Effect of interactions on the admittance of ballistic wires

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Effect of interactions on the admittance of ballistic wires

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Abstract. – A self-consistent theory of the admittance of a perfect ballistic, locally charge neutral wire is proposed. Compared to a non-interacting theory, screening effects drastically change the frequency behavior of the conductance. In the single-channel case the frequency dependence of the admittance is monotonic, while for two or more channels collective interchannel excitations lead to resonant structures in the admittance. The imaginary part of the admittance is typically positive, but can become negative near resonances.

The ac conductance (admittance) in mesoscopic systems attracted recently strong interest, mostly due to the finite-frequency measurements of Aharonov-Bohm oscillations in rings [1] and noise in diffusive metallic wires [2]. The theoretical investigation of the problem raises an important question: Standard calculations of the conductance, employing either a scattering (Landauer) or a linear response (Kubo) approach, describe the current in response to the *external* electric field, assuming the latter to be uniform (linear response) or ignoring the actual distribution of the potential (dc scattering approach). However, in realistic systems the potential is *not* uniform due to the screening effects. In dc transport the actual potential distribution is unimportant for the evaluation of the conductance due to the Einstein relation. In contrast, the ac response is strongly sensitive to the distribution of the potential inside the sample [3], [4]. In its turn, the potential profile is related via the Poisson equation to the electron density. Thus, the admittance has to be found self-consistently [5].

Indeed, early attempts to generalize dc conductance calculations to the ac case (see, *e.g.*, [6]) within the free-electron approach proved to be not self-consistent and failed to conserve current. The sensitivity of the ac conductance to different electric-field configurations is illustrated in ref. [7]. The construction of a current-conserving theory is not an easy task. Presently, two approaches are available in the literature. First, the self-consistent ac scattering approach [3], [4] allowed to study the low-frequency admittance and to express it through the scattering matrix for an arbitrary system [4], [8]. For arbitrary frequencies, the admittance is expressed through functional derivatives of the scattering matrix with respect to the local electric potential. Alternatively, one can employ the methods of non-equilibrium statistical mechanics, and express the ac response of an interacting system through the Green's functions in a Keldysh formalism [9], [10]. Although these methods are quite powerful, their application

to specific problems meets substantial technical difficulties. The high-frequency response of coupled infinitely extended one-channel liquids has been investigated by Matveev and Glazman [11]. Here we are particularly interested in wires connected to reservoirs. Compared to infinitely extended wires, the presence of reservoirs modifies the low-frequency response, possibly up to frequencies determined by a transit time.

Below we develop a self-consistent theory for the admittance of a perfect ballistic wire (directed along the x-axis) of length L and cross-section $S \ll L^2$ (3D) or width $W \ll L$ (2D), placed between two reservoirs [12], [13]. The wire is assumed to be shorter than all the lengths associated with inelastic scattering. Here we treat interactions in the random phase approximation (RPA). More specifically, we are interested in the limit where the wire is locally charge neutral. This is realistic in two situations: i) Suppose for a moment that the wire is coupled to a back-gate through a capacitance c per unit length. In the one-channel case, this system is equivalent to the Luttinger model with short-range interactions [14]; the interaction constant g and the capacitance are related by $g^2 = (1 + e^2/(\pi v_{\rm F} c))^{-1}$. Here $v_{\rm F}$ is the Fermi velocity which determines the density of states $1/\pi v_{\rm F}$. The charge neutral case corresponds to the zero-capacitance limit (or $g = 0$ limit), that is, the back gate is at a very large distance from the wire. This example also shows that the condition of applicability of RPA $e^2 \ll v_{\rm F}$ is compatible with a small interaction constant g since the capacitance can be very small. Based on this model the results presented below are valid for frequencies up to $v_{\rm F}/R$, where $R \ll L$ is the distance between the wire and the gate. The case of many channels cannot be reduced to the Luttinger model. ii) In the absence of a back-gate (or for $R \gg L$), the results presented below are valid for frequencies below the plasma modes of a wire. For one channel these frequencies are of order $\omega_{\rm p} \sim v_{\rm F}/L$ (see, *e.g.*, [11], [15]); for a multi-channel wire the highest lying plasmon branch ($v_{\rm F}/L$) provides the frequency which limits the results presented below.

We show that screening plays a crucial role for the frequency dependence of the admittance: a wire with only one transverse channel exhibits a monotonic frequency dependence, while a wire with several transverse channels shows resonant structures in the admittance, due to collective interchannel excitations in which an accumulation of charge in one channel is locally compensated by a charge depletion in another channel. As a consequence, the imaginary part of the admittance changes sign as a function of frequency.

General formulation. The ac transport in a 1D perfect wire is determined by the following system of equations for the local current j, the particle density ρ, and the electric potential φ/e:

$$j = j_{\rm p} - (4\pi e)^{-1}\nabla\partial_t\varphi; \quad j_{\rm p}(x) = e\sum_a v_a\left(\rho_a^+ - \rho_a^-\right), \tag{1}$$

$$\Delta\varphi = -4\pi e^2\sum_a\left(\rho_a^+ + \rho_a^-\right), \tag{2}$$

$$-\partial_x j_{\rm p} = e\partial_t\sum_a\left(\rho_a^+ + \rho_a^-\right), \tag{3}$$

$$\rho_a^\pm(x,t) = \rho_{0a}^\pm(x,t) - \int_0^L \Pi_a^\pm(x,x',t-t')\varphi(x',y=0,z=0,t')\mathrm{d}x'\,\mathrm{d}t'. \tag{4}$$

The index a labels transverse channels ($1 \le a \le N_\perp$), and $\rho_a^\pm$ denotes the excess density (with respect to the positive background) of right/left-movers in the channel a, which depends only on the coordinate x. The velocities v_a of left- and right-movers in the same channel coincide, $v_a = (\pi S\nu_a)^{-1}$. Here ν_a is the density of states in the channel a; the total density of states is given by $\nu = \sum\nu_a$. Equations (1)-(4) are valid within the ballistic wire extending from $x = 0$ to $x = L$. Furthermore, the quantities $\rho_{0a}^\pm$ are "bare" densities of particles in the channel a *injected* from the left/right reservoir; the distribution function of these particles is determined

by the distribution function of the left/right reservoir at the time of injection. Specifying the chemical potentials in the left/right reservoirs $\mu_{\rm L} = V(t)$, $\mu_{\rm R} = 0$, we obtain

$$\rho_{0a}^{\pm} = \frac{1}{2}\nu_a \int_0^\infty f_{\rm L,R}(\epsilon){\rm d}\epsilon,$$

and there follow $\rho_{0a}^{+}(x,t) = \nu_a V(t - x/v_a)/2$ and $\rho_{0a}^{-} = 0$. Note that for $x = L$ the bare density of right-movers does not vanish: We do not consider in detail the transition region between the wire and the reservoirs, where the electrons are distributed over many quantum channels.

Finally, $\Pi_a^{\pm}(x, x', t-t')$ is the polarization function, responsible for the density of right/left-movers induced in the channel a at the point x and time t due to a potential perturbation φ at x', t'. If the spatial and the temporal structure on the scale of $p_{\rm F}^{-1}$ (Friedel oscillations) and $\epsilon_{\rm F}^{-1}$, respectively, can be neglected, the polarization function quite generally has the form

$$\Pi_a^{\pm}(x, x', t) = (\nu_a/2)\left[\delta(x - x')\delta(t) - \partial_t P_a^{\pm}(x, x', t)\right], \tag{5}$$

where the function $P_a^{\pm}$ is the conditional probability to find a right/left-moving particle in the channel a at x at time t, if it was at x' at $t' = 0$. For a ballistic channel we have obviously

$$P_a^{\pm}(x, x', t) = \theta(t)\delta(x' \pm v_a t - x). \tag{6}$$

Now we return to our system of equations. First, we note that the Poisson equation (2) and the continuity equation (3) together with the definition of the current (1) imply $\partial j/\partial x = 0$, *i.e.* the current is conserved and does not depend on the space point. The particle current $j_{\rm p}$ is generally not conserved, and the displacement current is required for the self-consistent treatment. However, if the system is locally charge neutral, the case of interest here, the term $\Delta\varphi$ in eq. (2) vanishes simultaneously with the displacement current in eq. (1). Furthermore, we note that the system of equations (1)-(4) is excessive: $2N_\perp + 2$ equations (2), (3), (4) contain $2N_\perp + 1$ unknown fields φ and $\rho_a^{\pm}$. Equation (1) is already a consequence of the continuity equation and the Poisson equation. It is thus sufficient to consider eqs. (1), (2) and (4): The fact that the resulting current is position independent can be used as simple test of consistency.

It is convenient to Fourier-transform the equations with respect to time, and to introduce new variables: the full density in the channel a, $\rho_a = \rho_a^{+} + \rho_a^{-}$, and the density difference $\zeta_a = \rho_a^{+} - \rho_a^{-}$. The bare density injected by the reservoir into channel a in response to a potential oscillation V_ω in that reservoir is $\rho_{0a}(x) = (\nu_a V_\omega/2)\exp[i\omega x/v_a]$. The combined contribution of the probabilities $P^{\pm}$ gives after Fourier transform a term

$$p_a(x) = (i\omega\nu_a/2v_a)\exp\left[i\omega|x|/v_a\right]$$

in the polarization function. It is useful also to introduce the operator $\hat{Q}_a$,

$$\hat{Q}_a g = \nu_a g(x) + \int_0^L p_a(x - x')g(x'){\rm d}x'.$$

With these abbreviations, we obtain

$$j = e\sum_a v_a\zeta_a(x); \quad \zeta_a(x) = \rho_{0a}(x) - \int_0^L {\rm sign}(x - x')p_a(x - x')\varphi(x'){\rm d}x', \tag{7}$$

$$\sum_a \rho_a(x) = 0, \tag{8}$$

$$\rho_a(x) = \rho_{0a}(x) - \hat{Q}_a\varphi(x), \tag{9}$$

where $\varphi(x) \equiv \varphi(x, y = 0, z = 0)$. Combining eqs. (8) and (9) gives a closed integral equation for the potential, generated in response to the injected density,

$$\sum_a \hat{Q}_a \varphi = \sum_a \rho_{0a}. \tag{10}$$

Now we have to solve the equation (10) for the potential, and then calculate the current from eq. (7). The admittance is determined by $G(\omega) = ejS/V_\omega$. We re-emphasize again that the current and thus the admittance do not depend on the space point x, which is a check of the consistency of our approach.

One channel. For the case of one channel with the velocity v the solution of eq. (10) has the form

$$\varphi(x) = V_\omega \left(1 - \frac{2v}{i\omega L}\right)^{-1} \left(1 - \frac{x}{L} - \frac{v}{i\omega L}\right). \tag{11}$$

For $\omega \ll v/L$ the potential is screened, and everywhere in the wire is close to one-half of the external voltage. On the other hand, for $\omega \gg v/L$ the potential drops linearly. Equation (10) describes thus the crossover from a uniform potential at low frequencies to a uniform electric field at high frequencies. Calculating the admittance, we obtain

$$G(\omega) = \frac{e^2}{2\pi} \left(1 - \frac{i\omega L}{2v}\right)^{-1}. \tag{12}$$

The imaginary part of the admittance is positive (inductive). For low frequencies we reproduce the behavior found previously in ref. [8], $G(\omega) = e^2 N_\perp/\pi - i\omega E$, with an emittance

$$E = -\frac{e^2}{4} \nu L S. \tag{13}$$

Two channels. The Poisson equation for the potential (10) for a wire with two channels a and b can be solved by noticing that

$$(\omega^2 + v_a^2 \partial^2/\partial x^2) \hat{Q}_a \varphi = \nu_a v_a^2 \varphi''.$$

This implies that the potential has the form

$$\varphi(x) = \alpha + \beta x + \gamma \exp[i\xi x] + \delta \exp[-i\xi x],$$

where the coefficients α, β, γ and δ are determined by the substitution of this ansatz into eq. (10). Interestingly, now, in addition to the constant and linear part, the potential acquires also a spatially oscillating part with a wave vector given by $\xi = \omega/(v_a v_b)^{1/2}$. The oscillatory structure of the potential is also manifest in the admittance. Indeed, we find a non-trivial dependence of the conductance on the wave vector ξ and the density-of-states ratio $\eta = (v_a/v_b)^{1/2}$,

$$G(\omega) = \frac{e^2}{2\pi}(\eta^2 + 1) \left(-\frac{i\xi L \eta}{2} + \frac{(\eta^3 + 1)(\eta + 1) - (\eta^3 - 1)(\eta - 1)\exp[i\xi L]}{(\eta + 1)^2 - (\eta - 1)^2 \exp[i\xi L]}\right)^{-1}.$$

The conductance is symmetric with respect to the replacement $\eta \to \eta^{-1}$, as it must be. For low frequencies we reproduce again the static quantized conductance e^2/π and the emittance (13). The real part is strictly positive, although now both the real and imaginary part exhibit oscillations on top of the monotonic behavior found for the single-channel wire. In the limit $v_a = v_b$, which corresponds to a spin degenerate one-channel conductor, these oscillations

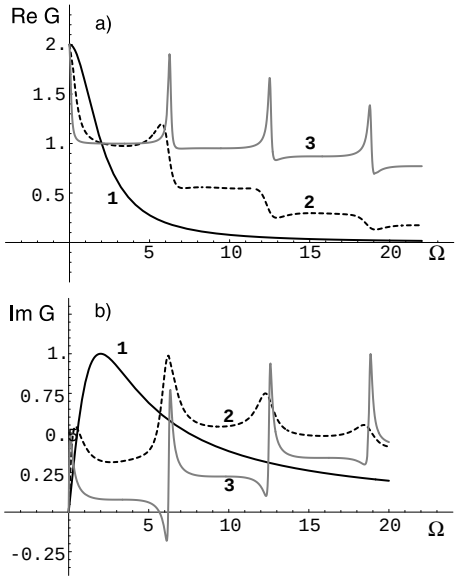

Fig. 1. – Real (a) and imaginary (b) parts of the conductance in units of $e^2/2\pi$ for the two-channel case as a function of the parameter $\Omega = \omega L/(v_a v_b)^{1/2}$; the parameter $\eta = (v_a/v_b)^{1/2}$ is equal to 1 (Curve 1), 5 (2) and 20 (3). Resonances are due to the interchannel excitations.

vanish. The behavior of the admittance as a function of frequency for different values of the parameter η is shown in fig. 1. Note that the imaginary part may change sign in the vicinity of the points $\omega \sim 2\pi n (v_a v_b)^{1/2}/L$ (see below), if η is large enough. A large η occurs for Fermi energies just above the threshold of the second conductance channel.

The oscillatory structure of the admittance can be understood by investigating the poles of the admittance. In the limiting case $\eta \ll 1$, we obtain for the spectrum of collective modes

$$\omega = \frac{(v_a v_b)^{1/2}}{L} \left[2\pi n(1-\eta^2) - 2i\eta\right], \qquad n \in Z.$$

The purely imaginary eigenvalue with $n = 0$, as for one channel, corresponds to the charge relaxation between the wire and the reservoirs via ballistic motion, while the resonances for $n \neq 0$ correspond to nearly neutral interchannel excitations. These modes are essentially standing waves induced in both channels simultaneously but with densities of opposite signs, $\rho_a = -\rho_b \propto \exp[i\xi x] - \exp[i\xi(L-x)]$. The decay of these excitations, Im $\omega = -2v_a/L$, is determined by the carriers in the channel with the lower velocity. Interestingly, we find that due to the coupling to reservoirs all the collective modes are damped with a relaxation constant which is the larger the shorter the wire is.

Many channels. For $N_\perp > 1$ channels the potential has the form

$$\varphi(x) = \alpha + \beta x + \sum_{i=1}^{N_\perp - 1} \left[\gamma_i \exp[i\omega\lambda_i x] + \delta_i \exp[-i\omega\lambda_i x]\right],$$

and the positive quantities λ_i are solutions of the equation

$$\sum_a \frac{v_a}{1 - v_a^2\lambda^2} = 0. \tag{14}$$

For arbitrary $N_\perp$ and ω, further analytical progress is hard, but the problem can be solved for

low frequencies $\omega \ll \min(v_a/L)$. We obtain

$$G(\omega) = \frac{e^2 N_\perp}{2\pi} + i\omega\frac{e^2}{4}\nu SL - \omega^2\frac{e^2\pi}{8}(SL)^2\sum_a \nu_a^2 + \dots \quad . \tag{15}$$

In conclusion, we investigated the admittance of a perfect ballistic wire in the frequency range below $v_{\rm F}/L$. We showed that the screening effects are very important for the admittance, and provide the current conservation. For one channel, the admittance is a monotonic function of frequency, whereas for two or more channels it contains also oscillatory components due to the density redistribution between different channels. In particular, the imaginary part of the admittance is generally positive (inductive-like), but can change sign and become capacitive in the vicinity of resonances due to the interchannel excitations. The wire exhibits resonances due to damped collective modes. The resonant effects predicted can be measured experimentally; the collective mode frequencies depend on the relative electron concentration in the different channels and near the threshold for a new quantum channel, where the resonances are most pronounced, can be made very small.

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