A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

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To cite this article: H. Fujisaki and K. Nakagawa 1996 *EPL* **35** 493

EUROPHYSICS LETTERS 1 September 1996

Europhys. Lett., **35** (7), pp. 493-497 (1996)

Global phase structure of the closed bosonic thermal string

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(received 20 May 1996; accepted in final form 11 July 1996)

PACS. 11.10Gh – Field theories: Renormalization.
PACS. 11.25−w – Theory of fundamental strings.
PACS. 12.10−g – Unified field theories and models.

Abstract. – The global phase structure of the bosonic-thermal-string ensemble is described in proper reference to the thermal duality symmetry as well as the thermal stability of modular invariance for the dimensionally regularized, $D = 26$ closed-bosonic-thermal-string theory within the framework of the thermofield dynamics.

Building up thermal-string theories based upon the thermofield dynamics (TFD) [1] has gradually been endeavoured in leaps and bounds [2]-[10]. In previous papers of ourselves [6], [9], the physical significance of the thermal duality relation [11], [12] has been examined in relation to the infrared behaviour of the one-loop cosmological constant for closed-thermal-string theories of the critical dimension within the TFD framework. In the present communication, the TFD algorithm of the $D = 26$ closed-bosonic-thermal-string theory is commented and exemplified at any finite temperature *à la* ref. [6] and ref. [9] through the infrared one-loop self-energy amplitude of the dilaton, graviton and antisymmetric tensor particle. Physical aspects of the TFD thermal-string amplitude are then described in connection with the global phase structure of the thermal-string ensemble.

The one-loop self-energy amplitude $A(k_1;\zeta_1,\zeta_2;\beta)$ of the massless thermal tensor boson is expressed as $A(k_1;\zeta_1,\zeta_2;\beta) = A(k_1;\zeta_1,\zeta_2) + \bar{A}(k_1;\zeta_1,\zeta_2;\beta)$ at any finite temperature in the $D = 26$ closed-bosonic-thermal-string theory based upon the TFD algorithm, where $k_r^\mu; r = 1,2$ and $\zeta_r^{\mu\nu} = \zeta_r^\mu\bar{\zeta}_r^\nu;\ r = 1,2$ read external momenta and polarization tensors, respectively. The $D = 26$ zero-temperature amplitude $A(k_1;\zeta_1,\zeta_2)$ is written in the modular invariant fashion as follows [13]:

$$\begin{aligned} A(k_1;\zeta_1,\zeta_2) &= (\pi\kappa)^2(\alpha')^{-D/2}\zeta_1^{\mu\nu}\zeta_2^{\sigma\rho}\int_F \mathrm{d}^2\tau\int_P \mathrm{d}^2\nu\,\tau_2^{-D/2}\times\\ &\times\ \exp\left[\pi\tau_2\cdot\frac{D-2}{6}\right]|f(\exp[2\pi i\tau])|^{-2(D-2)}\times\\ &\times\ \left[\Big(\frac{\alpha'}{8\pi\tau_2}\Big)^2(\eta_{\mu\nu}\eta_{\sigma\rho}+\eta_{\mu\rho}\eta_{\nu\sigma})+\Big(\frac{\alpha'}{8\pi^2}\Big)^2\eta_{\mu\sigma}\eta_{\nu\rho}\Big|\frac{\pi}{\tau_2}+\frac{\partial}{\partial\nu}\Big\{\frac{\vartheta_1'(\nu-\tau|\tau)}{\vartheta_1(\nu-\tau|\tau)}\Big\}\Big|^2\right], \qquad (1)\end{aligned}$$

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where $f(w) = \prod_{n=1}^{\infty}(1-w^n)$; $w = q^2 = \exp[2\pi i\tau]$, $\eta_{\kappa\tau}$ is the space-time metric, ϑ_1 reads the Jacobi theta-function, F denotes the fundamental domain of the modular group $SL(2,Z)$ in the complex τ-plane and the integration over the complex ν-plane is restricted to cover a single parallelogrammatic region P [14]. It is almost needless to mention that the slope and intercept of the closed-string reggeon are $\alpha'/2$ and $2\alpha = (D-2)/12$, respectively, and κ reads the string coupling constant. Since the soft domain $k_1 \simeq 0$ is necessary and sufficient at any finite temperature for the dynamical mass shift of the massless thermal tensor boson, the present discussion is confined to the asymptotic behaviour of the $D = 26$ temperature-dependent amplitude $\bar{A}(k_1;\zeta_1,\zeta_2;\beta)$ at the low-energy limit $k_{10} \simeq 0$. We are then eventually led to the "proper-time" integral representation of $\bar{A}(k_1;\zeta_1,\zeta_2;\beta)$ as follows:

$$\begin{aligned}
\bar{A}(k_1;\zeta_1,\zeta_2;\beta) = {} & 4\pi\Big(\frac{\kappa}{4\pi}\Big)^2(\alpha')^{-D/2}\int_0^{\infty}\mathrm{d}\tau_2\,\tau_2^{-D/2}\exp\Big[\pi\tau_2\cdot\frac{D-2}{6}\Big]\times \\
& \times\int_{-\pi}^{\pi}\mathrm{d}\phi_1\int_{-\pi}^{\pi}\mathrm{d}\phi_2\int_0^1\frac{\mathrm{d}x_1}{x_1}\theta(x_1-\exp[-2\pi\tau_2])\times \\
& \times\sum_{l=1}^{\infty}\exp\Big[-\frac{\sigma l^2\beta^2}{2\tau_2}\Big]F(\tau_2;z_2,\bar{z}_2,z_1z_2,\bar{z}_1\bar{z}_2)+ \\
& + i\pi^2\Big(\frac{\kappa}{4\pi}\Big)^2(\alpha')^{-D/2}\int_0^{\infty}\mathrm{d}\tau_2\,\tau_2^{-D/2}\exp\Big[\pi\tau_2\cdot\frac{D-2}{6}\Big]\times \\
& \times\int_{-\pi}^{\pi}\mathrm{d}\phi_1\int_{-\pi}^{\pi}\mathrm{d}\phi_2\sum_{l=1}^{\infty}l\exp\Big[-\frac{\sigma(l+1)^2\beta^2}{2\tau_2}\Big]\theta(-|w|)\times \\
& \times\{f(\tau_2;0,0,|w|\exp[i(\phi_1+\phi_2)],|w|\exp[-i(\phi_1+\phi_2)])+ \\
& + F(\tau_2;0,0,-|w|\exp[i(\phi_1+\phi_2)],-|w|\exp[-i(\phi_1+\phi_2)])\}
\end{aligned} \qquad (2)$$

at $k_{10} \simeq 0$, where $z_r = x_r\exp[i\phi_r]$; $r = 1,2$, $|w| = \exp[-2\pi\tau_2]$ and θ is the step function. We do not go into details of the real function F but merely refer to ref. [6] and ref. [9]. As a consequence, the zero-energy thermal amplitude $\mathrm{Im}\bar{A}(k_1 \simeq 0;\zeta_1,\zeta_2;\beta)$ vanishes identically at $D = 26$ as well as at $\alpha = 1$ for any finite temperature because $|w| > 0$ as expected from gauge invariance.

All we have to do is now reduced to carrying out the regularization of the $D = 26$ thermal amplitude $A(k_1;\zeta_1,\zeta_2;\beta)$ at $k_{10} \simeq 0$. The $D = 26$ one-loop mass shift $A(k_1 \simeq 0;\zeta_1,\zeta_2;\beta)$ of the dilaton, graviton and antisymmetric tensor boson is then described at any finite temperature in the standard fashion [13] which is manifestly free of ultraviolet divergences at $\tau_2 \sim 0$ and $|\nu| \sim \infty$ for any value of β and D due to modular invariance and double periodicity. The standard integral representation thus obtained is still annoyed with infrared divergences near the endpoints $\tau_2 \sim \infty$ and $|\nu| \sim 0$, however, unless $D < 2$. The regularization of the ν integration has already been brought to realization in the modular invariant fashion [15], [16]. Moreover, the infrared divergence of the one-loop TFD self-energy amplitude at $\tau_2 \sim \infty$ can be remedied at any finite temperature through the dimensional regularization in the sense of analytic continuation which is, of course, modular invariant as well as double periodic. The dimensionally regularized, $D = 26$ one-loop dual symmetric mass shift $\hat{A}(k_1 \simeq 0;\zeta_1,\zeta_2;\beta)$ of the dilaton, graviton and antisymmetric tensor boson in then reduced to

$$\hat{A}(k_1 \simeq 0;\zeta_1,\zeta_2;\beta) = 4\pi(\pi\kappa)^2(4\pi^2)^{(D-1)/2}\zeta_1^{\mu\nu}\zeta_2^{\sigma\rho}\{\tilde{D}_{\mu\nu\sigma\rho}+\tilde{G}_{\mu\nu\sigma\rho}+\tilde{T}_{\mu\nu\sigma\rho}\}\times$$

$$\times\ \frac{\sqrt{\pi\alpha'}}{\beta}(4\pi\alpha')^{-D/2}\sum_{m,n\in Z}\int_{-\frac{1}{2}}^{\frac{1}{2}}\mathrm{d}\tau_1\exp[2\pi imn\tau_1]\Big(\frac{\beta^2}{4\pi^2\alpha'}m^2+\frac{4\pi^2\alpha'}{\beta^2}n^2-\frac{D-2}{6}\Big)^{(D-1)/2}\times$$

$$\times\ \Gamma\Big[-\frac{D-1}{2},\pi\sqrt{1-\tau_1^2}\Big(\frac{\beta^2}{4\pi^2\alpha'}m^2+\frac{4\pi^2\alpha'}{\beta^2}n^2-\frac{D-2}{6}\Big)\Big],\quad D=26\,, \qquad (3)$$

where $\tilde{D}_{\mu\nu\sigma\rho}=(\alpha'/8\pi)^2\eta_{\mu\nu}\eta_{\sigma\rho}$, $\tilde{G}_{\mu\nu\sigma\rho}=0$, $\tilde{T}_{\mu\nu\sigma\rho}=(\alpha'/8\pi)^2(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho})$. It is a matter of course that $\tilde{D}_{\mu\nu\sigma\rho}$, $\tilde{G}_{\mu\nu\sigma\rho}$ and $\tilde{T}_{\mu\nu\sigma\rho}$ describe the factors of the dilaton, graviton and antisymmetric tensor boson contribution, respectively, to the one-loop thermal amplitude $\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)$. The thermal duality symmetry $\beta\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)=\tilde{\beta}\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\tilde{\beta})$ then follows for any value of β, where $\tilde{\beta}=4\pi^2\alpha'/\beta$. In accordance, the dimensionally regularized, thermal amplitude $\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)$ yields the non-vanishing one-loop dual symmetric mass shift for the dilaton and antisymmetric tensor boson which is literally proportional at any finite temperature to the dimensionally regularized, $D=26$ one-loop dual symmetric thermal cosmological constant $\hat{\Lambda}(\beta)$ [6]. The dimensionally regularized, one-loop dual symmetric mass shift of the graviton, on the other hand, is of course guaranteed to vanish identically at any finite temperature. It will be possible to argue that these observations based upon the TFD paradigm are in full consonance with the thermal stability of renormalizability, factorizability, duality and gauge invariance, which is in turn substantiated at the soft limit $k_1\simeq 0$ as an immediate consequence of the thermal stability of both modular invariance and double periodicity.

Let us examine the singularity structure of the dimensionally regularized, $D=26$ dual symmetric thermal amplitude $\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)$. The position of the singularity $\beta_{|m|,|n|}$ is determined by solving $\beta/\tilde{\beta}\cdot m^2+\tilde{\beta}/\beta\cdot n^2-4=0$ of every allowed (m,n) in eq. (3). We then obtain infinitely many branch point singularities of the square-root type as follows: $\beta_{|m|,0}=\beta_{\mathrm{H}}/|m|$; $\tilde{\beta}_{|m|,0}=\tilde{\beta}_{\mathrm{H}}\cdot|m|$; $\beta_{1,1}=\beta_{\mathrm{H}}\cdot(\sqrt{3}+1)/2\sqrt{2}$; $\tilde{\beta}_{1,1}=\beta_{\mathrm{H}}\cdot(\sqrt{3}-1)/2\sqrt{2}$, where m is nonzero integral and β_{H} ($\tilde{\beta}_{\mathrm{H}}$) reads the inverse (dual) Hagedorn temperature. In particular, $\beta_{1,0}$ and $\tilde{\beta}_{1,0}$ form the leading branch points at $\beta_{\mathrm{H}}=4\pi\sqrt{\alpha'}$ and $\tilde{\beta}_{\mathrm{H}}=\pi\sqrt{\alpha'}$, respectively. It is of crucial importance to note that $\beta^{-1}=\beta_{\mathrm{H}}^{-1}$ ($\tilde{\beta}_{\mathrm{H}}^{-1}$) represents the lowest-temperature singularity for the physical β ($\tilde{\beta}$) channel in proper reference to the infrared behaviour of the dual symmetric thermal amplitude $\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)$. Moreover, there appears the self-dual leading branch point at $\beta_{2,0}=\tilde{\beta}_{2,0}=\beta_0=2\pi\sqrt{\alpha'}$ as an inevitable consequence of the thermal duality symmetry. In addition, $\beta_{1,1}$ and $\tilde{\beta}_{1,1}$ yield the non-leading branch points at $(\sqrt{3}+1)\pi\sqrt{2\alpha'}$ and $(\sqrt{3}-1)\pi\sqrt{2\alpha'}$, respectively. Finally, all the residual secondary branch points at $\beta_{|m|,0}$ ($\tilde{\beta}_{|m|,0}$) with $m=\pm3;\ \pm4;\ldots$ are, of course, removed onto the unphysical sheet of the physical $\tilde{\beta}$ (β) channel across the leading branch cut mentioned above. If the thermal duality symmetry had been manifestly violated for the TFD self-energy amplitude $\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)$, on the contrary, we would then have obtained infinitely many finite-temperature branch point singularities at $\beta_l=\beta_{\mathrm{H}}/l$, $l=1,2,3,\ldots$ of the square-root type above the Hagedorn temperature β_{H}^{-1} and there would eventually have appeared the essential singularity at the finite temperature $\beta=0$ in natural consonance with the breaking of the thermal duality. It is to be remembered that the existence of an inverse critical temperature β_0 in addition to the inverse (dual) Hagedorn temperature β_{H} ($\tilde{\beta}_{\mathrm{H}}$) for the TFD self-energy amplitude $\hat{A}(k_1\simeq 0;\zeta_1,\zeta_2;\beta)$ is simply and naturally inherent in the singularity structure of the dimensionally regularized, one-loop free energy amplitude $\hat{\Lambda}(\beta)$ in the $D=26$ closed-bosonic-thermal-string theory based upon the TFD calculus [6], [9]. Consequently, the present aspects of the TFD thermal-string amplitude will afford active confirmation to the interesting argument *à la* ref. [11] and ref. [17] on the global phase structure of the heterotic thermal-string ensemble.

Let us now turn our attention to the statistical ensemble of the $D = 26$ closed bosonic thermal string. The thermodynamical properties of the bosonic-thermal-string excitation can be analyzed in the same fashion as ref. [11], ref. [17] and ref. [18] through the microcanonical ensemble paradigm outside the analyticity domain of the canonical ensemble. Substantial use is made of the thermal duality relation not only for the canonical region but also for the microcanonical region. There will then exist three phases as follows [11], [17]: I) the β channel canonical phase in the range $4\pi\sqrt{\alpha'} = \beta_{\rm H} \leqq \beta < \infty$, II) the dual $\tilde{\beta}$ channel canonical phase in the range $0 < \beta \leqq \tilde{\beta}_{\rm H} = \beta_{\rm H}/4$ and III) the microcanonical phase in the range $\tilde{\beta}_{\rm H} < \beta < \beta_{\rm H}$. Moreover, there will occur an effective splitting in half of the microcanonical region as in the following [17]: III-i) the β channel microcanonical domain $\beta_{\rm H}/2 = \beta_0 \leqq \beta < \beta_{\rm H}$ and III-ii) the dual $\tilde{\beta}$ channel microcanonical domain $\tilde{\beta}_{\rm H} < \beta \leqq \tilde{\beta}_0 = \beta_0$. As a consequence, it will be possible to claim that the so-called maximum temperature of the $D = 26$ closed-bosonic-thermal-string theory is asymptotically described in the sense of the thermal duality relation as the self-dual temperature $\beta_0^{-1} = \tilde{\beta}_0^{-1} = 2 \cdot \beta_{\rm H}^{-1} = 1/2\pi\sqrt{\alpha'}$. It is parenthetically mentioned that the present observation might lead up to a novel hypothesis of the "extended" β (dual $\tilde{\beta}$) channel microcanonical phase for the range $\beta_0 \leqq \beta < \beta_{1,1}$ ($\tilde{\beta}_{1,1} < \beta \leqq \tilde{\beta}_0 = \beta_0$). Another newfangled hypothesis of the β (dual $\tilde{\beta}$) channel microcanonical phase confined to the region $\beta_{1,1} \leqq \beta < \beta_{\rm H}$ ($\tilde{\beta}_{\rm H} < \beta \leqq \tilde{\beta}_{1,1}$) might not be abandoned yet, however, in which the maximum temperature of the bosonic-thermal-string excitation will be effectually reduced at least at the one-loop level to $\beta_{1,1}^{-1}$ in replacement of β_0^{-1}. The future exploration of the thermodynamical properties of string excitations will be inevitable for the manifest materialization of the physical significance of $\beta_{1,1}$ as well as $\tilde{\beta}_{1,1}$, anyhow.

Let us next touch upon the physical significance of the τ_1 integral in the asymptotic estimation of the infrared behaviour of the TFD thermal amplitude $\hat{A}(k_1 \simeq 0; \zeta_1, \zeta_2; \beta)$. Any solution $\beta_{|m|,|n|}$ as well as $\tilde{\beta}_{|m|,|n|}$ with $mn \neq 0$ might temporarily be spurious in the sense that the τ_1 integration vanishes, unless either $m = 0$ or $n = 0$, due to the factor $\exp[2\pi i m n \tau_1]$ in eq. (3) under the tentative replacement of $\sqrt{1-\tau_1^2}$ by 1, or equivalently some appropriate positive constant, in the incomplete gamma-function Γ. Such a hypothetical, contaminative prescription for the leading-order evaluation in the infrared domain of the moduli space has thoroughly been left out of consideration in the present context, of course, because of the manifest breaking of modular invariance. Accordingly, it will be of pratical interest to emphasize that the present argument on the global phase structure of the bosonic-thermal-string ensemble is in full accordance with the fundamental properties such as modular invariance and double periodicity within the general framework of the TFD algorithm of closed-thermal-string theories of the critical dimension.

The present TFD paradigm might deserve more than ephemeral consideration in the thermodynamical investigation of the thermal-string excitation in general.

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