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Dynamic time reversal of randomly backscattered acoustic waves

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Abstract. – We report the first experiments using the reversibility of a transient acoustic wave in a multiple-scattering medium to simulate either a stationary or a dynamic acoustic lens. The method is based on time reversal experiments performed in a backscattering configuration. In the stationary case, we show that we take advantage of multiple scattering to focus better than with a perfect reflecting interface. In the dynamic case, we explain the refocused spot time evolution by a simple model based on the time-dependent ability to recover the angular spectrum thanks to both single- and multiple-scattering paths.

Because of the extreme sensitivity to initial conditions that lies at the heart of chaotic phenomena, a time reversal experiment with particles is impossible. In wave physics however, the amount of information necessary to describe a wave field unambiguously is limited, by the shortest wavelength of the field, and a time reversal experiment becomes theoretically possible. In practise, in acoustics, we take advantage of the technology of reversible arrays of piezoelectric transducers to perform such experiments. A time reversal experiment with acoustic waves is equivalent to complex conjugation for a single frequency signal which has already been extensively studied in optics [1]. The big difference lies in the fact that optical phase conjugated mirrors cannot produce the true time reversal for a broadband wave form. They are limited by the time responses of optical detectors which are long compared to the period of optical waves. For a broadband pulse, a non-linear process is used to conjugate the envelope of the optical wave form, which is equivalent to a true time reversal only in the limit of zero bandwidth.

Three years ago, the first experiments showing the reversibility of acoustic waves through high-order multiple-scattering media were reported [2,3]. These experiments were performed in a transmission mode by means of a Time Reversal Mirror (TRM), a device that can record a wave form $f(t)$, time reverse it, and retransmit $f(-t)$ inside the medium.

In this letter, we show that an analogous experiment made in a backscattering configuration leads to recreate an image of the source despite disorder, what we call the "mirror effect".

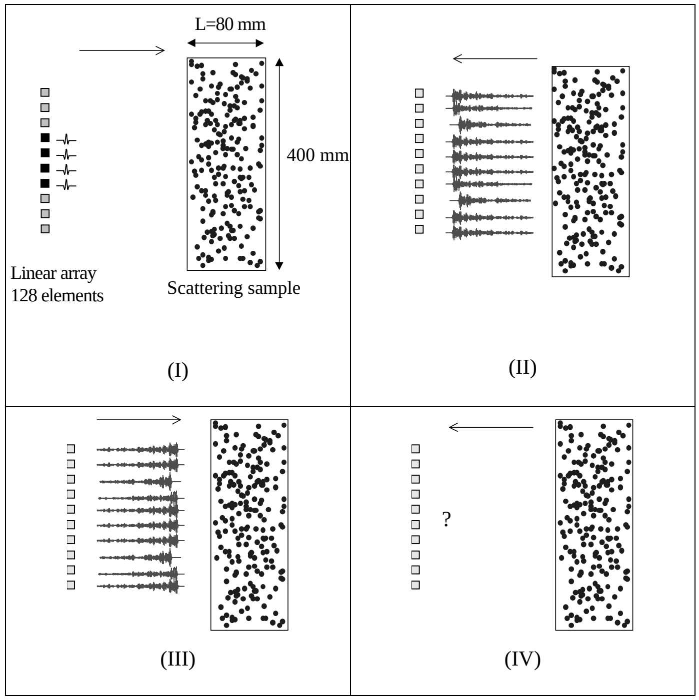

Fig. 1. – Experimental setup. (I) A few elements, in black on the figure, transmit a short pulse into the sample. (II) The backscattered signals are recorded on each transducer. (III) Each signal is time-reversed and retransmitted. (IV) We look at what comes back to the array.

Furthermore, in a stationary experiment, we show that we take advantage of disorder to focus back with a better resolution and lower side lobes than in the case of a perfect planar reflector. Finally, we highlight that the refocusing process is highly time-dependent and propose a model to explain the evolution of the directivity pattern as a function of time.

Let us first describe a typical backscattering time reversal experiment. The multiple-scattering sample consists of 6000 steel rods randomly distributed in the plane. It is immersed in a water tank. The sample thickness is 80 mm. The characteristic measured transport parameters are $l^* = 4.4$ mm for the transport mean free path and $D = 3.3$ mm$^2/\mu$s for the diffusion coefficient. These two quantities were determined using the recording of the Coherent Backscattering Effect [4] recently discovered in acoustics [5, 6]. The Time Reversal Mirror (TRM) we use is a linear array of 128 transducers with central frequency 1.5 MHz (which corresponds to an average wavelength of 1 mm in water) and *pitch half-wavelength*. Each of these reversible transducers has its own electronics: detection amplifier, analog-to-digital converter, digital memory and a programmable generator capable of synthetizing the temporally time-reversed signal stored in the memory.

Initially, ten elements (from transducers 58 to 67) transmit a pulsed wave (3.5 cycles, which means that the duration of the initial pulse is about 1 μs) into the sample, 140 mm from the array. The backscattered wave is spatially sampled on the array. On each transducer, the received signal is temporally sampled at a 20 MHz rate, digitized on 8 bits and recorded. Because of multiple scattering, the duration of these recorded signals is very long ($\approx 250\ \mu$s). The 250 μs are then time-reversed and retransmitted, thus recreating an ultrasonic wave which propagates back in the same medium (fig. 1). Finally, an amazing time compression is

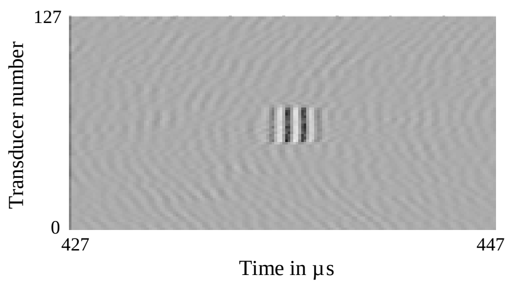

Fig. 2

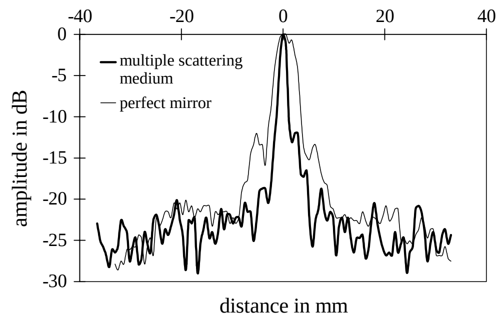

Fig. 3

Fig. 2. – Image of the source recreated by time reversal. The signals received on each transducer after back propagation of the time-reversed wave are represented on a grey scale image. Time t is in abscissa, transducer number is in ordinate. The time origin corresponds to the emission of the time-reversed wave. We notice that the time-reversed wave has reconverged to the ten emitting transducers (from #58 to 67), at time 437 μs.

Fig. 3. – Directivity patterns obtained in a stationary experiment with the multiple-scattering sample (thick line), with a perfect planar reflecting steel sample (thin line). The initial source was one transducer (element # 64) of the array emitting 3.5 cycles. With the multiple-scattering sample, the directivity pattern is thinner and the side lobes level is lower.

observed on each of the ten emitting elements that were previously used as sources. Indeed, the signals received on these transducers last about 1 μs against 250 μs for the time-reversed backscattered signals. The image of the source has thus been recreated (fig. 2). It is what we call the "mirror effect".

To have a better understanding of the refocusing process, we have performed the same experiment using a single array element (transducer #64) as a source emitting 3.5 cycles. The directivity of the time-reversed wave is obtained keeping the maximum pressure on each transducer (fig. 3). If the multiple-scattering sample is replaced by a perfect reflecting interface with same width, such as a steel sample, the directivity pattern is essentially limited by the aperture of the array. In this case, it is larger and present higher side lobes level (fig. 3). Thus, using the multiple-scattering sample, we do not only manage to refocus back despite disorder,

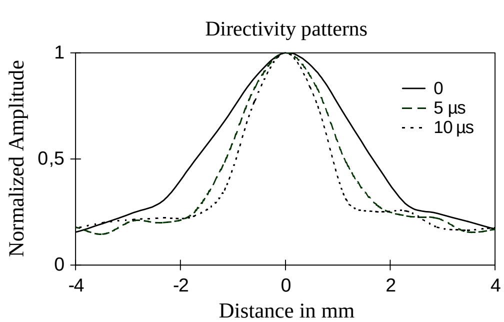

Fig. 4. – The initial source is the 64-th transducer of the array emitting 3.5 cycles. We select in the 128 backscattered signals various time reversal windows (width $\Delta T = 5$ μs) translated by 1 μs step. We present here the directivity patterns obtained with three different time reversal windows.

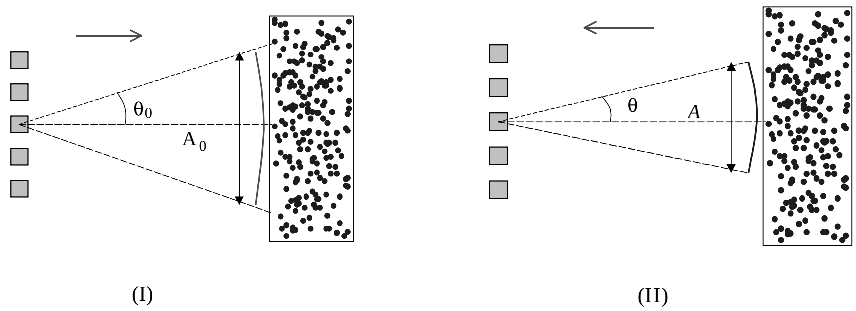

Fig. 5. – (I) A wave is transmitted by one element whose typical angular directivity is θ_0. When it intercepts the medium, the incoming wave front is characterized by some aperture A_0. (II) After backpropagation of the time-reversed wave, the recreated front at the exit of the sample is characterized by a new aperture $A \leq A_0$. This aperture acts as a secondary source which will focus on the initial transducer with a directivity pattern depending on A.

but we also take advantage of scattering to focus better.

We can also account for the focal spot size. According to diffraction laws, spatial frequencies lower than $1/\lambda$ are in any case lost during propagation, which defines an incompressible limit to the refocusing on the source. Taking this limitation into account, the question is to know how able the TRM is to reconstruct the initial angular spectrum (which is in our case a sinc function). In other words, to what extent can we reconstruct the initial source in both size and shape? The dynamic experiment brings us an answer.

Instead of using the whole 128 signals, we now select short time windows, called the *time reversal windows* (width $\Delta T = 5$ μs), whose origins are moved from $t = 0$ to $t = 12$ μs, by 1 μs step.

For one time origin, each of the 128 windows is time-reversed and retransmitted. Then, keeping the maximum pressure received on each transducer, we can define the directivity pattern for that particular time. In fig. 4, we have plotted the resulting directivity patterns for three different time reversal windows.

We notice that the directivity pattern narrows with time which indicates inversely that the wave front coming out from the sample after time reversal contains a larger angular spectrum. So the system mimics an acoustic lens whose aperture is time varying. The question is to determine quantitatively its evolution as a function of the selected window in the signal.

Figure 5 illustrates the simple idea on which our theory is based. In the best case, if time reversal is perfect, what we recreate at the exit of the sample, after backpropagation of the time-reversed wave, is simply the initial front. Now, depending on the selected time reversal window, what we actually recreate is a front with a smaller aperture than the initial one; this recreated front behaves as a secondary source which produces around the initial source a focal spot depending on its aperture and its directivity. The question is therefore to relate this directivity angle θ with the selected time-reversed window. The angle θ is simply the maximum angle which has been recorded in the given window.

Around 1.5 MHz, the scattering cross-section of one scatterer is nearly isotropic. Then at least a part of a pulse reaching a point on the first row of scatterers is reradiated exactly in the backward direction (fig. 6). On the one hand, this fact explains that the array aperture does not limit the resolution any more as is the case if the multiple-scattering sample is replaced by a perfect reflecting interface as previously discussed. Indeed, provided that the sample is large enough to entirely intercept the initial incoming wave front, the latter can be completely

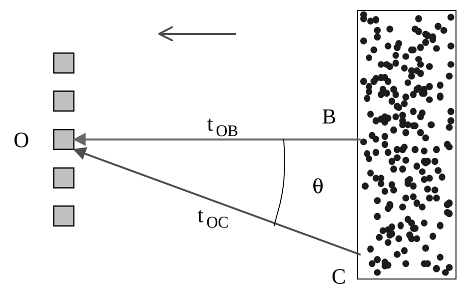

Fig. 6. – Because of the isotropic scattering cross-sections of the rods, any point C reached by the incident wave reradiates it in all directions and especially in the backward direction. So it is possible to relate simply the angle θ with the time $t = t_{OC} - t_{OB}$.

reconstructed if the whole received signal is used. On the other hand, the isotropic scattering cross-section of the scatterers justifies a simple model based on ray trajectories to relate the angle θ with a corresponding time. In this case, the time t_{OC} necessary to recover the angle θ is simply the time for the corresponding ray to go forth and back and be recorded by the TRM (ray OC in fig. 6). Compared to the shortest time t_{OB} needed for a pulse to go forth and back between the array and the sample (ray OB in fig. 6), this time is given by

$$t = t_{OC} - t_{OB} = 2\frac{F}{c}\left(\frac{1}{\cos(\theta)} - 1\right), \tag{1}$$

where c denotes the sound speed in water and F the distance source / sample.

With $F = 140$ mm and $c = 1.5$ mm/μs, we obtain the numerical expression

$$\frac{1}{\cos(\theta)} = 1 + 0.0053t\,. \tag{2}$$

Indeed, once information from this angle has been recorded, it can be time-reversed, back propagated and therefore participate to the reconstruction of the initial front. Now, the point is to relate the angle θ (corresponding to a time t) with the measured resolution.

Up to now, we have based our model only on the information scattered by the first row. But, in the case of one single row of scatterers, taking a *time-limited window* at time $t > 0$,

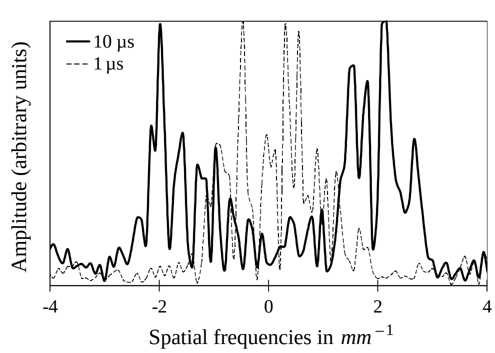

Fig. 7. – Spatial Fourier transforms of the direct front (dotted line) and of a window taken 10 μs behind the first echoes (full line) in the case of a single row of scatterers.

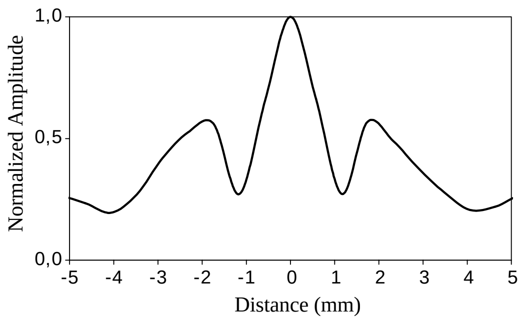

Fig. 8

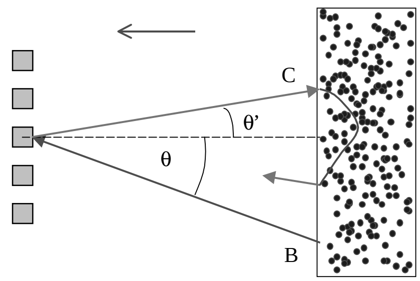

Fig. 9

Fig. 8. – Refocused spot obtained using a 5 μs window taken 10 μs behind the first echoes. The sample is made of one row of rods only. The distance between two maxima is $\lambda F/D$.

Fig. 9. – In a time window located at time t after the direct echo, we get back the angular information θ according to relation (1), which enables to recreate point B of the initial wave front. Thanks to multiple scattering, we record at the same time information corresponding to shorter angles θ' of the angular spectrum. Thus, in our example, we are also able to recreate point C.

corresponding to some angle $\theta > 0$, eliminates the information corresponding to shorter angles. Indeed, we receive on the array only the waves coming from the two extreme regions corresponding to the adequate back and forth time. In this case, we do not reconstruct the whole angular spectrum but only its extreme sides (fig. 7) and the expected focal spot results from interferences between two finite sources, separated by a distance $D = 2F\sin(\theta)$.

Indeed, if the experiment with one single row is performed *with a time-limited window* taken at 10 μs, the obtained focal spot shows very high grating lobes (fig. 8), the distance between two maxima being $\lambda F/D$.

But with the multiple-scattering sample, the refocused spot is quite different (fig. 4) in

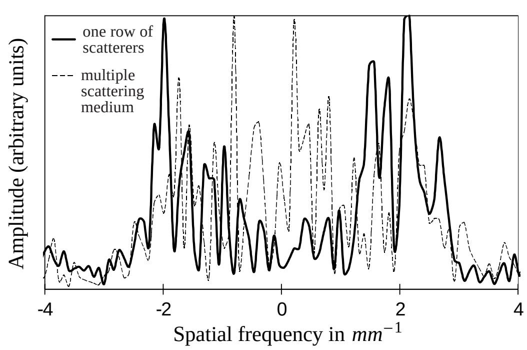

Fig. 10

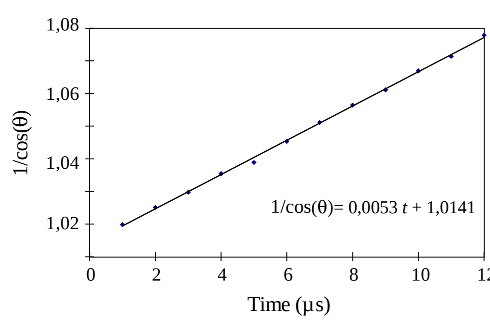

Fig. 11

Fig. 10. – Spatial Fourier transforms of windows taken 10 μs after the first echoes: for a single row of scatterers (solid line), for the multiple-scattering sample (dotted line). With the multiple-scattering sample, the spatial frequencies distribution is more uniform.

Fig. 11. – Time evolution of the reconstructed angular spectrum. Correlation coefficient of the fit: 0.999.

both size and resolution. Actually, thanks to the multiple-scattering paths, we record in all windows, even the farthest, the information coming from all the points of the front (fig. 9), not only from a particular point. Whatever the time reversal window is, all spatial frequencies are present with the same weight (fig. 10). Therefore, for any time reversal window, time reversal is able to reconstruct the whole spatial frequency spectrum.

In this case, the aperture of the secondary source can be modelled by a concave radiator with focal distance F and aperture $D = 2F\sin(\theta)$.

Then, it becomes legitimate to assume that the angle appearing in relation (1) is related to the experimentally measured FWHM δ by the classical relation

$$\delta = 1.2\frac{\lambda F}{D} \quad \text{with} \quad D = 2F\sin(\theta)\,. \tag{3}$$

This theory is found to be in very good agreement with experiment. Indeed, the curve $\frac{1}{\cos(\theta)}$ as a function of time is very well fitted by a line with expected slopes and intercepted values (fig. 11).

Finally, we have shown that a time reversal experiment is possible in a backscattering mode with ultrasonic waves propagating in a disordered medium. Besides, we take advantage of disorder to get for the refocused wave a better directivity pattern and a better side lobes level than with a perfect reflecting interface. Furthermore, the spatial resolution is improved when time windows are taken at later times. For a given time-reversed window, scattering on the first row enables to record the extreme angles θ corresponding to the adequate forth and back time. Thanks to the diffusive halo created in the multiple-scattering regime, all the angular spectrum between these two extreme angles is also recovered in the chosen time reversal window. Then, the recreated aperture behaves as a secondary source which can be well modelled by a concave radiator. The predicted focal spot size is found to be in very good agreement with experimental results.

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