A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

Selected elevation in quantum tunnelling

EUROPHYSICS LETTERS 15 March 2003

Europhys. Lett., **61** (6), pp. 817–823 (2003)

Selected elevation in quantum tunnelling

E. Granot(*)
Department of Electrical Engineering, College of Judea and Samaria
Ariel 44837, Israel

(received 13 September 2002; accepted in final form 7 January 2003)

PACS. 73.40.Gk – Tunneling.
PACS. 66.35.+a – Quantum tunneling of defects.

Abstract. – The tunnelling through an opaque barrier with a *strong* oscillating component is investigated. It is shown that in the non-weak perturbations regime (in contrast to the weak one), higher perturbations rate do not necessarily improve the activation. In fact, in this regime two rival factors play a role, and, as a consequence, this tunnelling system behaves like a sensitive frequency-shifter device: for most incident particles' energies activation occurs and the particles are *energetically elevated*, while for specific energies activation is depressed and the transmission is very low. This effect is unique to the *strong* perturbation regime, and it is *totally absent* in the weak-perturbation case. Moreover, it cannot be deduced even in the adiabatic regime. It is conjectured that this mechanism can be used as a frequency-dependent transistor, in which the device's transmission is governed by the external field *frequency*.

When a quantum particle propagates through an opaque barrier in the classically forbidden domain, it tunnels. This conduct is manifested by the exponentially small transmission probability. Nevertheless, when some part of the barrier weakly oscillates, the particle will be activated to higher energies, and the transmission will increase substantially [1–4]. However, when the temporal change is not merely a perturbation but rather a strong variation, the dynamics are too complicated to be described by such a simple rule of thumb, and most of the interesting phenomena belong to this category.

While it is well known that even a very weak external oscillating field may considerably increase the tunnelling current [3, 4], changes which are comparable to the initial system parameters can cause elevator resonant activation and activation assisted tunnelling [5, 6], charge quantization pumping [7], coherent destruction of tunnelling (CDT) [8, 9] as well as wave function collapse [10]. That is, large changes reveal a wealth of physical phenomena.

When the tunnelling particle energy (Ω) is close to the potential barrier height (V), the dynamics becomes more complicated since the perturbation amplitude (ΔV) can exceed $V-\Omega$, *i.e.*, the perturbation "strength" may be larger than the effective tunnelling barrier. This case is extremely sensitive, since the dynamics is governed by two rival factors. On the one hand, the oscillation's amplitude is so large that for a finite segment of the oscillation period the alternating potential blocks the particles' transmission. Hence, following this reasoning, the tunnelling rate should decrease. On the other hand, energy quanta generated by the oscillating barrier can be absorbed by the tunnelling particle to assist it in the due course of tunnelling.

(*) E-mail: scipio@inter.net.il

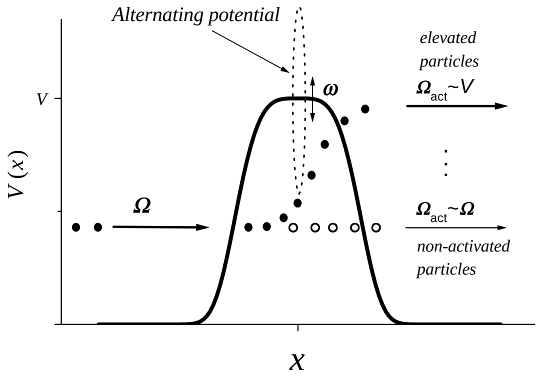

Fig. 1 – An illustration of the system: The incident particles' energy Ω (and the oscillating frequency ω) determines whether the incident particles will be elevated to higher-energy states (dark circles, most cases) or will remain in their initial state (open circles, specific cases).

In this work, we investigate the interplay between these two rival factors, and show that the competition between them is responsible for the high sensitivity to the system's parameters. For example, it so happens that the incident particles are not always activated to higher energy: when their initial energy is equal to one of a series of specific energies the activation is frustrated, elevation to higher energies does not occur and the transmission is decreased accordingly.

The problem of tunnelling in the presence of an external time-dependent potential has been extensively studied [1–10]. However, in this paper the strong perturbation regime is addressed and no approximate assumptions are made in the solution analysis. We give this case both an *exact numerical* solution, and an *analytical* solution, using an approximation, to show that the general conclusions can be deduced even in the slowly varying regime. However, we also show that the oscillations frequency cannot be arbitrarily small, and in fact it should be larger than the spectral bandwidth of the resonances, otherwise (and, in particular, the adiabatic regime) the effect vanishes.

A few words should be added here about the plausibility of the effect's practical implementation. The industry has a special interest in resonant tunnelling devices (RTD), which allow miniaturizing electronic circuits and improving their performance. Ordinary RTD are very sensitive to manufacturing processes, temperature, and impurities, and, as a consequence, a reliable device is highly costly. The presented tunnelling device allows for the fabrication of a low-cost device where any resonant refinement can be done by variations in the external field *frequency*, and no special geometry or manufacturing restrictions are needed.

In this paper we discuss the tunnelling dynamics of a beam of particles which are activated by strong harmonic perturbations. The tunnelling takes place through a very opaque potential barrier (high and wide). In order to have the barrier opaque at all times, we follow [5] and discuss the extreme case of an oscillating point potential $-\beta\delta(x)\cos(\omega t+\eta)$ (see fig. 1).

In terms of the Schrödinger equation, the dynamics can be expressed by

$$-\psi'' - \beta\delta(x)\cos(\omega t+\eta)\psi + V(x)\psi = i\dot{\psi}, \tag{1}$$

where we adopt the units $2m=1$, $\hbar=1$, and the notations $\psi'' \equiv \partial^2\psi/\partial x^2$, $\dot{\psi} \equiv \partial\psi/\partial t$, and $V(x)$ is the potential barrier which vanishes quickly for $|x| \to \infty$.

The time-dependent solution can be written as a discrete Fourier transform:

$$\psi(x<0) = \varphi_\Omega^+ e^{-i\Omega t} + \sum_n r_n \varphi_{\Omega+n\omega}^- e^{-i(\Omega+n\omega)t}, \tag{2}$$

$$\psi(x>0) = \sum_n t_n \varphi_{\Omega+n\omega}^+ e^{-i(\Omega+n\omega)t}, \tag{3}$$

where $\varphi_\omega^\pm$ are the solutions of the stationary state Schrödinger equation, which does not include the oscillating term, *i.e.*

$$-\varphi_E^{\pm\prime\prime} + [V(x) - E]\varphi_E^\pm = 0. \tag{4}$$

The homogeneous solutions φ_E^+ describe waves that propagate from $-\infty$ to $+\infty$, while φ_E^- describes the waves that are incoming from $+\infty$ and outgoing to $-\infty$. Thus, $\varphi_E^+ \to \tau_E e^{i\sqrt{E}x-iEt}$ (for $x\to\infty$), while $\varphi_E^- \to \tau_E e^{-i\sqrt{E}x-iEt}$ (for $x\to-\infty$), and $|\tau_E|^2$ is the probability of penetrating the barrier.

By taking care of the matching conditions of the solutions in eqs. (2) and (3) at $x=0$, we easily obtain

$$2s_n\chi_n + \beta(s_{n-1}+s_{n+1}) = 2\chi_0\varphi_0^+\delta_{n0}, \tag{5}$$

when using the following notations:

$$\chi_n \equiv \frac{\varphi_n^{+\prime}}{\varphi_n^+} - \frac{\varphi_n^{-\prime}}{\varphi_n^-} \tag{6}$$

and

$$s_n \equiv e^{i\eta n}\varphi_n^+ t_n, \tag{7}$$

where $\varphi_n^\pm \equiv \varphi_{\Omega+n\omega}^\pm(x=0)$, $\varphi_n^{\pm\prime} \equiv \partial\varphi_{\Omega+n\omega}^\pm/\partial x|_{x=0}$, δ_{n0} is the Kronecker delta and $t_n \equiv t_{\Omega+n\omega}$.

This difference equation can readily be solved. Notice that

$$\chi_n = 1/g_n(0), \tag{8}$$

where $g_n(x)$ is the Green function of the equation $-\psi'' + V(x)\psi = (\Omega+n\omega)\psi$.

Thus, in the case of a perfectly symmetric rectangular barrier, χ_n comes directly from [11]

$$g_n(0) = -\coth\left[\rho_n L + i\arctan(k_n/\rho_n)\right]/2\rho_n, \tag{9}$$

where $k_n \equiv \sqrt{\Omega+n\omega}$, $\rho_n \equiv \sqrt{V-k_n^2}$, $2L$ is the barrier length and finally V is its potential height.

In fig. 2 we present the exact numerical solution of eq. (5) in the case of a rectangular barrier (in the figure the absolute value of s_n is presented), which is the spectral solution of eq. (1) at $x=0$. This figure illustrates the solution's sensitivity to the incoming particles' energy: a 2% change in Ω causes a severe reduction in activation.

To formulate an analytical expression for this solution, we take advantage of the fact that the perturbations are strong, *i.e.*, we can assume that $\beta^2 \gg V-\Omega \gg \omega$ (notice that in fig. 2 $\beta^2 \simeq V-\Omega$). Moreover, although in the numerical analysis we used the exact form of the Green function (eq. (9)), in the case of an opaque barrier, the approximation $g_n(0) \simeq -(2\rho_n)^{-1}$ may be used with great accuracy.

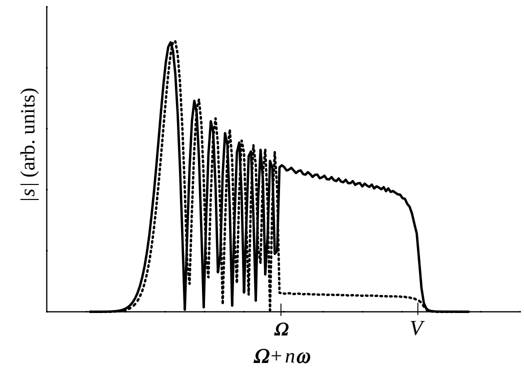

Fig. 2 – A plot of the transmission coefficient $|s_n|$ (defined in eq. (7)) as a function of the transmitted particles frequency (the activated energy) $\Omega + n\omega$. The solid line represents the case $\Omega/V = 0.625$ and the dotted one represents the case $\Omega/V = 0.6125$. The other parameters are $\omega/V = 0.0075$, $\sqrt{V}L = 10.75$ and $\beta L = 9.35$.

In this strong perturbation amplitude and low-frequency regime, the difference equation (eq. (5)) can be transformed to a differential equation. By using the definitions

$$G(n) \equiv \beta s_n / \chi_0 \varphi_0^+ \tag{10}$$

and $c(n) \equiv 1 + \chi_n/\beta$, one easily obtains the differential equation

$$\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}n^2} G(n) + c(n) G(n) = \delta(n), \tag{11}$$

where $\delta(n)$ is the Dirac delta-function.

Hence, one can regard $G(n)$ as having a Green function properties (including the singularity at $n = 0$).

When the Green function $G(n)$ is known, the transmitted solution of eq. (1) follows directly from eqs. (3), (7) and (10):

$$\psi\,(x \geq 0) = \frac{\chi_0 \varphi_0^+}{\beta} e^{-i\Omega t} \int \mathrm{d}n G(n) \frac{\varphi^+_{\Omega+n\omega}(x)}{\varphi^+_{\Omega+n\omega}(0)} e^{-in(\omega t + \eta)}. \tag{12}$$

In particular, the scattered wave function at $x = 0$ is proportional to the Fourier transform of the Green function.

Since $V - \Omega \gg \omega$, and $\beta \gg \rho$ one can approximate

$$c \simeq 1 + \omega n/(\beta\rho), \tag{13}$$

where $\rho \equiv \sqrt{V - \Omega}$.

Then, we can define for convenience the variable

$$\xi \equiv \left(n + \rho\frac{\beta}{\omega}\right)\left(\frac{\omega}{2\beta\rho}\right)^{1/3} \tag{14}$$

and the Green function is then

$$G(\xi) = \; -i\pi\left(\frac{2\beta\rho}{\omega}\right)^{1/3} \begin{cases} \mathrm{Ai}(-\xi)\left[\,\mathrm{Ai}(-\xi_0) + i\,\mathrm{Bi}(-\xi_0)\right], & \text{for } \xi < \xi_0, \\ \mathrm{Ai}(-\xi_0)\left[\,\mathrm{Ai}(-\xi) + i\,\mathrm{Bi}(-\xi)\right], & \text{for } \xi > \xi_0, \end{cases} \tag{15}$$

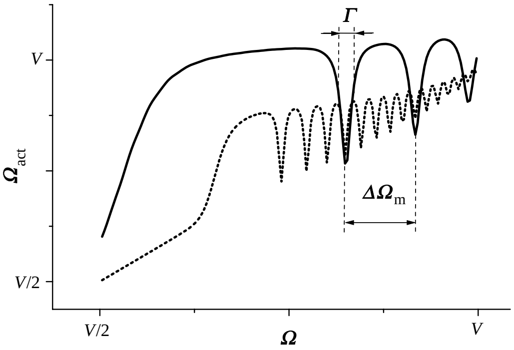

Fig. 3 – Two characteristic plots (for arbitrary parameters) of the mean activation energy ($\Omega_{\rm act}$) as a function of the incident particles' energy Ω (see eq. (18)). The two curves represent the same system except for ω, which is five times smaller in the dotted curve.

where $\xi_0 \equiv \xi\,(n=0) = (\beta\rho/\omega\sqrt{2})^{2/3}$ and Ai and Bi are the Airy functions of the first and second kind, respectively.

For frequencies which are lower than Ω (negative n's) the *amplitude* oscillates like a simple Airy function (see fig. 2 for a typical plot of $|s_n|$, which is related to the Green function by eq. (10)):

$$|G(\xi)|^2 \simeq \pi(2\beta\rho/\omega)^{2/3}\xi_0^{-1/2}\,\mathrm{Ai}^2(-\xi) \quad \text{for } n<0, \tag{16}$$

since $|\,\mathrm{Ai}(-\xi_0) + i\,\mathrm{Bi}(-\xi_0)|^2 \simeq \pi^{-1}\xi_0^{-1/2}$. That explains the insensitivity of the amplitude of G (for a specific $n<0$ or $\xi<\xi_0$) to small variations in the incoming energy Ω (see, for example, fig. 2). However, for a specific incoming energy Ω, the amplitude of G oscillates with respect to the *transmitted* energies (*i.e.*, $\Omega + n\omega$). In this regime (*i.e.*, $\xi > \xi_0$)

$$|G(\xi)|^2 \simeq \pi(2\beta\rho/\omega)^{2/3}\xi^{-1/2}\,\mathrm{Ai}^2(-\xi_0) \quad \text{for } n>0, \tag{17}$$

which means that for an incident particles' energy Ω, the amplitude of the Green function has a very mild dependence on the *transmitted* particles' energies (*i.e.*, on n), while it is strongly dependent on the *incident* particles' energies Ω. This can explain the result presented in fig. 2, where a two percent change in the incoming particles' energy was enough to frustrate the activation to higher energies.

It is clear from eq. (17) and from the periodicity of the Airy function, that the probability of an incident particle being activated to higher energy is very sensitive to its initial energy. This sensitivity is manifested in the following calculation of the mean activation energy, $\Omega_{\rm act} = \Omega + \omega\langle n\rangle$, *i.e.*,

$$\Omega_{\rm act} = \Omega + \omega \int \mathrm{d}n n P_n \bigg/ \int \mathrm{d}n P_n, \tag{18}$$

where $P_n \equiv |s_n\varphi_n^+\,(x>L)/\varphi_n^+(0)|^2$.

A plot of the mean activation energy ($\Omega_{\rm act}$) as a function of the incident one (Ω) is shown in fig. 3 (an exact numeric solution).

The opaqueness of the barrier is the cause for the sharp changes in $\Omega_{\rm act}$. Since the tunnelling coefficient

$$\varphi^+_{\Omega+n\omega}\,(x>L)/\varphi^+_{\Omega+n\omega}(0) \simeq \exp\left[-\sqrt{V-\Omega-n\omega}L\right] \tag{19}$$

is exponentially small for non-activated particles, there is a great advantage for a particle to be activated to higher energies $\Omega + n\omega \simeq V$. However, since $G(n)$ is an oscillating function of the incoming energy Ω, there are some energies Ω_m for which $G((V - \Omega_m)/\omega)$ vanishes. In these cases, according to eq. (17), not only is the transition to energy V forbidden, but the transition to all the other higher energies (which correspond to $n > 0$) is, as well.

Therefore, the particles must tunnel out with energy which is very close to their initial one (*i.e.*, $\Omega_{\rm act} \leq \Omega - \omega$). This explains the source of these oscillations and the sharp transitions between the activated energies $\simeq V$ and the non-activated ones $\simeq \Omega$. The transitions occur when $\mathrm{Ai}^2(-\xi_0) \simeq 0$. Taking the low-frequency limit ($\omega \to 0$), the Airy function can be expressed by simple trigonometric functions to obtain the transition criterion $\cos^2[\frac{2}{3}\frac{\rho_m \beta}{\omega} - \frac{\pi}{4}] \simeq 0$, where $\rho_m \equiv \sqrt{V - \Omega_m}$. Therefore, the incoming energies for which $\Omega_{\rm act} \simeq \Omega$, and thus no activation occurs, are approximately

$$\Omega_m \simeq V - \left[\frac{3}{2}\left(\frac{3}{4} + m\right)\pi \frac{\omega}{\beta}\right]^2 \tag{20}$$

and since the ratio (19) is exponentially larger for higher n's, the difference between the incident energy and (20) must be exponentially small to prevent activation (otherwise, the value of the Airy function will not be small enough to suppress the dominance of the high energies in the average (18)), and therefore the spectral width of these regions is exponentially small, $\Gamma \simeq \exp[-\rho_m L/2]$ (see fig. 3).

In fig. 3 the non-activated energies Ω_m, for which $\Omega_{\rm act} \simeq \Omega$, are the minima of the plot while the maxima correspond to full activation (*i.e.*, $\Omega_{\rm act} \simeq V$).

This effect occurs *only* when the perturbations are strong; otherwise (*i.e.*, $\beta^2 \ll V - \Omega$), the solution of eq. (5) is reduced to $s_n \sim \prod_{n'=1}^{n} \beta/2\chi_{n'}$, which is always (for large n) an exponentially small quantity. In fig. 3 one can see that when $V - \Omega$ increases (and β^2 remains fixed) the effect vanishes.

Moreover, when the oscillations rate decreases (*i.e.*, ω decreases) the spectral difference between two successive valleys shrinkage, *i.e.*, $\Delta\Omega_m \equiv \Omega_m - \Omega_{m-1}$ also decreases. It is therefore clear that, when ω is small enough so that $\Delta\Omega_m < \Gamma$, the effect vanishes and elevation to higher energies is depressed. In fig. 3 this is shown as the dotted line.

Hence, this effect cannot be anticipated either in the adiabatic regime or in the weak-perturbation approach. In both regimes the effect vanishes.

This effect can be qualitatively understood with the aid of the following argument. In order to achieve maximum activation, the incident particle must remain in a quasi-localized state inside the barrier for a relatively long period. However, it is known that CDT [8, 9] can occur for specific energies. In these cases barrier's penetration is depressed and the particle cannot accumulate energy quanta, and therefore cannot be activated. In our case these energies can be *qualitatively* evaluated in the two-level approximation as the zeros of the Bessel function $J_0(\hat{\varepsilon}/\omega)$ (see ref. [9]), where $\hat{\varepsilon}$ can be approximated in our case by $\hat{\varepsilon} \propto \beta\rho$. In the limit of small ω this result resembles eq. (20). However, the main point is that since activation is a gradual process, *i.e.*, from energy Ω to $\Omega + \omega$ and all the way up to $\simeq V$, then it suffices to destruct tunnelling at the incident energy Ω in order to eliminate activation *completely*.

In general, the exact geometry and shape of the barrier and the impurity are of no essential importance (see, *e.g.*, generalization in ref. [6]). One can always (for any given geometry) control the activation energy and the transmission by adapting the external field frequency.

To summarize, tunnelling transmission through an opaque barrier with an oscillating section was investigated. It was shown that in the *strong* perturbation (and *non*-adiabatic) regime a new selection rule appears. Not all the incident particles are activated, as could be anticipated in a tunnelling process. Although in most cases the incident particles are elevated to higher-energy states, for some incident particles' energy activation is depressed and the particles remain approximately in their initial states. The spectral width of these energy domains is exponentially small.

* * *

I would like to thank Prof. M. Azbel for enlightening discussions.

REFERENCES

[1] Buttiker M. and Landauer R., *Phys. Rev. Lett.*, **49** (1982) 1739.
[2] Zangwill A. and Soven P., *Phys. Rev. Lett.*, **45** (1980) 204.
[3] Ivlev B. I. and Mel'nikov V. I., *Phys. Rev. Lett.*, **55** (1985) 1614.
[4] Fisher M. P. A., *Phys. Rev. B*, **37** (1988) 75.
[5] Azbel M. Ya., *Phys. Rev. Lett.*, **68** (1992) 98.
[6] Azbel M. Ya., *Europhys. Lett.*, **18** (1992) 537.
[7] Aleiner I. L. and Andreev A. V., *Phys. Rev. Lett.*, **81** (1998) 1286.
[8] Grossmann F., Dittrich T., Jung P. and Hänggi P., *Phys. Rev. Lett.*, **67** (1991) 516.
[9] Grifoni M. and Hänggi P., *Phys. Rep.*, **304** (1998) 243.
[10] Azbel M. Ya., *Phys. Rev. Lett.*, **73** (1994) 138.
[11] Merzbacher E., *Quantum Mechanics* (Wiley) 1970.