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Self-squeezing states of magnons in an antiferromagnet

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Abstract. – The self-squeezing states of magnons in an antiferromagnet are studied in this paper. These particular states allow a reduction in the quantum fluctuations of the spin components to below the zero-point quantum noise level of the coherent magnon states. The conditions of achieving magnon self-squeezing states are given by calculating the expectation values of the spin fluctuations, and a possible detection scheme based on a polarized neutron-scattering technique is suggested.

The squeezed states of bosons including photons [1], polaritons [2, 3] and phonons [4–9] have attracted much attention during the past few years. These states are interesting because they can have lower quantum noise than the vacuum or coherent states. Compared to both photons and phonons, the elementary excitations of the spin waves, magnons, are also bosons, and their Hamiltonian in a particle-number representation is similar to that of the photons or the phonons. Also, the spin vectors are similar to the lattice displacements or the light field vectors. Therefore, it is possible that the magnon squeezing states exist. Here we study the properties of the magnon squeezing states and explore the possibility of generating these states through magnon-magnon interaction. By investigating the dynamical and quantum fluctuation properties of the spin waves, in analogy with the modulation of quantum noise in light and lattice waves, we derive the condition of achieving magnon squeezing states.

A common requirement of achieving the squeezing states for the photons or the phonons is that there are two-mode bosons excited by the external fields with different incident directions [9]. In the antiferromagnet, the spins of all electrons in each unit cell in the crystal lattices compensate one another (in the equilibrium state in the absence of a magnetic field). These spins, periodically repeated in all cells, form the magnetic sublattices of the antiferromagnet. The external magnetic field with single direction and the internal anisotropy field can excite two different sub-spin waves or magnons. The two types of magnons interfere with each other and generate the self-squeezing states.

The magnon self-squeezing states are attractive because they have new statistical and quantum-mechanical properties and a potential application such as the microwave emission with lower noise. In terms of this effect, we may understand the interaction between atoms in the antiferromagnet.

In the antiferromagnet, there are two sublattices and each unit cell of the crystal contains two atoms. Let $\vec{S}_a$ represent any atomic spin on one sublattice and $\vec{S}_b$ any spin on the other. The external magnetic field $\vec{B}$ is parallel to the z-axis. The Heisenberg Hamiltonian of the system is [9]

$$H = |J| \sum_{\mu,\nu} \left(\vec{S}_{a\mu} \cdot \vec{S}_{b\nu} + \vec{S}_{b\nu} \cdot \vec{S}_{a\mu} \right) - g\beta(B + B_{\rm A}) \sum_{\mu} S^z_{a\mu} - g\beta(B - B_{\rm A}) \sum_{\nu} S^z_{b\nu}, \tag{1}$$

where B is the strength of the external magnetic field, $B_{\rm A}$ the anisotropy field, β the Bohr magneton, and J the exchange integrals. The exchange interaction is assumed to be limited to nearest neighbors of each other.

First of all, we introduce the raising and lowering operators:

$$S^+_{i\mu} = S^x_{i\mu} + iS^y_{i\mu}, \quad S^-_{i\mu} = S^x_{i\mu} - iS^y_{i\mu} \quad (i = a, b) \tag{2}$$

and then make use of the Holstein-Primakoff substitutions

$$S^+_{a\mu} \longrightarrow (2S)^{1/2} a_\mu, \qquad S^-_{a\mu} \longrightarrow (2S)^{1/2} a^+_\mu, \qquad S^z_{a\mu} \longrightarrow S - a^+_\mu a_\mu, \tag{3}$$

for atom μ on sublattice a, and

$$S^-_{b\nu} \longrightarrow (2S)^{1/2} b_\nu, \qquad S^+_{b\nu} \longrightarrow (2S)^{1/2} b^+_\nu, \qquad S^z_{b\nu} \longrightarrow b^+_\nu b_\nu - S, \tag{4}$$

for atom ν on sublattice b. Finally, we introduce a transformation to spin wave variables through

$$a_k = N^{-1/2} \sum_{\mu} e^{i\vec{k}\cdot\vec{R}_\mu} a_\mu, \qquad b_k = N^{-1/2} \sum_{\nu} e^{i\vec{k}\cdot\vec{R}_\nu} a_\nu, \tag{5}$$

and the corresponding conjugates. The summations are restricted to the sublattices a and b, respectively, each of which contain N atoms. Equations (2)–(5) are substituted into (1), and the Hamiltonian can be rewritten by the creation and annihilation operators a_k's and b_k's as

$$H = E_0 + \sum_k \left[\hbar\omega_a a^\dagger_k a_k + \hbar\omega_b b^\dagger_k b_k + \eta_k \left(a_k b_k + a^\dagger_k b^\dagger_k \right) \right], \tag{6}$$

where

$$E_0 = -2N\left(ZS^2|J| + g\beta S B_{\rm A}\right), \qquad \hbar\omega_a = 2ZS|J| + g\beta(B_{\rm A} + B),$$
$$\hbar\omega_b = 2ZS|J| + g\beta(B_{\rm A} - B), \qquad \eta_k = 2ZS|J|\gamma_k,$$

and $\gamma_k = \sum_s e^{i\vec{k}\cdot\vec{R}_s}$, here $\vec{R}_s$ being a vector connecting an atom with one of its nearest neighbors, and the sum includes all such vectors.

By solving the Schrödinger equation $i\hbar\frac{\partial|t\rangle}{\partial t} = H|t\rangle$, we can obtain the time-dependent state vector $|t\rangle = U|0\rangle$; here $|0\rangle$ is the initial state vector, and $U = e^{-\frac{i}{\hbar}Ht}$ is the time evolution operator. In terms of the transformation formula

$$e^{-\frac{i}{\hbar}(H_0+V)t} = e^{-\frac{i}{\hbar}H_0 t} \exp\left[-\frac{i}{\hbar} \int_0^t {\rm d}t_1 \left(e^{\frac{i}{\hbar}H_0 t_1} V e^{-\frac{i}{\hbar}H_0 t_1} \right) \right], \tag{7}$$

the time evolution operator can be re-expressed as

$$U = e^{-i(\hbar\omega_a a_k^\dagger a_k + \hbar\omega_b b_k^\dagger b_k)} e^{\xi_k^\dagger a_k b_k + \xi_k a_k^\dagger b_k^\dagger}, \tag{8}$$

where

$$\xi_k^* = \frac{\eta_k}{\hbar(\omega_a + \omega_b)} \left[e^{-i(\omega_a+\omega_b)t} - 1\right] \tag{9}$$

and

$$\xi_k = \frac{\eta_k}{\hbar(\omega_a + \omega_b)} \left[e^{i(\omega_a+\omega_b)t} - 1\right] \tag{10}$$

are the squeezing factors. We can obtain the time-dependent operators (Heisenberg operators) as

$$a_k(t) = U^{-1} a_k U = e^{-i\omega_a t}\left(\cosh\sqrt{\xi_k^*\xi_k}\, a_k - \sqrt{\frac{\xi_k}{\xi_k^*}} \sinh\sqrt{\xi_k^*\xi_k}\, b_k^*\right), \tag{11}$$

$$b_k(t) = U^{-1} b_k U = e^{-i\omega_a t}\left(\cosh\sqrt{\xi_k^*\xi_k}\, b_k - \sqrt{\frac{\xi_k}{\xi_k^*}} \sinh\sqrt{\xi_k^*\xi_k}\, a_k^*\right) \tag{12}$$

and the corresponding conjugates.

In analogy to the definition of the photon squeezed states, the self-squeezing state of the magnons is defined as a particular state in which the fluctuation of some component of the spins is lower than the one of the vacuum state. In the following, we will calculate the fluctuation of the x-axis component of the spins and try to find out the condition of achieving the self-squeezing states. Of course, other components of the spins may be discussed in similar way.

The fluctuation of the x-axis component of the spins is expressed as

$$\langle \Delta S_x^2 \rangle = \langle S_x^2 \rangle - \langle S_x \rangle^2 = \sum_{\mu\nu} \langle (S_{a\mu}^x + S_{b\mu}^x)(S_{a\nu}^x + S_{b\nu}^x) \rangle, \tag{13}$$

where $\langle \cdots \rangle$ denotes an expectation value on the squeezed states. The expectation value of the x-axis component of the spins is zero, *i.e.* $\langle S_x \rangle = 0$. Equations (2)–(5), (11) and (12) are substituted into (13), and we obtain

$$\begin{aligned}
\langle \Delta S_x^2 \rangle = &\frac{S}{2} \sum_k \sum_m e^{i\vec{k}\cdot\vec{R}_m} \langle t | a_k a_k^\dagger + a_k^\dagger a_k + b_k b_k^\dagger + b_k^\dagger b_k + 2(a_k b_k + a_k^\dagger b_k^\dagger) | t \rangle = \\
= &\frac{S}{2} \sum_k \sum_m e^{i\vec{k}\cdot\vec{R}_m} \langle 0 | \{ a_k(t) a_k^\dagger(t) + a_k^\dagger(t) a_k(t) + b_k(t) b_k^\dagger(t) + b_k^\dagger(t) b_k(t) + \\
&+ 2[a_k(t) b_k(t) + a_k^\dagger(t) b_k^\dagger(t)] \} | 0 \rangle.
\end{aligned} \tag{14}$$

Therefore we obtain the fluctuation of the x-axis component of the spins

$$\langle \Delta S_x^2 \rangle = S \sum_k \sum_m e^{i\vec{k}\cdot\vec{R}_m} \left[1 + 2\sinh^2\sqrt{\xi_k^*\xi_k} - \cos\left(\frac{\omega_a + \omega_b}{2} t + \frac{\pi}{2}\right) \sinh 2\sqrt{\xi_k^*\xi_k}\right]. \tag{15}$$

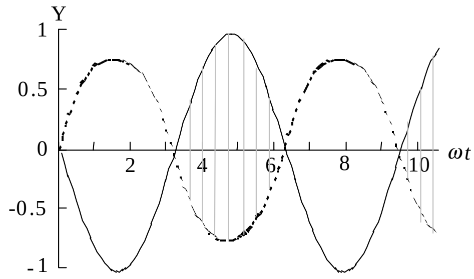

Fig. 1 – Schematic diagram of the magnon self-squeezing states. The dotted line corresponds to the function $Y = \tanh(\frac{2ZS|J|\gamma_k}{\omega}\sin\omega t)$, and the solid line to the function $Y = \cos(\omega t + \frac{\pi}{2})$. The hatched parts are the regions of the self-squeezing states.

The condition of achieving the self-squeezing states is

$$\cos\left(\frac{\omega_a + \omega_b}{2}t + \frac{\pi}{2}\right) > \tanh\sqrt{\xi_k^*\xi_k},$$

or

$$\cos\left(\omega t + \frac{\pi}{2}\right) > \tanh\frac{2ZS|J|\gamma_k}{\omega}\sin\omega t, \tag{16}$$

where

$$\omega = \frac{2ZS|J| + g\beta B_\mathrm{A}}{\hbar}. \tag{17}$$

As shown in fig. 1, the hatched parts are the regions that satisfy the squeezing condition, while in other regions squeezing states do not exist. The squeezing states can achieve smaller fluctuation for the spin component during certain time intervals and are therefore helpful for decreasing quantum noise. Because the squeezing condition is not related to the external magnetic field, the corresponding states are called self-squeezing states. The self-squeezing states always occur on alternative half-periods. Their time period

$$\tau = \frac{2\pi\hbar}{2ZS|J| + g\beta B} \tag{18}$$

is related to the exchange integral and the anisotropy field, which are the physical quantities that describe material properties. It is helpful to understand the material properties if we are able to measure the period of occurrence of magnon self-squeezing states. In order to estimate the order of magnitude of τ, we take the crystal MnF_2 as an example. The material parameters [10] are taken to be $|J| = 11.9\,\mathrm{meV}$, $Z = 6$, $S = 1$, and $B_\mathrm{A} = 0.88\,\mathrm{T}$, and we obtain $\tau = 2.896 \times 10^{-14}\,\mathrm{s}$.

The self-squeezing magnons of the antiferromagnet could be detected by a polarized-neutron scattering technique. Suppose that the neutron polarization is parallel to the x-axis, and the external magnetic field is parallel to the z-axis. According to the general theory of neutron scattering, the differential cross-section for the scattering of the neutron by the antiferromagnet is related to the fluctuation of the spins [11] and proportional to $\langle\Delta S_x^2\rangle$. Therefore, it is possible to observe the periodicity of the self-squeezing states by the neutron scattering experiment.

In summary, our calculations have shown that in an antiferromagnet there are magnon self-squeezing states that can reduce the fluctuation of the spin component. The magnon self-squeezing states occur periodically with time at $T = 0\,\mathrm{K}$, and they represent a basic characteristic of the antiferromagnet. The mechanism of generating the self-squeezing states is due to a modulation interaction between two sublattice spins. We may obtain useful information about the material properties, such as the exchange integral and the anisotropy field, by measuring the time periods of the self-squeezing states.

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