



Emergence of memory

To cite this article: K. Klemm and P. Alstrøm 2002 *EPL* **59** 662

View the [article online](#) for updates and enhancements.

You may also like

- [Computational modelling of memory retention from synapse to behaviour](#)
Mark C W van Rossum and Maria Shippi
- [Improving brain-machine interface performance by decoding intended future movements](#)
Francis R Willett, Aaron J Suminski, Andrew H Fagg et al.
- [Synaptic electronics: materials, devices and applications](#)
Duygu Kuzum, Shimeng Yu and H-S Philip Wong

Emergence of memory

K. KLEMM and P. ALSTRØM

Niels Bohr Institute - Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

(received 5 September 2001; accepted in final form 6 June 2002)

PACS. 05.65.+b – Self-organized systems.

PACS. 87.18.Sn – Neural networks.

PACS. 05.45.-a – Nonlinear dynamics and nonlinear dynamical systems.

Abstract. – We propose a new self-organizing mechanism behind the emergence of memory in which temporal sequences of stimuli are transformed into spatial activity patterns. In particular, the memory emerges despite the absence of temporal correlations in the stimuli. This suggests that neural systems may prepare a spatial structure for processing information before the information itself is available. A simple model illustrating the mechanism is presented based on three principles: 1) Competition between neural units, 2) Hebbian plasticity, and 3) recurrent connections.

Memory is believed to be a universal feature of the nervous system [1] and exciting results improving our understanding of molecular as well as organizational mechanisms underlying memory have been obtained in recent decades [2]. On the organizational level significant work has been devoted to the study of “brain maps” underlying the ability to recognize patterns or features from a given sensory input [3,4]. Many intriguing suggestions have been given as to how a memory emerges that is able to extract and recall features from a spatial pattern of neural activity [5,6].

In this letter, we focus on the mechanism behind self-organization from a *temporal* sequence of activity. Time is important in many cognitive tasks, *e.g.* vision, speech, signal processing and motor control. The crucial point is how to represent time, and methods often involve time delays in one form or another [7,8]. How does a structured memory emerge that can cope with temporal sequences of activity? For example, the information we receive through a temporal sequence of input must at least to some extent be memorized spatially in the neuronal activity pattern. Here we present a simple conceptual model for the time-to-space transformation, from which a memory emerges.

The fundamental assumptions of the model presented here are the following: 1) *Competition* between neural units; excited neural units have an inhibiting effect on other units. In the limit of strong inhibition this is winner-take-all [9], where only the region of units with the strongest excitation remains active, suppressing all surrounding units. 2) *Hebbian Plasticity* is an abstract formulation of long-term potentiation depending on pre- and postsynaptic activity: If activity of unit A is followed by activity of unit B, the connection from A to B is strengthened [10,12]. 3) *Recurrent* connectivity opens up the possibility for ongoing information processing in the network by internal feedback.

Features 1) and 2) mentioned above are employed by the *self-organizing map* model formulated by Kohonen [4]. Recently, it has been argued [6] that the self-organizing map is a biologically plausible large-scale model of cortical information processing. However, the self-organizing map has a purely unidirectional information flow without internal dynamics. We know of few attempts to explicitly integrate memory of past stimuli into the self-organizing map [13–17]. These approaches have been shown to work well on specific tasks.

The scope of the current paper is to investigate generally, *i.e.* task-independent, the formation of an internal dynamics that can lead to formation of memory. In our approach, memory is not designed but emerges as a result of the self-organized dynamics of the neural system.

Consider M neural units arranged as a one-dimensional lattice with periodic boundary conditions (a ring). The model describes the time-discrete evolution of the real-valued activities $y_0(t), \dots, y_{M-1}(t)$ of the units. At a given time step t each unit i receives a recurrent excitation $h_i^{\text{rec}}(t) = \sum_j w_{ij} y_j(t-1)$ through connections w_{ij} . Additionally, there is an S -dimensional input $\mathbf{x} = (x_1(t), \dots, x_S(t))$ to the system causing an external excitation $h_i^{\text{ext}}(t) = \sum_j v_{ij} x_j(t)$ through connections v_{ij} . The total excitation is $h_i(t) = h_i^{\text{rec}}(t) + h_i^{\text{ext}}(t)$. Next, we define the centre of activity i^* as the unit with the largest total excitation: $i^*(t) = \arg \max_i h_i(t)$. The updated unit activities form a Gaussian profile around the centre of activity (we suppress t in the notation here),

$$y_i = c \exp \left[-\frac{\text{dist}^2(i, i^*)}{2\sigma^2} \right], \quad (1)$$

where $\text{dist}(i, i^*)$ denotes the distance between units i and i^* in lattice points. The model parameter σ is a measure of the width of the neural-activity field. The constant value $c > 0$ is chosen such that the normalization $\sum_i (y_i)^2 = 1$ holds. Finally, all connections are updated according to a Hebb-rule with a saturation term. Each recurrent connection w_{ij} is changed by

$$\Delta w_{ij} = \eta y_i(t) (y_j(t-1) - w_{ij}), \quad (2)$$

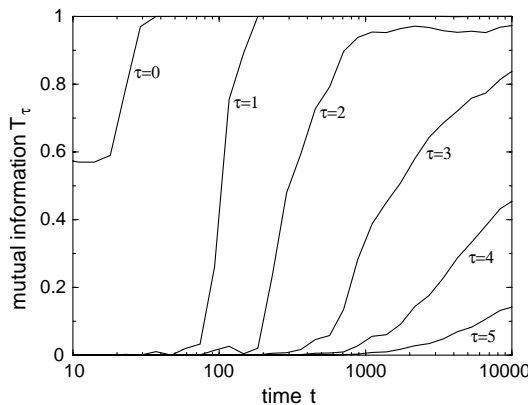


Fig. 1 – The time evolution of the mutual information T_τ indicates the formation of memory. The stream of stimuli $\mathbf{x}(t)$ contains 1 bit of information per time step (two different stimuli presented with equal probability). Thus $T_\tau = 1$ means that the network perfectly remembers the stimulus presented τ time steps before, whereas $T_\tau = 0$ means statistical independence between the stimulus and the network state. The displayed results were obtained as averages over 100 independent simulation runs with networks of size $M = 64$ units.

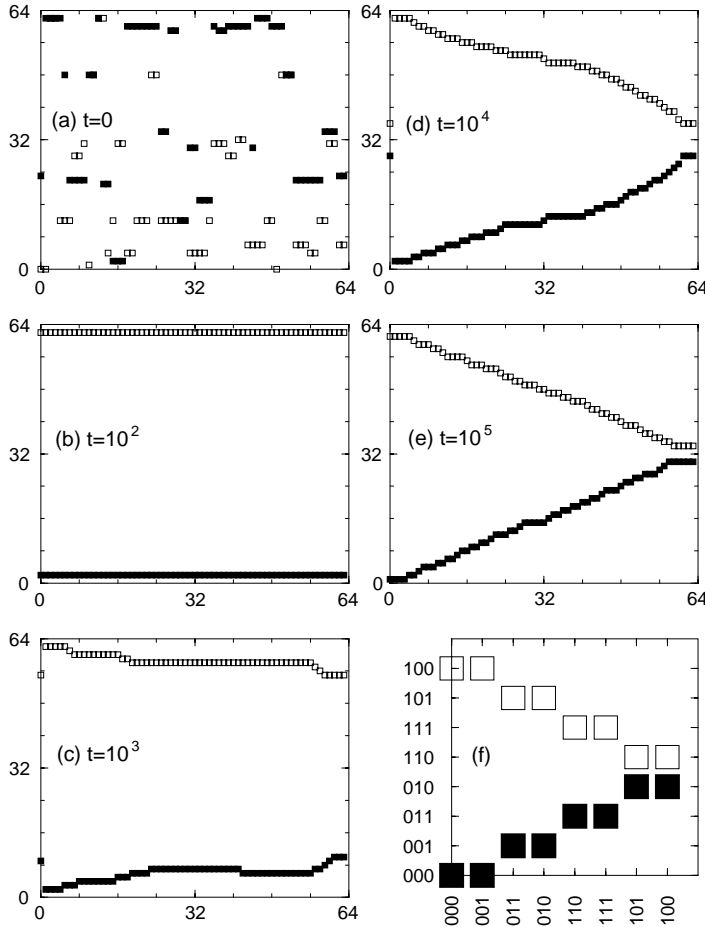


Fig. 2 – Spatial representation of memory. (a)-(e) The return-map of the centre of activity i^* after $0, \dots, 10^5$ steps of learning. The diagrams show $i^*(t)$ as a function $i^*(t-1)$. The function has two branches (filled and unfilled squares) corresponding to the two different values the stimulus $x(t)$ can assume. (f) Idealized return-map for a network with $M = 8$ units. Each unit represents a certain history of stimuli. The histories of the units are given as bit strings on the axes.

where $\eta > 0$ is a constant learning rate. Correspondingly, the increment for the input connections is

$$\Delta v_{ij} = \eta y_i(t) (x_j(t) - v_{ij}) . \quad (3)$$

This completes one time step of the dynamics. The learning rate has a value $\eta = 0.2$ in all the simulations presented in the following. The length scale is taken to be $\sigma = 1.0$. The connections w_{ij} and v_{ij} are initialized with random values in the interval $[0; 0.001]$.

The memorization ability of the network is the degree to which the state of the network, given by i^* , depends on the past stimuli. A suitable measure of statistical dependence between the two stochastic variables is their mutual information [18]. Given a discrete set X of possible stimuli, the mutual information between the current centre of activity $i^*(t)$ and the stimulus

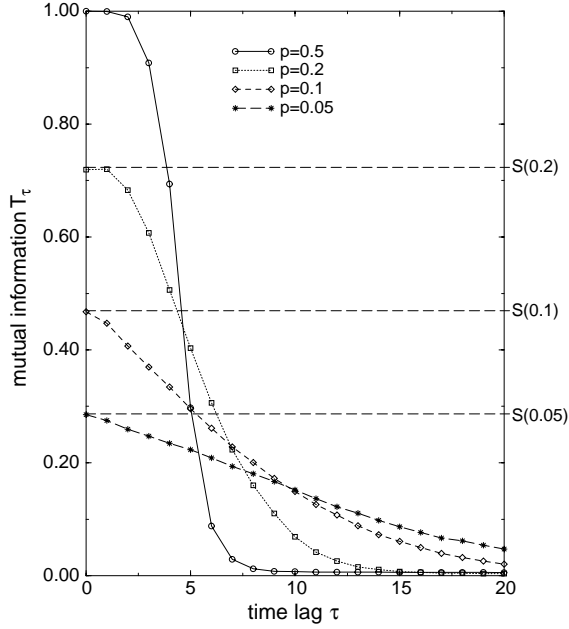


Fig. 3 – Adaptation to asymmetry in the occurrence of the stimuli, as the probability p of presenting the stimulus **1** deviates from 0.5. The network adapts to the value of p by varying the memory length. For $p = 0.5$ only a few time steps can be remembered with the available number of units. Lowering $p < 0.5$ decreases the information per time step in the stream of stimuli. Then longer memories are possible. Each plotted value is an average over 100 independent runs with networks of size M . The networks had learnt for 100000 time steps before mutual information was estimated.

$\mathbf{x}(t - \tau)$ presented τ time steps before reads

$$T_\tau = \sum_{i=1}^M \sum_{\mathbf{x}' \in X} p_\tau(i, \mathbf{x}') \log_2 \frac{p_\tau(i, \mathbf{x}')}{\Pr(i^* = i) \Pr(\mathbf{x} = \mathbf{x}')}, \quad (4)$$

where $p_\tau(i, \mathbf{x}') = \Pr(i^*(t) = i \wedge \mathbf{x}(t - \tau) = \mathbf{x}')$ is the joint distribution of the centre of activity and the past stimulus. When estimating the joint probability distribution p_τ and its marginals for a given network at a certain time, the dynamics is sampled over 5000 time steps with $\eta = 0$. Consequently, these time steps are not included in the learning time measured.

Let us now demonstrate the emergence of memory by simulations where the network is presented with a random time series. The considered networks have $M = 64$ units and $S = 2$ inputs. We present only two different orthogonal stimuli, $\mathbf{x} = (2, 0)$ and $\mathbf{x} = (0, 2)$. We use **0** and **1** as shorthand for the two stimulus vectors. At each time step one of the vectors **0** and **1** is drawn randomly with probability $p = 0.5$.

Figure 1 shows the time evolution of the mutual information T_τ . Originally, the state of the network depends only on the current stimulus. This means $T_0 > 0$, but $T_\tau = 0$ for all $\tau > 0$. After approximately 40 time steps the two stimuli are always discriminated by different network states ($T_0 = 1$). Before step $t = 200$ we observe the emergence of memory: $T_1 = 1$ indicates full discrimination between stimuli presented the previous time step. With further learning the memory length expands to more time steps, hence $T_2 > 0$, $T_3 > 0$ and so on. The maximum information content of the network is bounded by the number M of possible

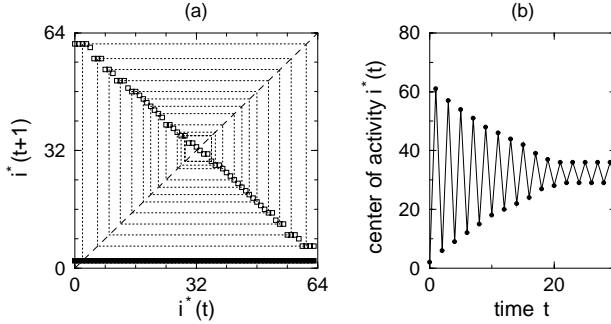


Fig. 4 – Typical simulation results for the case that one of the two stimuli occurs very seldom (here with probability $p = 0.05$). (a) Return map of i^* , after 10^6 learning steps. When the seldom stimulus $\mathbf{x}(t) = \mathbf{1}$ occurs, $i^*(t) = 2$ becomes the centre of activity (filled horizontal branch). Presentation of the other stimulus $\mathbf{x} = \mathbf{0}$ in the subsequent time steps leads to the iteration dynamics indicated by the dotted lines. (b) Corresponding time series of the centre of activity i^* . 20 time steps after the last presentation of stimulus $\mathbf{1}$ the dynamics reaches a two-cycle.

states (centres of activity). Thus the condition $\sum_{\tau=0}^{\infty} T_{\tau} \leq \log_2 M = 6$ causes a saturation in the formation of the memory.

More insight can be gained by considering the geometrical structure of the memory. In fig. 2 we have plotted the evolution of the return-map of the network dynamics for a typical simulation run configured as in the previous section. The diagrams are to be interpreted as follows: The abscissa is the centre of activity $i^*(t-1)$ in the previous time step. The ordinate is the subsequent centre of activity $i^*(t)$. Depending on the given stimulus $\mathbf{x}(t)$ either the filled or the unfilled squares represent the mapping $i^*(t-1) \rightarrow i^*(t)$. We observe that the two branches of the return map tend to become straight lines with slopes $1/2$ and $-1/2$, respectively. Panel (f) of fig. 2 shows an idealized version for the case of $M = 8$ units. The emerging return map f can be interpreted as the inverse of a tent map where the ambiguity of the two branches is resolved by the given stimulus,

$$i^* \mapsto f_{\mathbf{x}}(i^*) = \begin{cases} \lfloor i^*/2 \rfloor, & \text{if } \mathbf{x} = \mathbf{0}, \\ M-1 - \lfloor i^*/2 \rfloor, & \text{if } \mathbf{x} = \mathbf{1}. \end{cases} \quad (5)$$

By $\lfloor \cdot \rfloor$ we denote the integer part of the argument. In order to understand how the stimuli are stored in the network state, it is convenient to write the centre of activity as a binary number $i^* = \sum_{k=0}^{L-1} 2^k i_k =: (i_{L-1}, \dots, i_0)$, where $L = \log_2 M$ denotes the number of bits used. Writing also the stimulus \mathbf{x} as a binary value $x \in \{0, 1\}$, the return map eq. (5) reads

$$f_{\mathbf{x}}(i_{L-1}, \dots, i_0) = (x, i_{L-1} \oplus x, i_{L-2} \oplus x, \dots, i_1 \oplus x). \quad (6)$$

The operation \oplus is the exclusive-or ($a \oplus b = 0$, if $a = b$, otherwise $a \oplus b = 1$). Thus the operation f_0 shifts all bits of the argument to the right, discards the least significant bit and inserts x as the highest significant bit. f_1 additionally inverts all bits of the argument. Applying $f_{\mathbf{x}}$ iteratively $\tau \geq L$ times, we obtain

$$i^*(t) = f_{\mathbf{x}(t)} \circ f_{\mathbf{x}(t)} \circ \dots \circ f_{\mathbf{x}(t-\tau+1)}(i^*(t-\tau)) \quad (7)$$

$$= \left(x(t), x(t-1) \oplus x(t), x(t-2) \oplus x(t-1) \oplus x(t), \dots, \bigoplus_{s=0}^{L-1} x(t-s) \right). \quad (8)$$

Thus at any time t the values $x(t), x(t-1), \dots, x(t-L+1)$ of the L previous stimuli can be extracted from $i^*(t)$. Note that, due to the non-linear superposition of the stimuli by the exclusive-or, the memory effect in general cannot be observed when applying purely linear measures. In particular, the linear correlation function between $i^*(t)$ and $x(t-\tau)$ vanishes for $\tau > 0$. However, using the mutual information T_τ (eq. (4)) one detects the memorization of past stimuli in $i^*(t)$. For more than two discrete stimuli the emergence of memory is observed accordingly, forming a return map with more than two branches.

We now consider the case of asymmetry in the presentation of stimuli. We use the same two stimuli as in the preceding sections. Unlike before, we admit the probability p of presenting stimulus **1** to assume values different from the symmetric case $p = 0.5$. The amount of information per time step in the stream of stimuli is then given by the Shannon function $S(p) = p \log_2(p) + (1-p) \log_2(1-p)$.

Figure 3 shows the mutual information as a function of the time-lag τ for different values of p . For small τ the mutual information is close to $S(p)$ for all considered values of p . This means that in any case the network almost perfectly memorizes a few preceding time steps. However, varying p causes a redistribution of memory: As the parameter p decreases, the decay of T with growing τ becomes weaker: The smaller p , the “longer” the memory. Thus the neural network automatically adapts to the statistics of the stimuli.

Again we consider the emerging return map as done before in fig. 2 for the special case of $p = 0.5$. Lowering p reduces the number of units stimulus **1** is mapped to, thereby increasing the number of units stimulus **0** is mapped to. Comparing with fig. 2(e) the unfilled branch of the return map becomes steeper whereas the filled branch becomes flatter. For values of $p \leq 0.1$ typically a return map as shown in fig. 4(a) develops. Here one branch of the map is a constant (horizontal line) such that after presentation of the infrequent stimulus the centre of activity $i^*(t)$ does not depend on the previous one $i^*(t-1)$. As illustrated by fig. 4(b) the network state i^* passes a transient and reaches an attractor provided persistent presentation of the frequent stimulus. The network state is a measure of the time having passed since the last presentation of the infrequent stimulus.

In summary, we have formulated and examined a simple model of memory dynamics based on a few assumptions. We have shown that the dynamics based on these assumptions readily builds up a structure for systematic storage of recent stimuli. We have also demonstrated the adaptation of the memory in reaction to the information contained in the stimuli. Importantly, no correlations in the stream of stimuli are required for the structure to emerge. A neural network can learn a basic spatial representation of temporal information before temporally correlated information itself is available. Noise is enough in order to build up a memory.

REFERENCES

- [1] THORPE W. H. (Editor), *Learning and Instinct in Animals*, 2nd edition (Harvard University Press, Cambridge, Mass.) 1963.
- [2] KANDEL E. R., SCHWARTZ J. H. and JESSELL T. M. (Editors), *Principles of Neural Science*, 4th edition (McGraw-Hill) 2000.
- [3] HERTZ J., KROGH A. and PALMER R. G., *Introduction to the Theory of Neural Computation* (Addison-Wesley) 1991.
- [4] KOHONEN T., *Self-Organizing Maps*, 3rd edition (Springer) 2001.
- [5] HAMPSON R. E., SIMERAL J. D. and DEADWYLER S.A., *Nature*, **402** (1999) 610.
- [6] KOHONEN T. and HARI R., *Trends Neurosci.*, **22** (1999) 135.
- [7] UNNIKRISSNAN K. P., HOPFIELD J. J. and TANK D. W., *IEEE Trans. Signal Process.*, **39** (1991) 698.

- [8] HAYKINS S., *Neural Networks*, 2nd edition (Prentice Hall) 1999.
- [9] YUILLE A. L. and GEIGER D., *The Handbook of Brain Theory and Neural Networks*, edited by ARBIB M. A. (MIT Press, Cambridge, Mass.) 1995, p. 1056-1060.
- [10] HEBB D. O., *The Organization of Behavior* (Wiley, New York) 1949. Reprinted in parts in [11].
- [11] ANDERSON J. A. and ROSENFELD E. (Editors), *Neurocomputing: Foundations of Research* (MIT Press, Cambridge, Mass.) 1988.
- [12] BLISS T. V. P. and COLLINGRIDGE G. L., *Nature*, **361** (1993) 31
- [13] KANGAS J., *Proc. Internat. Joint Conf. Neural Networks*, **2** (1990) 331.
- [14] SOMERVUO P., *Proc. Internat. Joint Conf. Neural Networks*, **3** (1999) 1900.
- [15] KOSKELA T., VARSTA M., HEIKKONEN J. and KASKI K., *Int. J. Knowledge-based Intelligent Engineering Systems*, **2** (1998) 60.
- [16] LIU Q., RAY S., LEVINSON S., HUANG T. and HUANG J., *Proc. Internat. Joint Conf. Neural Networks*, **5** (1999) 2970.
- [17] EULIANO N. R., *Proc. IEEE International Conference on Neural Networks*, **2** (1998) 1063.
- [18] SHANNON C. E. and WEAVER W., *The Mathematical Theory of Information* (University of Illinois Press, Urbana) 1963.