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Measurement of gravitomagnetism

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Abstract. – Gravitomagnetism is a fundamental, weak-field, prediction of Einstein's theory of general relativity, is generated by currents of mass and owes its name to its formal analogies with magnetism, generated by currents of electric charge. Then, according to general relativity, Earth's spin should influence the motion of its orbiting satellites. Indeed, we analysed the laser ranging observations of the orbits of the satellites LAGEOS and LAGEOS II and obtained the first direct measurement of the gravitomagnetic orbital perturbation due to the Earth's spin, known as the Lense-Thirring effect.

For centuries the origin of inertia has been discussed and debated by scientists and philosophers. Mach's ideas on the origin of inertia inspired Einstein to develop the theory of general relativity [1], [2]; Mach thought that the inertial forces arise from some interaction with the distant masses in the universe. A posteriori, general relativity did not unambiguously incorporate these ideas. However, the theory of general relativity satisfies [2] at least a kind of "weak manifestation" of Mach's ideas: the "dragging of inertial frames" by a spinning mass, also known as Lense-Thirring effect. In general relativity the concept of inertial frame has only a local meaning and a local inertial frame is "rotationally dragged" by mass-energy currents, in other words moving masses influence and change the orientation of the axes of a local inertial frame (gyroscopes). Thus, an external current of mass, such as the spinning Earth, "drags" and changes the orientation of gyroscopes. A test gyroscope precesses with respect to "an asymptotic inertial frame" with angular velocity: $\dot{\Omega} = -1/2\mathbf{H} = [-\mathbf{J} + 3(\mathbf{J} \cdot \hat{\mathbf{x}})\hat{\mathbf{x}}]/|\mathbf{x}|^3$, where **J** is the angular momentum of the central object and **H** its gravitomagnetic field (see fig. 1). This is the "rotational dragging of inertial frames", or "frame-dragging" ("dragging", as Einstein named it).

Since 1896 several experiments have been discussed and proposed to measure the rotational dragging of inertial frames by a spinning body [1]-[4]. The NASA Gravity Probe-B experiment [3] is aimed to measure with great accuracy this phenomenon on the orientation of the

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Fig. 1. – The gravitomagnetic field, \mathbf{H} , generated by the angular momentum, \mathbf{J} , of a central rotating body. In general relativity, for a localized, stationary, mass-energy distribution, in the weak-field and slow-motion limit, the three "vector" components of the metric tensor are given by: $\mathbf{h} \cong -2(\mathbf{J} \times \mathbf{x})/r^3$, where \mathbf{J} is the angular momentum of the central body and \mathbf{h} is known as the gravitomagnetic potential. The gravitomagnetic field \mathbf{H} is given by $\mathbf{H} = \nabla \times \mathbf{h}$. To characterize the gravitomagnetic field generated by the angular momentum of a body and the Lense-Thirring effect, and distinguish it from other relativistic phenomena such as the de Sitter effect —due to the motion of a gyroscope in a static gravitational field— one may give a description of the gravitomagnetic field in terms of spacetime-curvature invariants. The pseudoinvariant * $\mathbf{R} \cdot \mathbf{R}$, built from the Riemann tensor \mathbf{R} and its dual * \mathbf{R} , gives an invariant characterization of gravitomagnetism since it is non-zero in the field of a central body if and only if the body is rotating. Indeed the pseudoinvariant * $\mathbf{R} \cdot \mathbf{R}$ is proportional to the angular momentum of the central body. Thus, one may describe gravitomagnetism as that phenomenon of nature such that spacetime curvature is generated by the spin of a body [2].

axis of spin of a small orbiting gyroscope. However, the whole orbital plane of a satellite is itself a kind of enormous gyroscope dragged by the gravitomagnetic field. Indeed, in addition to the rotational dragging and precession of a test gyroscope due to the angular momentum **J** of a central object, the orbit of a test particle around a central body with angular momentum **J** has a secular rate of change of the longitude of the line of the nodes (intersection between the orbital plane of the test particle and the equatorial plane of the central object), discovered by Lense-Thirring (1918): $\dot{\Omega}^{\text{Lense-Thirring}} = 2\mathbf{J}/[a^3(1-e^2)^{3/2}]$; where *a* is the semimajor axis of the test particle, and *e* its orbital eccentricity. The proposed LAGEOS III experiment [4] should measure with great accuracy the gravitomagnetic dragging of the nodal lines of the orbits of two non-polar, passive, laser-ranged satellites with supplementary inclinations.

However, the orbit of a test particle also has a secular rate of change of the mean longitude of the orbit and of the longitude of the pericenter $\dot{\omega}$ (defining the Runge-Lenz vector): $\dot{\omega}^{\text{Lense-Thirring}} = 2J (\hat{\mathbf{J}} - 3\cos I \hat{\mathbf{l}})/[a^3(1-e^2)^{3/2}]$; where $\hat{\mathbf{l}}$ is the orbital angular momentum, unit vector, of the test particle, and I its orbital inclination (angle between the orbital plane and the equatorial plane of the central object).

Our direct measurement of the Lense-Thirring effect was obtained by laser ranging observations of the satellites LAGEOS and LAGEOS II [5]. The gravitomagnetic field has changed the point of closest approach to Earth —perigee— of the satellite LAGEOS II by about 11 meters during our period of observation of about 3.1 years.

Laser ranging to the Moon and to artificial satellites is an impressive technique to measure distances from a laser tracking station on Earth to retro-reflectors placed on the Moon, or on satellites orbiting Earth. By the use of short laser pulses ranges can be measured with accuracies of less than 1 cm from emitting lasers on Earth to retro-reflectors on a satellite, and with accuracies of about 2 cm to retro-reflectors on the Moon. The NASA-ASI (Italian Space Agency) satellite LAGEOS II is a high-altitude, small cross-sectional area-to-mass ratio, spherical, laser-ranged satellite. It is made of heavy brass and aluminum, is completely passive and covered with laser retro-reflectors. It acts as a reference target for ground-based laser tracking systems to measure —via laser ranging— crustal movements, plate motion, polar motion and Earth rotation. LAGEOS II is essentially identical to the NASA satellite LAGEOS (LAser GEOdynamics Satellite) but they have different orbital parameters. The semimajor axis of LAGEOS is $a \cong 12270$ km, the period $P \cong 3.758$ hr, the eccentricity $e \cong 0.004$, and the inclination $I \cong 109.9^{\circ}$. The semimajor axis of LAGEOS II is $a_{\rm II} \cong 12163$ km, the eccentricity $e_{\rm II} \cong 0.014$, and the inclination $I_{\rm II} \cong 52.65^{\circ}$.

Cornerstones of our analysis were: the NASA launch in 1976 of the LAGEOS satellite; the development by the NASA/Goddard Space Flight Center of the powerful program GEODYN, and of its new version GEODYN II, for satellite orbit determination, geodetic parameter estimation, tracking instrument calibration, satellite orbit prediction and other applications in geodesy; the determination of highly accurate Earth's gravity field solutions, including GEML1, GEML2, GEMT1, GEMT2, GEMT3, GEMT3S.

The other new basic elements that made our direct measurement of the Lense-Thirring effect possible were: 1) the launch in October 1992 by NASA and ASI of the laser-ranged satellite LAGEOS II; 2) the new Earth's gravity field solutions JGM-2 and JGM-3, jointly developed by NASA-Goddard and by the CSR (Center for Space Research) of the University of Texas at Austin [6]; 3) the continuous laser ranging to the satellites LAGEOS and LAGEOS II from several stations around the world, the ranging data from the best stations have a precision of a few millimeters; and 4) the use of a new method [7] to measure the gravitomagnetic field.

We analysed the orbits of the satellites LAGEOS and LAGEOS II using existing laser ranging observations, a highly accurate modeling of their orbital perturbations including the gravity field solution JGM-3, and the 1994 version of GEODYN II. All the general relativistic perturbations due to the masses of Earth and Sun, including the de Sitter or geodetic effect (today measured with accuracy of the order of 10^{-2}), were incorporated in the GEODYN equations of motion and then computer-integrated; we did not, however, include in our model the orbital perturbations due to the Earth's angular momentum, that is the Lense-Thirring, gravitomagnetic, effect to be determined. In order to measure the frame-dragging effect from our residuals we introduced a new parameter μ , which, by definition, is one in general relativity, $\mu^{\text{GR}} \equiv 1$, and zero in Newtonian theory [2].

The residuals of the orbital elements of a satellite give a measure of any perturbation that is not modeled accurately enough or that is not included in the model. The orbital elements we analysed are: the node of LAGEOS I, the node of LAGEOS II, and the perigee of LAGEOS II. The nodes of LAGEOS and LAGEOS II are both dragged by the Earth's angular momentum; according to the Lense-Thirring formula one has: $\dot{\Omega}_{\rm I}^{\rm Lense-Thirring} \cong 31$ milliarcsec/year and $\dot{\Omega}_{\rm II}^{\rm Lense-Thirring} \cong 31.5$ milliarcsec/year. The argument of pericenter (perigee in our analysis), ω , of a test particle, that is the angle on its orbital plane measuring the departure of the pericenter from the equatorial plane of the central body, also has a Lense-Thirring drag; for LAGEOS I one has: $\dot{\omega}_{\rm I}^{\rm Lense-Thirring} \cong 32$ milliarcsec/year, and for LAGEOS II: $\dot{\omega}_{\rm II}^{\rm Lense-Thirring} \cong -57$ milliarcsec/year. The nodal precessions of LAGEOS and LAGEOS II can be determined with an accuracy of the order of 1 milliarcsec/year, or less. In fact, we obtained a root mean square of the node residuals of about 2 milliarcsec for LAGEOS and of about 3 milliarcsec for LAGEOS II, over a total period of observation of about 3.1 years. Regarding the perigee, the observable quantity is $ea\dot{\omega}$, where e is the orbital eccentricity of the satellite. Thus, for LAGEOS the perigee precession $\dot{\omega}$ is an extremely difficult quantity to measure; its

orbital eccentricity is in fact about 4×10^{-3} . The orbit of LAGEOS II is more eccentric, its orbital eccentricity is about 0.014, and the Lense-Thirring drag of the perigee of LAGEOS II is almost twice as large, in magnitude, as that of LAGEOS. In fact, we obtained a root mean square of the residuals of the LAGEOS II perigee of about 35 milliarcsec over about 3.1 years, whereas the total effect of frame-dragging on the perigee, over about 3.1 years, is ≈ -176 milliarcsec.

The most critical source of error in our measurement arises from uncertainties in the Earth's even zonal harmonics and in their temporal variations. Using only the satellites orbiting today, one cannot eliminate the unmodeled orbital perturbations due to all the even zonal harmonics; in particular, unmodeled orbital effects due to the harmonics of lower order are of a size comparable to or larger than the Lense-Thirring effect. However, by analysing the JGM-3 solution with its uncertainties in the even zonal harmonic coefficients and by calculating the secular effects of these uncertainties on the orbital elements of LAGEOS and LAGEOS II, we found that the main sources of error in the determination the frame-dragging effect are concentrated in the first 2 or 3 even zonal harmonics, that is J_2 , J_4 and J_6 . To further test [8] the order of magnitude of the real errors in the estimated value of the J_{2n} coefficients, we took the difference [7] between two different gravity field solutions: JGM-3 and GEMT-3S [9]. We then found that by far the largest uncertainties, on the nodes of LAGEOS and LAGEOS II and on the perigee of LAGEOS II, arise from ΔJ_2 and ΔJ_4 , a smaller error is due to ΔJ_6 , and much smaller errors arise from the differences in the other J_{2n} coefficients. However, we have the three observable quantities: the node of LAGEOS, the node of LAGEOS II, and the perigee of LAGEOS II, and we want to determine the parameter μ , measuring the frame-dragging effect. Then, we can use these three observable quantities $\hat{\Omega}_{\rm I}$, $\hat{\Omega}_{\rm II}$ and $\dot{\omega}_{\rm II}$ to determine μ , thereby eliminating the two largest sources of error arising from the uncertainties in J_2 and J_4 . This new method leads to a value of μ unaffected by the errors due to δJ_2 and δJ_4 , by far the largest, but sensitive only to the smaller errors due to δJ_{2n} with $2n \ge 6$. As regards tidal, secular and seasonal changes in the geopotential coefficients, we stress that the main effects on the nodes and perigee of LAGEOS and LAGEOS II due to tidal and other temporal variations in the Earth's gravity field are due to changes in the first two even zonal harmonic coefficients, J_2 and J_4 . Any tidal error in J_2 and J_4 , and any error due to other unmodeled temporal variations in J_2 and J_4 , including their secular and seasonal variations, is eliminated using our combination of residuals of nodes and perigee. In particular, most of the errors due to the 18.6 year and 9.3 year tides, associated with the Moon node, are eliminated in our measurement. Thus, using three observable quantities, the two nodes and the perigee, one can solve for μ and eliminate δJ_2 and δJ_4 :

$$\delta \, \dot{\Omega}_{\rm LageosI}^{\rm Exp} + k_1 \delta \dot{\Omega}_{\rm LageosII}^{\rm Exp} + k_2 \delta \dot{\omega}_{\rm LageosII}^{\rm Exp} \cong$$

 $\cong \mu(31 + 31.5k_1 - 57k_2) \text{milliarcsec/year} + (\text{contributions from } \delta J_6, \delta J_8, \ldots), \tag{1}$

where $k_1 = 0.295$ and $k_2 = -0.35$ are obtained (in order to eliminate the δJ_2 and δJ_4 errors) from the system of the three equations for the nodal rates of LAGEOS and LAGEOS II and for the perigee rate of LAGEOS II. The best fit lines of the residuals of the nodes of LAGEOS and LAGEOS II had a slope of respectively ≈ -11 milliarcsec and ≈ 40 milliarcsec, and the best fit line of the residuals of the perigee of LAGEOS II had a slope of ≈ -188 milliarcsec. In fig. 2 we plotted the sum of the residuals of the nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II according to our formula to eliminate the δJ_2 and δJ_4 errors and after having removed a few small periodical residual signals (corresponding to the main tidal effects, to the odd zonal harmonics perturbation of the perigee and to the largest solar radiation pressure perturbation) and the small observed inclination residuals. In other words each point of fig. 2



Fig. 2. – Sum of the residuals of the nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II from November 1992 to December 1995, using the method described in the text. On the vertical axis we plotted (node residuals of LAGEOS) + 0.295 (node residuals of LAGEOS II) – -0.35 (perigee residuals of LAGEOS II). In our analysis we included polar motion from VLBI (IERS), Earth's solid and ocean tides and Earth's gravity field, GM and spherical harmonics up to order 50, from the JGM-3 gravity field model, solar, lunar and planetary perturbations and non-gravitational perturbations including solar radiation pressure, Earth's radiation pressure, anisotropic thermal radiation effects, and atmospheric drag. For each 15-day arc we estimated all station coordinates except the latitude of Goddard Space Flight Center and the latitude/longitude of Hawaii (maintained fixed), the spacecrafts' initial conditions (initial positions and velocities), the satellites' reflectivities and 15-day along-track accelerations. The best fit line shown through these combined residuals has a slope of about 66 milliarcsec/year (the total integrated effect corresponds to about 12 meters at the LAGEOS altitude), that is $\mu \cong 1.1$ (whereas $\mu \equiv 1$ in general relativity), and the corresponding root mean square of the residuals is about 13 milliarcsec. Due to systematic (secular and periodical) errors and random errors, we estimated the total error in our measurement of μ to be less than 30% of μ .

was obtained by one residual of the node of LAGEOS, plus the corresponding residual of the node of LAGEOS II times the factor 0.295, plus the corresponding residual of the perigee of LAGEOS II times the factor -0.35. By fitting a straight line through these combined residuals of nodes and perigee (obtained using the JGM-3 gravity field model) we finally found

$$\mu \cong 1.1. \tag{2}$$

This combined, measured, gravitomagnetic perturbation of the satellites' orbits corresponds to about 12 meters at the LAGEOS altitude, that is about 205 milliarcsec. The root mean square of the post-fit combined residuals is about 13 milliarcsec.

The main error sources affecting the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II are: errors due to uncertainties in the even zonal harmonics, C_{2n0} , with $2n \ge 6$, errors in the modeling of the perigee rate of LAGEOS II due to uncertainties in the odd zonal harmonics, errors due to unmodeled tidal perturbations and to variations in the gravity field, random and stochastic observational errors, errors due to non-gravitational perturbations. By

a preliminary error analysis we found: even zonal harmonics, $\delta \mu^{\text{even zonals}:J_{2n} \geq J_6} \leq 17 \% \mu$; odd zonal harmonics, $\delta \mu^{\text{odd zonals}} \leq 3 \% \mu$; tides plus other variations of the Earth's gravity field, $\delta \mu^{\text{tides} +} \leq 6 \% \mu$; non-gravitational perturbations, $\delta \mu^{\text{non-gravitational}} \leq 13 \% \mu$.

Therefore, by taking into account the effects of all the systematic and random error sources, we finally found

$$\delta \mu/\mu \stackrel{<}{\sim} 30\% \,. \tag{3}$$

In conclusion, we obtained the result

$$\mu = 1.1 \pm 0.30 \tag{4}$$

(whereas $\mu = 1$ in general relativity).

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