact parameter. The contracted proton pattern approaches from behind the "door". The pattern shows a contraction along the beam axis —perpendicular to the door— for a specific gluonic density (48) with a randomly oriented intrinsic space.

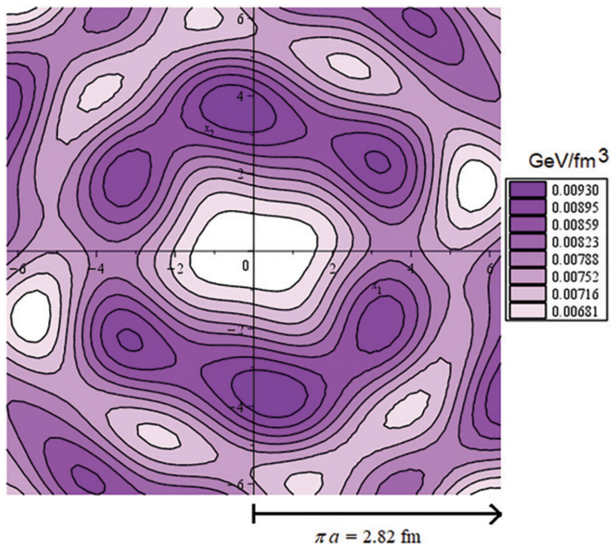

Fig. 3: Transverse gluon densities for the target proton averaged over 60 random orientations of the intrinsic toroidal angular space. The transverse scale is commented in the text following eq. (54). The figure shows the structure of $\mid \mathcal{T}_{ij} \mid$ from (48) contracted along the beam axis according to (51). The transverse size is comparable to [5] whereas [6] has only half the size. Both [5] and [6] display isotropy in the azimuthal angle. The azimuthal anisotropy in the present prediction reflects the toroidal structure of the intrinsic configuration space according to our model (1).

Note that the toroidal wavefunction is used in (17) for generating colour quark fields, but it also contributes to the six off-diagonal gluon densities. Consider namely the squared energy-momentum tensor density [33]

$$\mathcal{T}_{ij}^2 = \int \mathrm{d}\alpha^3\mathrm{d}\beta^3\left(\frac{-i\hbar c}{a}\mathrm{d}\Phi_u(iE_{ij})\right)^2, \quad i, j = 1, 2, 3. \quad (47)$$

After the integration in (47) over the six off-toroidal degrees of freedom $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ we have [33]

$$\mathcal{T}_{ij}^2 = \frac{\hbar^2 c^2}{a^2 V^2} \frac{(\mathbf{S}^2 + \mathbf{M}^2)/(3\hbar)^2}{4} R^2(\theta_1, \theta_2, \theta_3), \quad i \neq j, \tag{48}$$

where V is the volume over which the wave function is normalised in laboratory space. For neutral flavour N-baryons $\mathbf{S}^2 + \mathbf{M}^2 = 4\hbar^2$ corresponding to isospin three-component $i_3 = \pm\frac{1}{2}$, hypercharge $y = 1$ and $n = 1$ in (9). The diagonal terms in the energy-momentum tensor are [33]

$$\mathcal{T}_{jj}^2 = \frac{\hbar^2 c^2}{a^2 V^2} \left(\frac{\partial R}{\partial \theta_j}\right)^2, \quad \mathcal{T}_{00} = \frac{\mathcal{E}}{V} R^2. \tag{49}$$

They should represent colour quark densities[3] contributing to the interior pressure in the proton in the laboratory system and to the total mass density respectively [33].

Figure 3 shows an average of 60 transverse gluon densities as calculated from an exemplar toroidal wave function for the proton [24] (normalisation over $\theta_j \in [-2\pi, 2\pi]$ gives $N^2 = 96\pi^3$)

$$R = \frac{1}{N} \begin{vmatrix} 1 & 1 & 1 \\ \sin\frac{1}{2}\theta_1 & \sin\frac{1}{2}\theta_2 & \sin\frac{1}{2}\theta_3 \\ \cos\theta_1 & \cos\theta_2 & \cos\theta_3 \end{vmatrix}. \tag{50}$$

The average density for $I = 60$ contractions in fig. 3 is constructed by random rotations D through angles α, β, γ in the space of toroidal angles as

$$R_\perp^2 = \frac{1}{I} \sum_{i=1}^{I} \int_{-2\pi}^{2\pi} \mathrm{d}\left(\frac{z}{a}\right) \; R^2 \left(D^{-1}(\alpha_i, \beta_i, \gamma_i) \begin{pmatrix} x/a \\ y/a \\ z/a \end{pmatrix} \right),$$
$$\alpha_i, \beta_i, \gamma_i = \text{random} \in [-\pi, \pi]$$
$$D(\alpha, \beta, \gamma) =$$
$$\begin{pmatrix} \cos\gamma & \sin\gamma & \\ -\sin\gamma & \cos\gamma & \\ & & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & & \sin\beta \\ & 1 & \\ -\sin\beta & & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & & \\ & \cos\alpha & \sin\alpha \\ & -\sin\alpha & \cos\alpha \end{pmatrix}. \tag{51}$$

The volume V over which to distribute the gluon densities underlying fig. 3 needs some comments. The wave function R has period doubling on $U(3)$ and is therefore normalised over the angular box $[-2\pi, 2\pi]^3$ with $N^2 = 96\pi^3$. But we want to distribute over the intrinsic torus using our length scale a from (2). The angular box is mapped by the exponential function into the maximal torus of $U(3)$

$$U_0 = \begin{pmatrix} e^{i\theta_1} & & \\ & e^{i\theta_2} & \\ & & e^{i\theta_3} \end{pmatrix}, \quad \theta_j \in [-\pi, \pi]. \tag{52}$$

[3] The two diagonal gluon degrees of freedom also contribute here.

With J^2 from (6) included in the volume element [26] and $x_j = a\theta_j$ from (2) we find the torus volume in physical units

$$V_0 = a^3 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} J^2 \, \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}\theta_3 = a^3 \cdot 96\pi^3. \tag{53}$$

The density scale from (48) in fig. 3 is then

$$\frac{\hbar c/a}{3V} = 0.102\,\mathrm{GeV/fm}^3, \qquad \text{for} \quad V = V_0/N^2 = a^3. \tag{54}$$

I have inserted the scale $\pi a = 2.82\,\mathrm{fm}$ in fig. 3 to get a sense of the order of magnitude corresponding to the relation (2) between spatial and angular variables. It should be stressed, though, that fig. 3 represents the angular space and not directly laboratory space size for which (54) implies a box normalisation on $V = a^3$.

Given the average over random orientations in (51), one might had expected[4] azimuthal isotropy in fig. 3. However, we observe azimuthal anisotropy which reflects the toroidal structure of the configuration space used for the proton mass eigenstate in (1) when projected on laboratory space in scattering experiments.

Figure 3 for proton gluon densities seems apt for comparing with scattering data analyses based on hot spot models [9]. In the energy-dependent edition of the hot spot model the profile factor $T_p(\vec{b})$ for particle p at impact parameter $\vec{b}$ is written as a sum over a varying number N_{hs} of hot spots increasing with energy and the hot spots located at different positions $\vec{b}_i$ inside the particle

$$T_p(\vec{b}) = \frac{1}{N_{hs}} \sum_{i=1}^{N_{hs}} T_{hs}(\vec{b} - \vec{b}_i). \tag{55}$$

Bendova *et al.* [9] use Gaussian profiles of sizes $B_{hs} = 0.8\,\mathrm{GeV}^{-2}$

$$T_{hs}(\vec{b}) = \frac{1}{2\pi B_{hs}} \exp\left(\frac{-\vec{b}^2}{2B_{hs}}\right) \sim r_{hs} \approx 0.18\,\mathrm{fm}. \tag{56}$$

It would be interesting to see extensions from "one-dimensional" energy-dependent charged particle multiplicities to transverse proton profiles analysed with the hot spot model (55) starting out, *e.g.*, with $N_{hs} = 6$ as implied in fig. 3.

Conclusion. – We have derived transverse gluon densities with azimuthal anisotropy for the proton interior. The densities result from a conception of the proton mass eigenstate as originating in a compact intrinsic configuration space, the Lie group $U(3)$ which carries a toroidal structure. Experimental analyses of scattering data for instance along the lines of hot spot models may disclose an intrinsic structural origin behind the Standard Model.

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* * *

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