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Interaction and control of optical localized structures

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Abstract. – We show the possibility of controlling the formation, the interaction and erasure of localised structures in a passive non-linear resonator. The localised structures , which have the character of stable 2D spatial solitons, are highly degenerate solutions which develop in the presence of modulational instabilities and their position in the transverse profile of the field can be decided by suitably tailoring the driving external field. The results indicate the possibility of realizing optical memories based on localised structures arrays.

The possibility of realizing two-dimensional spatial soliton-like structures in the transverse field profile of broad-area non-linear systems, contained in optical cavities, has recently attracted much interest for the realization of novel optical information encoding and processing procedures. While in the purely dispersive and Hamiltonian configuration, described by the non-linear Schrödinger equation, the 2D spatial solitons are unstable *vs.* diffractive catastrophic collapse, in absorptive-dispersive driven systems with saturation, evidence of stable soliton-like structures has been found [1]-[4]. We focus on the case where these phenomena arise in the presence of a modulational instability (as in [2],[3],[4]), and have the character of *Localized Structures* (LS) [5] where portions of the field-modulated profile coexist in the transverse plane with a homogeneous background.

The possibility of controlling the excitation process of one or more independent LS having the character of 2D solitons, the investigation of LS interaction and the procedures to erase a single LS without affecting the others, is the subject of this letter.

Precisely, we consider an optical ring cavity with plane mirrors, containing a homogeneously broadened collection of two-level atoms with transition frequency ω_a. We call ω_c the frequency of the cavity resonance closest to ω_a and assume that the free spectral range is large enough to ensure single longitudinal mode operation. The cavity is driven by a homogeneous input field at frequency $\omega_0 = \omega_\mathrm{a}$. The system is properly described by the Maxwell-Bloch equations in the paraxial and mean-field approximations which have been introduced in [6]. In the limit of fast atomic relaxation, after adiabatic elimination of the atomic variables, the model reduces to the field equation alone:

$$\frac{\partial}{\partial \tau} F(x,y,\tau) = -\left\{ \left[(1+i\theta) - i\nabla_\perp^2 \right] F(x,y,\tau) - Y + \frac{2CF(x,y,\tau)}{1+|F(x,y,\tau)|^2} \right\} , \tag{1}$$

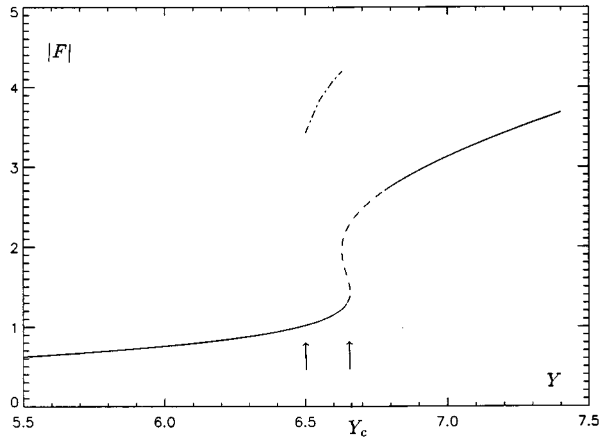

Fig. 1. – Steady-state curve for $C = 5.4, \theta = -1$. The broken line plots the part where the modulational instability develops and Y_c indicates its threshold . The dash-dotted line plots the modulated branch. Arrows indicate the region of coexistence between the homogeneous and the hexagonal solution. The instabilty region broadens with increasing C, with larger coexistence domains where LS are stable.

where F, Y are the normalized slowly varying amplitudes of the intracavity and driving field respectively; Y is real for definiteness. The time is defined as $\tau = \kappa t$, where κ is the cavity linewidth, the transverse coordinates x and y have been normalized to $\sqrt{\lambda L/4\pi T}$, where λ is the wavelength, L is the cavity length and T is the transmissivity of the cavity mirrors. C is the bistability parameter [7], the Laplacian operator $\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ describes diffraction and θ is defined as $\theta = (\omega_c - \omega_0)/\kappa$.

Numerical simulations have been performed by using a split-step code with at least a 128×128 square spatial grid and periodic boundary conditions. The homogeneous solutions at steady state are readily obtained as

$$Y^2 = I\left\{\left[1 + \frac{2C}{1+I}\right]^2 + \theta^2\right\} \qquad (I = |F_{st}|^2)\,. \tag{2}$$

Proper choice of the parameters leads to the well-known S-shaped curve, where modulational instabilities leading to pattern formation [6] have been studied in [8]: The instability leads to formation of a stationary regular hexagonal lattice above threshold, and there are wide parameter domains where this instability is such that there is an interval of the input intensity values where the heagonal branch coexists with a stable homogeneous solution. Figure 1 shows the stable and unstable portions of the steady-state curve and the interval of coexistence.

Previous works dealing with LS formation always used a suitable initial field profile to excite a localised structure [1]-[3],[9] with a homogeneous input field profile Y. However, to encode information via the LS, we must use an external control channel which is provided by the input field Y. Precisely, we introduce a suitable modulation of Y in order to create a localised structure in the transverse plane, *i.e.* we superimpose a Gaussian profile (control beam) to the homogeneous field (holding beam) Y_{hom} for a certain time t_{inj}, the maximum of the Gaussian being located at the transverse point (x_0, y_0) where we want the LS to be excited; after that time, the homogeneous field profile is restored. In practice, this is obtained by shining a narrow laser pulse in the optical cavity. Hence, the total input field Y appearing in eq.(1) has now the form

$$Y(x,y,\tau) = \begin{cases} Y_{hom} + \xi \exp\left[-\frac{1}{\beta^2}[(x-x_0)^2 + (y-y_0)^2]\right]\exp[i\varphi], & \tau \le \tau_{inj}, \\ Y_{hom}, & \tau > \tau_{inj}, \end{cases} \tag{3}$$

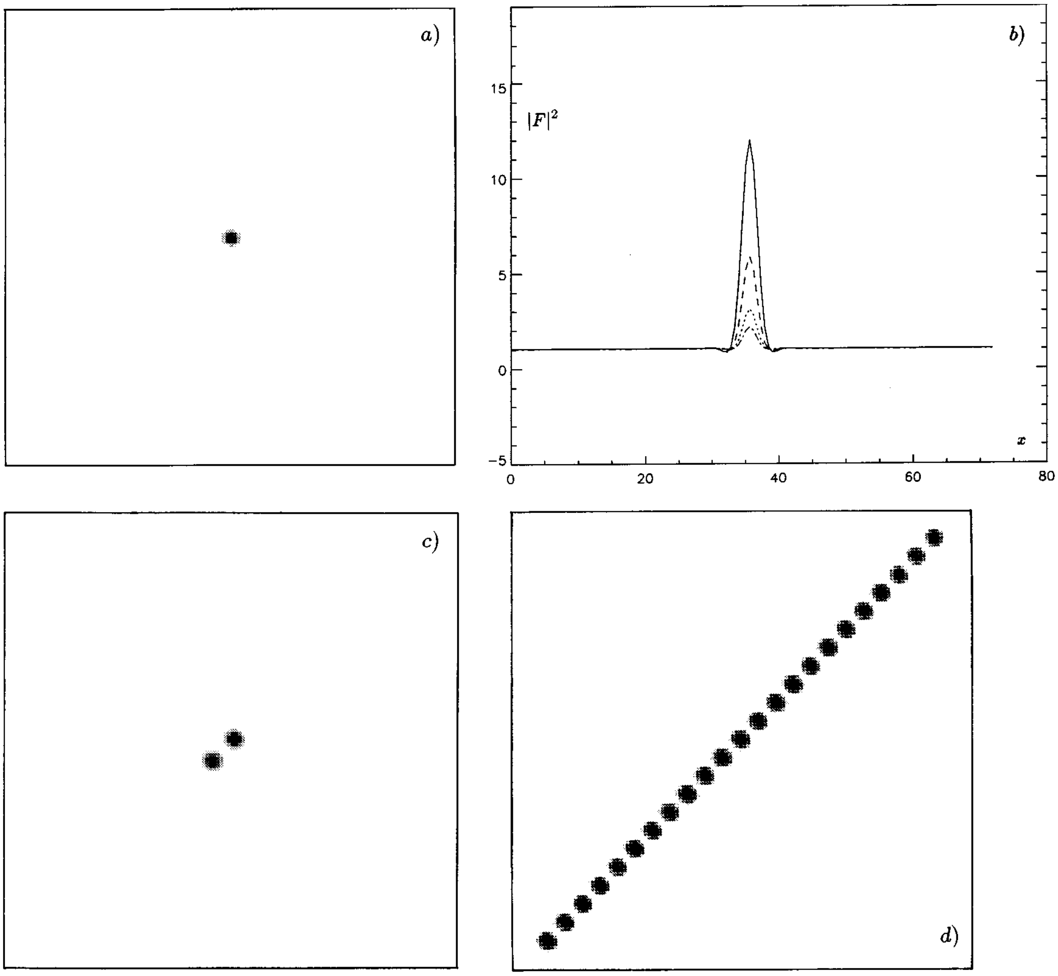

Fig. 2. – *a*), *c*), *d*): Output intensity transverse configuration with (respectively) one, two and 21 LS. Parameters are as in fig. 1, with $Y_{\text{hom}} = 6.5$. Black corresponds to high intensity, white to low. *b*) The build-up of the LS is plotted by different line styles (full, broken, dotted) corresponding to different times. The full line indicates the LS at regime. $\xi = 0.5$, $\beta = 1.6$, $\tau_{\text{inj}} = 120$.

with Y_{hom} in the interval of coexistence between the homogeneous solution and the hexagonal branch.

Under suitable choice of the Gaussian parameters which will be discussed below, the output field profile builds up a high-intensity peak at (x_0, y_0) (see fig. 2 *a*), *b*)). The characteristics of the resulting localised structure showed that its intensity, phase and radius match very well those of the hexagonal lattice's peaks; checks performed by integrating the full model of [6] in the limit of atomic relaxation rates much larger than κ, but without adiabatic elimination of the atomic variables, showed that identical LS can be excited with no discrepancy in their intensity or radius within the grid granularity.

By subsequently shining bright spots at different positions in the $Y(x, y)$ profile, several independent LS can be "written" at desired locations. Figures 2 *c*), *d*) show examples with 2 and 21 LS. We can heuristically regard the amplitude modulation as a local increase of the input intensity which *locally* brings the system above the bifurcation threshold, so that in a region of the plane around (x_0, y_0) the system can realize a modulated solution in the form of a number of LS. Thus, it is necessary (but not sufficient) that ξ is equal to or larger than

TABLE I. – *The values of the parameters are* $C = 5.4, \theta = -1$ (*hence* $Y_c = 6.75$), $Y_{hom} = 6.5, \tau_{inj} = 6$. *The table shows, for each value of* β, *the minimum value of* ξ *for developing a* LS. *For* $\beta > 4$ *more than one* LS *is created.*

β	0.8	1.6	2.4	4.0
$\xi + Y_{hom}$	7.5	6.9	6.8	6.75

$Y_c - Y_{hom}$ (Y_c is defined in fig. 1). Furthermore, τ_{inj} must be sufficient (depending on ξ and β) to develop in the output field a peak sufficiently high to locally bring the system state far from the attraction basin of the homogeneous solution and close to the modulated solution; hence the larger ξ, the shorter is the minimum injection time required to build a stable LS. The width β measures the region where the system is made able to develop a stable modulated solution. If β is too small, this process will be impossible or very difficult; if β is much larger than the typical wavevector of the hexagonal lattice, one obtains, for example, several LS or a whole portion of the hexagonal lattice. The dependence on β and ξ is illustrated in table I.

These results hold when the relative phase φ vanishes (see eq. (3)); there exist, however, large intervals of φ where the LS is easily obtained, as shown in table II. Hence some control of the phase is necessary, but it is not critical.

The question arises now about the maximum density of independent LS that can be achieved, and what happens when the LS interact. It is thus essential to evaluate the minimum distance from an existing localised structure at which a second one can be created, without interacting with the former.

A naive idea about the interaction of two (or more) LS can be grasped if one thinks that the LS is a single intensity peak, originally belonging to a hexagonal lattice. It is thus intuitive to assume that the two subelements of the lattice will interact when their distance is on the order of or smaller than the lattice transverse wavelength. Though this is a simplification, the idea is fundamentally substantiated by our results.

We base our picture on extended sets of simulations in which the second LS is excited at locations progressively closer to the first one, by using the same Gaussian form (3) with $\varphi = 0$. Two critical distances can be defined: we shall indicate them by D_{cr} and d_{cr} (with $D_{cr} > d_{cr}$). Let d be the distance between the existing localised structure and the transverse plane point where the Gaussian in the input field profile is centred, in order to excite a second structure; then we find the following:
1) if $d > D_{cr}$, a second independent LS is created;
2) if $D_{cr} > d > d_{cr}$, the two LS interact; the result of the interaction is that they move apart, until they reach a distance D_{cr};
3) if $d < d_{cr}$, the existing localised structure may be erased and in this case a fully homogeneous profile is left.

It turns out that D_{cr} is slightly larger (5–10%) than the hexagonal-lattice wavelength λ: for example, for $C = 5.4$, $\theta = -1$, $Y_{hom} = 6.5$, one finds $D_{cr} = 6.7$, while $\lambda = 2\pi$. The value of d_{cr} for the same set of parameters is 4.7, and in general, d_{cr} is somewhat smaller (70–90%) than λ.

TABLE II. – *The minimum* τ_{inj} *necessary to develop a* LS *is given as a function of the phase* φ. *Where such time diverges, the table reports* no. *Parameters are* $C = 5.4$, $\theta = -1$, $Y_{hom} = 6.5$.

φ	0	$\pi/10$	$2\pi/10$	$3\pi/10$	4–$16\pi/10$	$17\pi/10$	$18\pi/10$	$19\pi/10$
min τ_{inj}	4.8	4.8	5.2	7.1	no	12.0	6.4	5.2

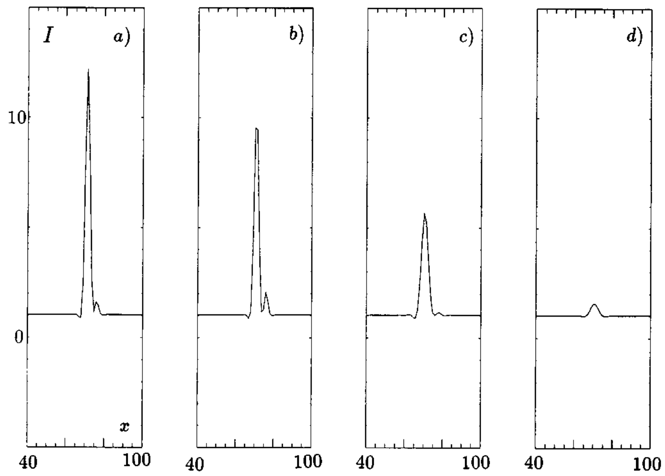

Fig. 3. – By injecting a narrow Gaussian beam closer to a LS than d_{cr}, one can erase it. *a*) and *b*) the new and the old soliton interacting; *c*) and *d*) annihilation of the LS and restoring of the homogeneous profile (the input Gaussian has been switched off by now). $C = 5.4, \theta = -1, Y_{\mathrm{hom}} = 6.5, \xi = 0.5$, $\beta = 0.8, \tau_{\mathrm{inj}} = 80$.

It is remarkable that for $d < d_{\mathrm{cr}}$ the existing LS may be cancelled, this process allows to *turn off* any soliton without influencing the others, locally restoring the homogeneous profile, as shown in fig. 3. However, this erasing procedure does not seem useful in practice, because it works only under a rather critical control of the parameters τ_{inj} and β. A much better procedure is identified by taking $\varphi = \pi$ in eq. (3), so that the inhomogeneous contribution is subtracted from the homogeneous background. When this "dark spot" is exactly superimposed to the existing LS, it locally creates the conditions to erase it as is shown in fig. 4. Here the value of ξ equals Y_{hom}, but also smaller values of ξ are enough to obtain the result. The relevant figure in this case is the minimum value of τ_{inj} which causes the erasure of the LS for a given value of ξ. We performed extended simulations to determine the couples $(\tau_{\mathrm{inj}}, \xi)$; an example is shown in table III.

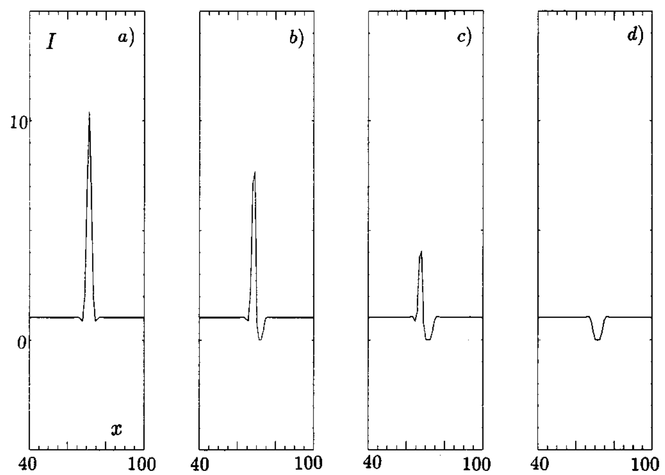

Fig. 4. – By local control of the input beam phase, the injected Gaussian has now $\varphi = \pi$. A "hole" is formed in the field profile which depletes the existing LS until it is cancelled. Parameters are as in fig. 3.

Table III. – *For each value of ξ we indicate the minimum value of τ_{inj} necessary to obtain erasure. Parameters are as in table I.*

ξ	0.5	0.4	0.3	0.2	0.1
τ_{inj}	3.6	4.2	5.4	8.6	20.0

As a rule of thumb, the product $\xi\tau_{\mathrm{inj}}$ is constant. The advantage of this procedure is its robustness relatively to the choices of β and τ_{inj}: τ_{inj} has to be larger than a minimum value, while it has no upper bound to satisfy, opposed to what happens in the case of fig. 3. When τ_{inj} is large, there is the formation of a dark spot also in the output field profile which finally returns to the homogeneous configuration. This process is robust also with respect to the Gaussian phase φ: a broad set of values for φ exists where the cancellation takes place (for the set of parameters of fig. 3, one finds cancellation for $-7\pi/10 \leq \varphi \leq 13\pi/10$).

In conclusion, we showed how recently discovered localised structures in a non-linear absorbing cavity (presumably the phenomenology is identical in the case of lasers with saturable absorbers) can be turned on and off by injecting narrow laser pulses. Basic mechanisms of structure excitation and interaction have been tailored to LS manipulation. The results have been illustrated for a specific choice of the values for C and θ, but the same picture holds over extended ranges of these parameters. Our analysis has been strictly deterministic, and in the presence of noise the LS undergo a slow random walk in the transverse plane. However, as shown by Firth and Scroggie [3] an appropriate phase modulation of the input field profile is able to pin down the position of the LS. Hence the results of [3] together with those of this work, demonstrate the possibility of realizing an array processor (which could be used, for example, as an optical memory) where the LS behave as pixels which can be turned on and off in a controlled way. For the first time in the field of Non-linear Optical patterns [10], we have shown here the possibility of operating full control of the transverse field configuration; this result follows from the fact that the LS are independent entities (when their distance is larger than D_{cr}). A key point for the future is to study the behaviour of LS when the background profile of the input field is not perfectly flat but is, *e.g.*, a shallow Gaussian, especially in the presence of phase modulation and noise.

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