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Dynamics of dilute disordered models:
A solvable case

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Abstract. – We study the dynamics of a dilute spherical model with two-body interactions and random exchanges. We analyze the Langevin equations and we introduce a functional variational method to study generic dilute disordered models. A crossover temperature replaces the dynamic transition of the fully connected limit. There are two asymptotic regimes, one determined by the central band of the spectral density of the interactions and a slower one determined by localized configurations on sites with high connectivity. We confront the behavior of this model to the one of real glasses.

Kirkpatrick, Thirumalai and Wolynes showed that a family of fully connected (FC) disordered spin systems realize schematically the phenomenology of the glass transition [1]. These models are very useful for several reasons: i) their statics and dynamics are solvable with a variety of techniques; ii) since their macroscopic dynamic equations are identical to those stemming from self-consistent approximations to more realistic interacting systems (*e.g.*, mode-coupling (MCA) [2]), they limit the range of validity of the latter; iii) their defects are clear (FC, infinite dimensions) and some improvements needed in a better description of real systems can be identified. The FC p spin disordered models are members of this group. Models of finite-dimensional manifolds embedded in infinite dimensions under quenched random potentials are generalizations that also yield real space information [3].

The main features of the FC spherical p spin model, on which we focus for concreteness, are the following. i) It has a dynamic transition at $T_{\rm d} < +\infty$. Above $T_{\rm d}$, an infinite system ($N \to \infty$) equilibrates with its environment [1]. Below $T_{\rm d}$, the equilibration time diverges with N [4]. Right above $T_{\rm d}$, the dynamics coincides [1] with the schematic MCA to super-cooled liquids [2] and correlations decay in two steps. Below $T_{\rm d}$, for any waiting time $t_{\rm w}$ two-time functions decay to zero after a sufficiently long subsequent time, with aging effects [4]. ii) It has a rich structure of metastable (TAP) states that are stationary points of a free-energy

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density f function of the relevant order parameters ($m_i \equiv \langle s_i \rangle$ in this case) [5, 6]. Within $(f_{\min}, f_{\mathrm{th}}]$, the number of metastable states, $\mathcal{N}(f)$, is exponential in N leading to a finite complexity $\Sigma(f) \equiv N^{-1} \ln \mathcal{N}(f)$ ($f_{\min} < f_{\mathrm{th}}$ when $p \geq 3$). Strictly below f_{th} minima are exponentially dominant with respect to saddles. Typically, the states on the threshold (TH) are marginal [5] and attract the dynamics when times diverge after $N \to \infty$ [4, 7]. iii) At T_{d} the liquid state ($m_i = 0 \, \forall i$) is fractured into an exponential in N number of TAP states with $m_i \neq 0$. At T_{s} the partition sum is dominated by the lowest states with $f_{\min} = f_{\mathrm{eq}}$ and, since $\Sigma(f_{\min}) = 0$, there is an entropy crisis associated to the Kauzmann paradox [1]. iv) The barriers between TAP states are expected to be O(N), leading to a separation in the time scales to descend below the threshold.

In finite-dimensional systems with finite-range interactions, however, the dynamic transition is only a crossover at T_{g} and it is not clear if the static transition exists at all. The dynamics must penetrate below the threshold to slowly descend towards equilibrium. Within this picture two distinct regimes would appear in the low-T isothermal dynamics of real systems: a FC-like one when the system approaches a pseudo-threshold of flat directions in phase space and a slower activated regime in which the system jumps over barriers to relax its excess energy density and very slowly progress towards equilibrium [3]. This view explains the cooling rate effects since crossing T_{g} with a slower rate allows to penetrate deeper below the threshold. How and if the aging properties in the first and second regime resemble is a very interesting open problem. One could attack it analytically by studying the dynamics of p spin models in time scales that diverge with N though this is very hard. Simulations of finite-size FC models [8] and Lennard-Jones mixtures [9, 10] support the existence of the two aging regimes.

In this letter we study an alternative model that realizes explicitly two nonequilibrium dynamic regimes, a FC-like one and a slower one that we relate to transitions between metastable states with *finite* lifetime. The merits of the model are manifold. Its dynamics is tractable analytically. Being defined on a random graph, the time-scale–dependent metastable states are determined by the "real space geometry": they are localized configurations on sites with high connectivity. The model has also some drawbacks that we discuss after presenting its solution. Still, we expect to use it as a starting point for the study of more realistic cases.

We solve the dynamics of the dilute spherical model:

$$H_J[\vec{s}] = -\frac{1}{2} \sum_{i \neq j} J_{ij} s_i s_j, \tag{1}$$

where the spins are continuous unbounded variables, spherically constrained $\sum_{i=1}^{N} s_i^2 = N$. The couplings J_{ij} are quenched random variables, independently and identically distributed with

$$
\begin{aligned}
P(J_{ij}) &= \left(1 - p/N\right) \delta\left(J_{ij}\right) + p/N \pi\left(J_{ij}\right), \\
\pi\left(J_{ij}\right) &= \frac{1}{2} \left[\delta\left(J_{ij} - 1/\sqrt{p}\right) + \delta\left(J_{ij} + 1/\sqrt{p}\right) \right],
\end{aligned} \tag{2}
$$

that satisfy $J_{ii} = 0$ and $J_{ij} = J_{ji}$. When p is finite and N goes to infinity, this distribution law generates a random graph, so that the connectivity of a given site is distributed according to a Poisson law with average p. Contrary to the FC case, *i.e.* the complete graph obtained in the $p = N \to \infty$ limit, each spin interacts with a finite number of "neighbors", as in a finite-dimensional system. Yet, the "neighbors" are chosen randomly among the other spins and there is no geometrical distance. The model is of mean-field type. In all the following we take $p \gg 1$ but finite with respects to N. The spins evolve with the Langevin equation

$$\dot{s}_i(t) = -\delta H_J[\vec{s}]/\delta s_i(t) - z(t) s_i(t) + \xi_i(t), \tag{3}$$

where the Lagrange multiplier $z(t)$ enforces the spherical constraint and $\xi_i(t)$ is a white noise with zero mean and variance $\langle \xi_i(t)\xi_j(t_{\rm w})\rangle = 2T\delta_{ij}\delta(t - t_{\rm w})$. ($k_{\rm B} = 1$ and we rescaled time to absorb the friction coefficient.)

We solve the model in two ways. By rotating the spin $\vec{s} = (s_1, \ldots, s_N)$ onto the basis of eigenvectors of the random matrix J_{ij}, the Langevin equations of any continuous quadratic model decouple and can be completely solved [11,12]. Many interesting dynamic observables depend only on the density of states $\rho(\mu)$ of the interaction matrix J_{ij}. We estimate their scaling in different regimes analytically and we confront them to the numerical solution of the dynamical equations where we approximate $\rho(\mu)$ with eq. (29) in [13]. Then we introduce a more general method that can also be applied to nonquadratic models and/or soft spins. It is a dynamic generalization of the iterative method used to find $\rho(\mu)$ for dilute random matrices [14].

For future reference, let us summarize the main properties of $\rho(\mu)$ [13, 15]. It has a symmetric central band in $[-\lambda_{\rm c}(p), \lambda_{\rm c}(p)]$, a crossover extending beyond $|\lambda_{\rm c}(p)|$ that is not known in detail, and two tails that vanish as $\rho(\mu) \sim \exp[-p\mu^2 \ln \mu^2]$ when $\mu \to \pm\infty$. The tails are due to large fluctuations of the local connectivity. For $k \gg 2p$, a site with k neighbors gives rise to an eigenvalue $\mu \sim \sqrt{k/p}$ with a localized eigenvector $\vec{v}_\mu$ on it. When $p \to \infty$, $\lambda_{\rm c}(p) \to 2$ and the tails disappear.

At time t after preparation in the initial condition $s_\mu(0)$ the rotated spin $s_\mu(t) = \vec{v}_\mu \cdot \vec{s}(t)$ is given by [12]

$$s_\mu(t)\sqrt{\Gamma(t)} = s_\mu(0)e^{\mu t} + \int_0^t {\rm d}t' e^{\mu(t-t')}\sqrt{\Gamma(t')}\xi_\mu(t').$$

$\Gamma(t) \equiv \exp[2\int_0^t {\rm d}t' z(t')]$ is related to the energy density, $\epsilon(t) = T/2 - d_t \ln\Gamma(t)/4$, and it determines the decay of the self-correlation $NC(t, t_{\rm w}) = \sum_{i=1}^N [\langle s_i(t)s_i(t_{\rm w})\rangle]_J$ and linear self-response $NR(t, t_{\rm w}) = \sum_{i=1}^N \delta[\langle s_i(t)\rangle]_J/\delta h_i(t_{\rm w})|_{h=0}$. We choose the initial condition $s_\mu(0) = \pm 1 \ \forall\mu$ that mimics an instantaneous quench from $T = +\infty$ at $t = 0$. Imposing the spherical constraint $C(t, t) = \int {\rm d}\mu\rho(\mu)\langle s_\mu^2(t)\rangle = 1$ implies

$$\Gamma(t) = f(t) + 2T\int_0^t {\rm d}t' f(t - t')\Gamma(t'), \tag{4}$$

with $f(t) \equiv \int {\rm d}\mu\rho(\mu)e^{2\mu t}$. Asymptotically, $f(t)$ can be estimated with a saddle point evaluation of the integral. When it is dominated by $\mu \sim \lambda_{\rm c}(p)$, as in the FC limit, $f(t) \sim \exp[2\lambda_{\rm c}(p)t]/t^{3/2}$. After a crossover time $t_{\rm co}^0(p)$ the integral is dominated by μ's in the crossover in $\rho(\mu)$ and $f(t)$ grows faster than an exponential, though we cannot determine its functional form. Finally, when times are so long as to explore the tails of $\rho(\mu)$, $f(t) \sim \exp[t^2/(2p\ln t)]$.

We thus have a first crossover time $t_{\rm co}^0(p)$, where the behavior of f changes from exponential (with polynomial corrections) to faster than exponential. The more interesting function $\Gamma(t)$ is determined from $f(t)$ by eq. (4). Note that at $T = 0$, these two functions coincide. When T is raised to a positive value, $\Gamma(t)$ has still a crossover time $t_{\rm co}(T, p)$ beyond which it grows faster than exponential. The important feature which comes out of eq. (4) is that $t_{\rm co}(T, p)$ remains close to $t_{\rm co}^0(p)$, its zero-temperature value, until a crossover temperature $T_0(p)$ is reached. For higher temperatures $t_{\rm co}(T, p)$ grows with T. These time-temperature regimes are sketched in the left panel of fig. 1. More quantitatively, this behavior amounts to

– when $T < T_0(p)$: $\Gamma(t) \sim f(t)/(1 - T/T_0(p))^2$ for $1 \ll t < t_{\rm co}^0(p)$ and $\Gamma(t) \sim f(t)$ for $t \gg t_{\rm co}^0(p)$;

– when $T > T_0(p)$: $\Gamma(t) \sim \exp[b(T)t]$ for $1 \ll t < t_{\rm co}(T, p)$ and $\Gamma(t) \sim f(t)$ for $t \gg t_{\rm co}(T, p)$ with $t_{\rm co}(T, p)$ a growing function of T.
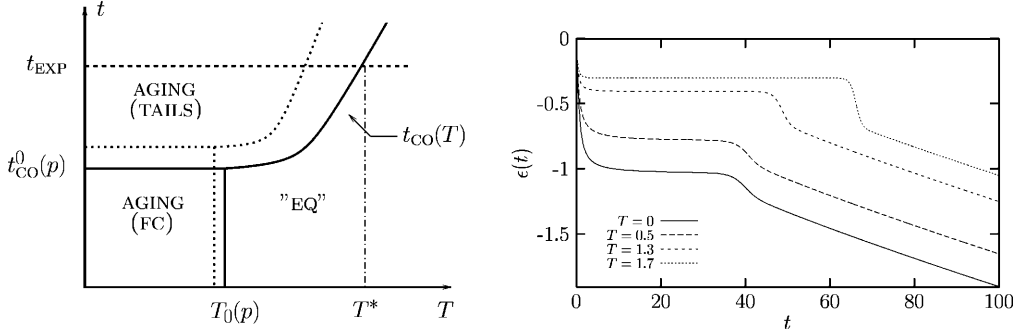
Fig. 1 – Left: the crossover time as a function of temperature. Solid line $p = p_1$, dashed line $p = p_2$; $p_1 < p_2$. Right: decay of the energy-density; $p = 10$. From bottom to top: $T = 0$, $T = 0.5$, $T = 1.3$ and $T = 1.7$.

$T_0(p)$ is the trace of the phase transition in the FC limit ($T_0(\infty) = 1$). It separates a low-T regime in which the model has a FC-like out-of-equilibrium decay for long time scales and a much slower one for still later epochs, from a high-T regime where the system crosses over from an equilibrium-like behavior at long time scales to the tail-dominated very slow out-of-equilibrium decay at much longer time scales. For all T, the system eventually reaches the tail-dominated regime but the higher the temperature ($> T_0(p)$) the longer the equilibrium-like regime lasts.

The time scales just described can also be visualized from the decay of $\epsilon(t)$ displayed in the right panel in fig. 1. For the two lower curves $T < T_0(p) \sim 1.1$ and $t^0_{\text{co}}(p) \sim 35$. The plateau occurs at $[T - \lambda_c(p)]/2$ as in the FC limit. For the two upper curves $T > T_0(p)$ and $t_{\text{co}}(T, p)$ increases with T. For all T, when $t \gg t_{\text{co}}(T, p)$, we have $\epsilon(t) \sim -t/(4p \ln t)$ meaning that the energy density diverges asymptotically, a result that is consistent with an unbounded $\rho(\mu)$. The dynamics in the tail-dominated regime is independent of T, so not activated.

We explain in this paragraph how these results can be viewed as a sketchy picture of a glass "transition", see the left panel of fig. 1. Imagine that experimentally one has access to times that are shorter than t_{exp}, choose a value of p such that $t^0_{\text{co}}(p) < t_{\text{exp}}$ and define T^* by $t_{\text{exp}} = t_{\text{co}}(T^*, p)$. T^* thus depends on the experimental time scale available, and is larger than $T_0(p)$. When $T < T_0(p)$ and times are shorter than $t^0_{\text{co}}(p)$ the system ages as in the FC limit, while beyond $t^0_{\text{co}}(p)$ aging is of a different kind, dominated by the tails of the distribution. When $T > T^*$, all accessible times are shorter than $t_{\text{co}}(T, p)$ and the dynamics is stationary as in the high-T phase of the FC limit. (Note, however, that these results are not accessible with a naive static calculation since ϵ is not bounded from below.) Finally, there is an intermediate temperature regime, $T_0(p) < T < T^*$, in which the dynamics crosses over from stationary to tail dominated. The width of this "unnatural" regime can be shrinked by using larger p's since $T_0(p)$ very weakly decreases with p, while $t^0_{\text{co}}(p)$ grows with p, cf. the two curves in the left panel of fig. 1. One concludes that the dynamics behaves similarly to what expected in real glasses in the sense that within the experimentally limited time window there are two aging regimes when $T < T_0(p)$ and a single stationary regime at high T.

From now on, in order to shorten the notation, we call T_0 the crossover temperature and t_{co} the crossover time at the considered T and p, i.e. $t_{\text{co}} = t^0_{\text{co}}(p)$ if $T < T_0(p)$ and $t_{\text{co}} = t_{\text{co}}(T, p)$ if $T > T_0(p)$. When $p \to \infty$, t_{co} moves to infinity for all T, $\rho(\mu)$ approaches the semi-circle law and $\Gamma(t)$ takes the FC form.

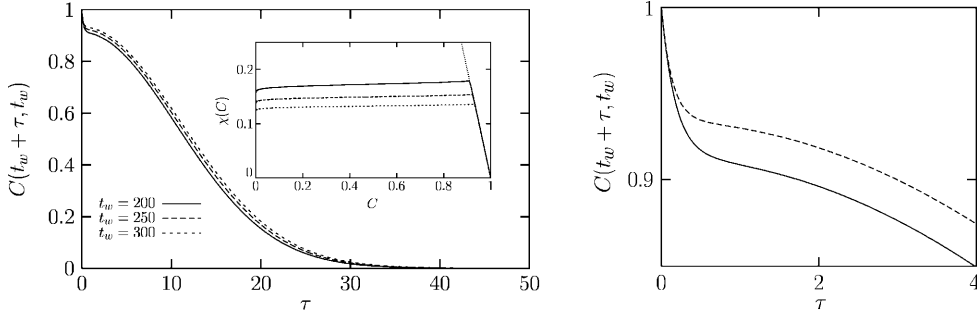The evolution in the FC limit was easy to grasp by following $\langle s_\mu(t) \rangle$, where the low-T

Fig. 2 – Left: the self-correlator in the tail-dominated regime for $t_{\rm w} = 200$ (solid) and 300 (dashed) at $T = 0.5$; in the inset $\chi(C)$ for the same $t_{\rm w}$'s, compared to FDT (dotted). Right: zoom over the initial decay of C.

nonequilibrium dynamics corresponds to an incomplete condensation on $\mu = 2$ [12]. Let us focus on $T < T_0$ and analyze $\langle s_\mu(t) \rangle$ in the dilute case. When t is in the FC-like regime, the projections on all eigenvectors with $\mu < \lambda_{\rm c}(p)$ are exponentially suppressed, the one with $\mu = \lambda_{\rm c}(p)$ increases as a power law in time and the ones with $\mu > \lambda_{\rm c}(p)$ increase exponentially in time. When t enters the tail-dominated regime, the cut-off $\mu_{\rm c}(t)$ which separates the eigenvectors that are exponentially suppressed ($\mu < \mu_{\rm c}(t)$) from those that increase exponentially ($\mu > \mu_{\rm c}(t)$) moves away from $\lambda_{\rm c}(p)$ and eventually grows as $\mu_{\rm c}(t) \sim t/(4p \ln t)$. In real space this means that localized states on all sites with connectivities ($k > p\mu_{\rm c}^2$) increase exponentially for times $< t(\mu_{\rm c})$, while all others are exponentially suppressed. At longer times sites with higher connectivities dominate.

Richer information about the nonequilibrium evolution is contained in the self-correlation, C, and linear self-response R [3,4]. An expression for C in terms of Γ that is also valid for the dilute model is given by eq. (2.12) in [12]. We identify several scalings. When t and $t_{\rm w}$ are long but shorter than $t_{\rm co}$, the dynamics resembles the one for the FC model. For $T > T_0$, the decay is stationary. For $T < T_0$ instead there is a first stationary decay towards an Edwards-Anderson plateau $q_{\rm EA} = 1 - T/T_0$ when $\tau \equiv t - t_{\rm w} \ll t_{\rm w}$ and a subsequent aging decay from $q_{\rm EA}$ towards zero with simple aging scaling, $C(t, t_{\rm w}) \sim \mathcal{C}(t_{\rm w}/t)$, when $t_{\rm w} \propto t$. Instead, when $t_{\rm w}$ falls in the tail-dominated regime $\Gamma(t) \sim f(t)$ and

$$C(t, t_{\rm w}) \longrightarrow \exp\left[-\frac{\tau^2}{8p \ln t_{\rm w}} \right] \left[1 - 2T \int_0^{\frac{\tau}{2}} {\rm d}y \exp\left[-\frac{y t_{\rm w}}{p \ln t_{\rm w}} \right] f(y) \right], \qquad (5)$$

when $t_{\rm w} \to \infty$ and $\tau \ll \ln t_{\rm w}$. At $T = 0$, we immediately identify the effective time $\tau_{\rm eff} \sim \sqrt{\ln t_{\rm w}}$ and a very slow subaging decay [16]. One can recast the correlation in the more familiar form

$$C(t, t_{\rm w}) = \exp\left[-\frac{\ln^2 x}{8} \right], \quad x = \frac{h(t)}{h(t_{\rm w})}, \quad h(t) = \exp\left[\frac{t}{\sqrt{p \ln t}} \right]. \qquad (6)$$

If $\tau \ll \sqrt{\ln t_{\rm w}/t_{\rm w}}$, we get $C \sim 1 - T\tau$. The duration of the $t_{\rm w}$-independent part of the decay decreases with $t_{\rm w}$ and eventually disappears. Thus, the finite-T corrections vanish for long $t_{\rm w}$ and the scaling (6) holds for all T. These features are shown in fig. 2.

The linear self-response is given by $R(t, t_{\rm w}) = \sqrt{\Gamma(t_{\rm w})/\Gamma(t)} f(\tau/2)$. When $t < t_{\rm co}$, the FC scaling holds. When $t_{\rm w} \gg t_{\rm co}$ and for $\tau \propto \sqrt{\ln t_{\rm w}}$ when C varies, $R(t, t_{\rm w}) =$

$\exp[-\tau t_{\mathrm{w}}/(2p\ln t_{\mathrm{w}})]$. R decays much faster than C, it approaches zero on the scale $\tau \sim \ln t_{\mathrm{w}}/t_{\mathrm{w}}$. An equivalent expression for this result is $R(t, t_{\mathrm{w}}) \sim \exp[(-\ln x)t_{\mathrm{w}}/\sqrt{p\ln t_{\mathrm{w}}}]$ with x given in eq. (6).

The modification of the fluctuation-dissipation theorem (FDT) is different in the two non-equilibrium regimes. In the FC-like scale the results are identical to those in [12] and the effective temperature [17] $T_{\mathrm{eff}} \equiv \partial_{t_{\mathrm{w}}}C/R$ diverges as $\sqrt{t_{\mathrm{w}}}$. In the tail-dominated scale T_{eff} diverges much faster, as $T_{\mathrm{eff}}[C] \sim \exp[a(C)t_{\mathrm{w}}/\sqrt{p\ln t_{\mathrm{w}}}]$ with $a(C) = \sqrt{(-2\ln C)}$. The inset in fig. 2 shows the parametric plot of the integrated response χ against C [3] when t_{w} is in the latter regime.

We solved (3) in a rather simple way thanks to the quadratic nature of the model. Unfortunately, this calculation cannot be adapted to treat cases with higher-order interactions as are models with soft spins. With the purpose of developing a more generic approach to the dynamics of dilute models, we introduce a method to derive macroscopic equations for C and R from the dynamic generating functional Z_{d}. We only sketch the method here, the details shall be given in a forthcoming publication [18]. The idea is to get benefit from the formal analogy between the static replica free energy and the dynamical action expressed with a supersymmetric (SUSY) formalism. Using the SUSY formulation of stochastic processes [19] the two-point functions are encoded in a correlator $NQ(a, b) = \sum_{i=1}^{N}[\langle\Phi_i(a)\Phi_i(b)\rangle]_J$ between two superfields $\Phi_i(a)$. a and b include time and two Grassmann coordinates $\theta, \overline{\theta}$. We define the fraction of sites with super-field Φ_i identical to a chosen value Φ for all coordinates, in analogy with the replica order parameter for dilute systems [20],

$$c[\Phi] \equiv N^{-1}\sum_{i=1}^{N}\prod_a \delta\big(\Phi(a) - \Phi_i(a)\big). \tag{7}$$

After introducing it in Z_{d} and averaging over disorder [18], we obtain the saddle point equation

$$c_{\mathrm{sp}}(\Phi) = \lambda \exp\left[-\frac{1}{2}\int \mathrm{d}a\Phi(a)\Big(- D_a^{(2)} + z(a)\Big)\Phi(a) - \frac{\delta H_{\mathrm{eff}}}{\delta c(\Phi)}\bigg|_{c_{\mathrm{sp}}} \right] \tag{8}$$

with λ a normalization constant, $D_a^{(2)}$ a differential operator, $\int \mathrm{d}a$ the integration over time and Grassmann coordinates, and

$$\exp\big[- NH_{\mathrm{eff}}\big] \;=\; \left[\exp\left[-\int \mathrm{d}a H_J[\Phi_i(a)] \right] \right]_J.$$

This equation determines $c[\Phi]$ for *any* model for which the saddle point evaluation is exact. (For models without spherical constraint, one removes the Lagrange multiplier $z(a)$.) It plays the role of eq. (20) in [13] for the replica order parameter $c_{\mathrm{sp}}(\vec{\phi})$ introduced to compute $\rho(\mu)$. It cannot be solved without assumptions, yet an iterative resolution similar to that of [13,14] taking into account more and more precisely the fluctuations in the local geometry of the interactions can be implemented. For the present model one recovers the same results obtained in this letter [18]. We expect that this approach will also be successful for non-quadratic models for which an exact resolution is not possible.

In short, we showed that the dilute spherical spin-glass realizes explicitly part of the scenario expected to hold beyond the "mean-field picture" of glassy dynamics. The main ingredient of the model is the unbounded distribution of local connectivities, regardless of the sign of the interactions. In the glassy context this model has, however, some flaws. Its equilibrium energy density is not bounded and, strictly speaking, it only has a low-T phase.

The dynamics in late epochs does not depend on T and when T is close to but above T_0 the stationary correlations do not decay in two steps. Nevertheless, the existence of a dynamic crossover around T_0 as opposed to a phase transition and the presence of two nonequilibrium regimes are expected features for real glasses.

Dilute magnets are important in several other contexts. They are prototypical models with Griffiths phases [21] and heterogeneous dynamics [22]. They are also related to several optimization problems, and their dynamics to that of optimization algorithms [23]. We shall report on studies of the dynamics of other dilute models, hopefully free of the defects mentioned above, in the future.

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