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To cite this article: M. Kerscher 2003 EPL 61 856

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Europhys. Lett., **61** (6), pp. 856–862 (2003)

## Luminosity segregation vs. fractal scaling in the galaxy distribution

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(received 21 August 2002; accepted in final form 17 January 2003)

PACS.  $98.65.\mbox{-r}$  – Galaxy groups, clusters, and superclusters; large scale structure of the Universe.

PACS. 98.62.Ve – Statistical and correlative studies of properties (luminosity and mass functions; mass-to-light ratio; Tully-Fisher relation, etc.).

PACS. 89.75.Da – Systems obeying scaling laws.

**Abstract.** – In this letter I present results from a correlation analysis of three galaxy redshift catalogs: the SSRS2, the CfA2 and the PSCz. I will focus on the observation that the amplitude of the two-point correlation function rises if the depth of the sample is increased. There are two competing explanations for this observation: one in terms of a fractal scaling, the other based on luminosity segregation. I will show that there is strong evidence that the observed growth is due to a luminosity-dependent clustering of the galaxies.

Introduction. – One of the problems in cosmology is to understand the formation of the large-scale structures in the Universe, as traced by the spatial distribution of galaxies. Theoretical models of large-scale structure and galaxy formation, whether involving analytical predictions or numerical simulations, are based on some form of random or stochastic initial conditions. This means that a statistical interpretation of the observed galaxy distribution is required, and that statistical tools must be deployed in order to discriminate between different cosmological models (e.q., [1]). The most frequently employed statistical measure is the two-point correlation function. Higher-order correlations are important, but already the observed two-point correlation properties of the galaxy distribution impose strong constraints on the models of structure formation. However, two basically different interpretations of the observed two-point properties are discussed: in the "standard" picture, the galaxy distribution is assumed to be homogeneous on large scales. The correlations of the deviations from this homogeneous distribution are quantified by the two-point correlation function  $\xi(r)$  (see, e.g., [2]). In the alternative picture, the galaxy distribution is modelled as a fractal. The two-point correlations are then measured with the conditional density  $\Gamma(r)$  (see, e.g., [3]). The inhomogeneous nature of a fractal challenges the standard picture. Clearly, these two models lead to different interpretations of observational results. In this letter I will focus on the growing of the amplitude of the two-point correlation function  $\xi(r)$  with the sample depth. This growing amplitude is either explained with the scaling properties of a fractal or with luminosity segregation, a luminosity-dependent clustering strength. By reanalysing three galaxy catalogues I will show that there are strong arguments in favour of luminosity segregation.

#### M. Kerscher: Luminosity segregation vs. fractal scaling

Two-point correlations. – Let me first discuss the stochastic picture where the galaxies positions in space are treated as a realization of a random process. The product density  $\rho_2(\boldsymbol{x}_1, \boldsymbol{x}_2) dV(\boldsymbol{x}_1) dV(\boldsymbol{x}_2)$  is the probability of finding two galaxies in the volume elements  $dV(\boldsymbol{x}_1)$  and  $dV(\boldsymbol{x}_2)$ , respectively. In a homogeneous and isotropic point distribution with mean number density  $\rho$ , one defines the two-point correlation function  $\xi(r)$  (e.g., [2]),

$$\rho_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho^2(\xi(r) + 1), \tag{1}$$

with  $r = |\mathbf{x}_1 - \mathbf{x}_2|$ . The conditional density can be defined as  $\Gamma(r) \equiv \rho(\xi(r) + 1)$ . For a point distribution on a fractal the mean number density  $\rho$  and also  $\xi(r)$  are not well defined. Thus, in this case, one investigates the two-point correlations with the conditional density  $\Gamma(r)$ : the density of galaxies at a distance of r as seen from another galaxy [4]. A scale-invariant cumulant  $\xi(r)$  is typically found in critical systems, whereas a scale-invariant conditional density  $\Gamma(r)$  is an indication for a fractal system. Clearly, only in the limit  $r \to 0$ , both  $\xi(r)$ and  $\Gamma(r)$  may show the same scaling behaviour. An instructive discussion of the different scaling regimes in the galaxy distribution is presented in [5].

Galaxy samples. – In a typical galaxy catalogue the position on the sky inside a given angular region  $\Omega$  and the flux in a given waveband are measured. The distance to our position is estimated from the redshift of the galaxy. A flux-limited sample consists of galaxies down to a limiting flux  $f_{\text{lim}}$ . To study the clustering properties of such a galaxy catalogue, I extract a sequence of volume-limited samples. A volume-limited subsample is constructed by introducing a limiting depth R and a limiting luminosity  $L_{\rm lim}$  and by admitting only galaxies within a distance  $s \leq R$  from our position and a luminosity  $L \geq L_{\text{lim}} (L_{\text{lim}} \propto R^2 f_{\text{lim}})$ , in the



Fig. 1

Fig. 2

Fig. 1 – Luminosity L of the galaxies in the Southern Sky Redshift Survey 2 (SSRS2) plotted against their distance s to our galaxy. Volume-limited samples comprise the galaxies in the upper left part enclosed by the solid lines  $(120h^{-1}\text{Mpc})$ , dotted lines  $(100h^{-1}\text{Mpc})$ , short-dashed lines  $(90h^{-1}\text{Mpc})$ , long-dashed lines ( $80h^{-1}$ Mpc), and dash-dotted lines ( $70h^{-1}$ Mpc). Low-luminosity galaxies at large distances are not observed, as can be seen from the empty region in the lower right part.

Fig. 2 – Sample geometry of a volume-limited sample with sample depth R (simplified sketch). Only galaxies inside the opening angle  $\Omega$  and with a distance s < R enter the sample.



Fig.  $3 - \xi(r)$  and  $\Gamma(r)$  for volume-limited samples from the SSRS2 with depth  $120h^{-1}$ Mpc (solid line),  $100h^{-1}$ Mpc (dotted line),  $90h^{-1}$ Mpc (short-dashed line),  $80h^{-1}$ Mpc (long-dashed line), and  $70h^{-1}$ Mpc (dash-dotted line). The limiting luminosity changes in these samples. The labels "LS" and "minus" mark the results obtained with the Landy and Szalay and the minus estimator, respectively. The solid dot marks  $r_0(120h^{-1}$ Mpc), the open dots mark  $r_0(100h^{-1}$ Mpc),  $r_0(90h^{-1}$ Mpc),  $r_0(80h^{-1}$ Mpc), and  $r_0(70h^{-1}$ Mpc) from right to left, according to eq. (2). The smooth solid line is  $\xi(r)$  for a fractal with D = 2 and  $r_0(70h^{-1}$ Mpc) according to eq. (2). Two- $\sigma$  error bars, determined from a Poisson process, are shown only for the  $120h^{-1}$ Mpc and  $70h^{-1}$ Mpc samples.

Euclidean case, see fig. 1). For a given sample one defines the sample-dependent number density  $\rho_{\rm S} = N/V$ , with the number of galaxies N, and the volume V. The two-point correlation function  $\xi_{\rm S}$ , and the conditional density  $\Gamma_{\rm S}$  determined from this sample satisfy the relation  $\Gamma_{\rm S}(r) = \rho_{\rm S}(1 + \xi_{\rm S}(r))$ . If our galaxy sample stems from a homogeneous distribution, neither  $\xi_{\rm S}$  nor  $\Gamma_{\rm S}$  should change if one increases the sample size (despite statistical fluctuations). If our sample stems from a point distribution on a fractal, the number density  $\rho_{\rm S}$  depends on the sample size, but the scaling exponent D-3 of  $\Gamma_{\rm S}(r) \propto r^{D-3}$  stays invariant (D is the correlation dimension). As a result,  $\xi_{\rm S}$  is changing with the size of the sample (see, *e.g.*, [3]). For the two-point correlation function often the following parameterisation is used  $\xi_{\rm S}(r) = (\frac{r_0}{r})^{\gamma}$  with the scaling index  $\gamma$ . The so-called "correlation length"  $r_0$  quantifies the amplitude  $r_0^{\gamma}$  of a scaleinvariant correlation function. Consider a large sample with a depth  $R_{\rm max}$  and several smaller samples  $R \leq R_{\rm max}$  within (see fig. 2). On a fractal  $r_0$  is proportional to R [6], specifically

$$r_0(R) = \frac{R}{R_{\text{max}}} r_0(R_{\text{max}}), \quad \text{and} \quad \xi(r) = 2\left(\frac{r}{r_0(R)}\right)^{D-3} - 1.$$
 (2)

In the following I will estimate the two-point correlation function as well as the conditional density for three galaxy samples. I checked that for the samples and the scales considered here, the estimators discussed by [7] give consistent results. I illustrate this by showing the results for  $\xi(r)$  both for the minus (reduced sample) estimator as favoured by [3], and for the estimator due to Landy and Szalay [8]. Reference [7] showed that the Landy and Szalay estimator has preferable variance properties.

Luminosity segregation but no fractal scaling in the SSRS2. – The Southern Sky Redshift Survey 2 (SSRS2 [9]) is 99% complete with a limiting magnitude of  $m_{\rm lim} = 15.5$  (magnitudes are logarithmic flux measures). The angular extent is  $-40^{\circ} \leq \delta \leq -2.5^{\circ}$  with  $b \leq -40^{\circ}$ and  $\delta \leq 0^{\circ}$  with  $b \geq 35^{\circ}$  (declination  $\delta$ , galactic latitude b). The magnitudes were Kcorrected as described in [10], and luminosity distances were used. Nearly identical results could be obtained using Euclidean distances and no K-correction. In fig. 3 both  $\xi(r)$  and  $\Gamma(r)$ 



Fig.  $4 - \xi(r)$  and  $\Gamma(r)$  for subsamples from the volume-limited samples from the SSRS2 with a depth of  $R_{\text{max}} = 120h^{-1}$ Mpc. The depth of the sub-samples is  $120h^{-1}$ Mpc (solid line),  $100h^{-1}$ Mpc (dotted line),  $90h^{-1}$ Mpc (short-dashed line),  $80h^{-1}$ Mpc (long-dashed line), and  $70h^{-1}$ Mpc (dash-dotted line). All samples have the same limiting luminosity. The smooth solid line is  $\xi(r)$  for a fractal with D = 2 and  $r_0(70h^{-1}$ Mpc) according to eq. (2). Marks as in fig. 3.

are shown for a sequence of volume-limited samples from the SSRS2. The number density in the volume-limited samples decreases from  $70h^{-1}$ Mpc to  $120h^{-1}$ Mpc. Consequently, the conditional density  $\Gamma(r)$  is decreasing with the sample depth. The amplitude of  $\xi(r)$  increases with the depth of the samples, and  $r_0$  roughly follows relation (2). This was interpreted as a sign of a fractal galaxy distribution (e.g., [3]). However, another explanation is possible. Due to the construction of the volume-limited sample the mean absolute luminosity of the galaxies in the sample increases with the depth of the sample (compare with fig. 1). Hence, the growing amplitude of the correlation function for deeper volume-limited samples may be a result of the stronger clustering of the more luminous galaxies. This is called luminosity segregation. Clearly, the two-point correlation function  $\xi(r)$  applied to a sequence of volumelimited samples is not able to distinguish between both claims. To test the scaling relation (2) independently of any luminosity dependence, I use a volume-limited sample with a depth  $R_{\rm max} = 120 h^{-1}$ Mpc. From this volume-limited sample I extract a sequence of subsamples with depths  $R \leq R_{\rm max}$  (see fig. 2). All these subsamples have the same lower limit in luminosity (see fig. 1). As can be seen from fig. 4, the estimated  $\xi(r)$  are consistent in these samples but inconsistent with the fractal prediction. There is no indication for a fractal scaling of the "correlation length"  $r_0(R)$  as given in eq. (2). Moreover, the conditional densities  $\Gamma(r)$  of these samples nearly overlap. Measurement errors, *e.g.* for the position of the galaxies, have no visible effect on the correlation functions. The dominant contribution is the statistical error. However, there is only one realisation of the galaxy distribution in the Universe. Therefore, I have to assume a model to quantify the statistical errors. The simplest model is a purely random distribution of points, the Poisson process. I estimate the errors from 100 realisations of a Poisson process with the sample geometry, and the number density as in the galaxy samples. Later on, I will show that a more realistic modelling leads to larger errors. These errors are within the same order as determined from a Poisson process. For both models, the errors are smaller than the predicted effects from fractal scaling.

Luminosity segregation but no fractal scaling in the CfA2. – As a spatially complementary sample to the SSRS2 I use a galaxy sample from the CfA galaxy catalogue (see [11] and references therein), with  $\delta \geq 0$ , and  $|b| \geq 35$  and a limiting magnitude of  $m_{\text{lim}} = 15.5$ . From this galaxy catalogue I extract similar volume-limited samples as for the SSRS2. The results



Fig. 5 –  $\xi(r)$  for a sequence of volume-limited samples with changing limiting luminosity  $L_{\text{lim}}$  from the CfA2 determined with the Landy and Szalay estimator. The smooth solid line is  $\xi(r)$  for a fractal with D = 2 and  $r_0(70h^{-1}\text{Mpc})$  according to eq. (2). Marks as in fig. 3.

Fig. 6 –  $\xi(r)$  for samples with varying depth but with the same limiting luminosity extracted from a volume-limited sample of the CfA2 with a depth of  $R_{\text{max}} = 120h^{-1}$ Mpc. Marks as in fig. 4.

shown in figs. 5, 6 lead to the same interpretation as for the SSRS2: the growing amplitude of  $\xi(r)$  is caused by luminosity segregation.

Neither luminosity segregation nor fractal scaling in the PSCz. – Both the galaxies in the SSRS2 and the CfA2 were selected in the optical waveband. The galaxies in the PSCz survey were selected according to their flux in the infrared as detected by the IRAS satellite. A detailed description of the IRAS PSCz galaxy catalogue may be found in [12]. I extract volumelimited samples from the PSCz survey using luminosity distances within the standard masked area. I approximate the sample geometry by two spherical caps with galactic latitude  $b \ge 5^{\circ}$  for the northern part and with  $b \leq -5^{\circ}$  for the southern part. Hence, I neglect some regions which were left empty due to galactic absorption or confusion in the IRAS PSC maps. I filled these empty regions with random points assuming the same number density as in the fully sampled region. No differences in the correlation properties between the filled and unfilled samples are visible in the two-point measures. The  $\xi(r)$  determined from a sequence of volume-limited samples is inconsistent with the fractal prediction and shows no significant variation of  $r_0$  with the sample size (fig. 7). As expected, extracting subsamples from one volume-limited sample with  $R_{\rm max} = 120h^{-1}$  Mpc does not change this behaviour, although the fluctuations increase in the sparser samples (fig. 8). Clearly, there is neither an indication for a fractal scaling of  $r_0$  with the sample depth nor for luminosity segregation (see also [13]). Due to the selection of galaxies in the infrared one does miss early-type (e.g. elliptical) galaxies. Because of this selection one does not find luminosity segregation in the PSCz [14]. To go beyond the error estimates relying on the Poisson process I estimate the errors for  $\xi(r)$  from the fluctuations between eleven mock galaxy catalogues, constructed from an N-body simulation based on a  $\Lambda CDM \operatorname{cosmology}(^1)$ . For such a clustered point distribution, the error bars are larger than in the case of the Poisson

 $<sup>(^{1})</sup>$ A description of the procedure and references to articles describing the simulation and the construction of the mock catalogues can be found in [15].



Fig. 7

Fig. 7 –  $\xi(r)$  for a sequence of volume-limited samples with changing limiting luminosity  $L_{\rm lim}$  from the PSCz determined with the Landy and Szalay estimator with marks similar to fig. 3. The smooth solid line is  $\xi(r)$  for a fractal with D = 2.2 and  $r_0(70h^{-1} \text{Mpc})$  according to eq. (2). Two- $\sigma$  error bars are shown only for the  $120h^{-1}$ Mpc sample. The larger error bars were determined from the mock catalogues. The Poisson error bars are slightly shifted to the left.

Fig. 8 –  $\xi(r)$  for samples with varying depth but with the same limiting luminosity  $L_{\text{lim}}$  extracted from a volume-limited sample of the PSCz with a depth of  $R_{\rm max} = 120h^{-1}$ Mpc similar to fig. 4. Marks as in fig. 7, but the two- $\sigma$  error bars are shown only for the 70 $h^{-1}$ Mpc sample.

process, but still of the same order (see figs. 7, 8). In both models, these errors are smaller than the predicted effects of fractal scaling on  $\xi(r)$  (two- $\sigma$  error bars are shown in the plots).

Summary. – By analysing three different galaxy catalogues I could show that the amplitude of the correlation function  $\xi(r)$  is depending on the luminosity. Luminosity segregation has already been found in the SSRS2, CfA2 and also the 2dF and the SDSS galaxy catalogues (see, e.g., [10, 16–21]). In these investigations, mainly volume-limited samples with varying depths or directly flux-limited samples have been used. Using such samples one is not able to separate the influence of luminosity segregation from a possible fractal scaling. In samples with a fixed lower limit in the luminosity and then reducing the depth of the samples I found no significant changes in the correlation function. Specifically, I found no sign for a growth of the correlation length  $r_0$  with increasing sample depths in these cases. This is a strong indication that the growth of  $r_0$  in standard volume-limited samples is caused by luminosity segregation, and a fractal explanation is disfavoured. Using a similar construction, the authors of [3] found a growing  $r_0$  in the Perseus Pisces survey (PPS). Our consistent results, both from the significantly larger SSRS2 and the CfA2 indicate that the PPS is too small in size to give reliable results. In the CfA2, the authors of [22] found also no significant growth of  $r_0$  using a comparable method. Reference [14] used mark correlation functions to quantify the luminosity dependency of the galaxy clustering in the SSRS2 in a scale-dependent way, uninfluenced by inhomogeneities.

I limited my investigations to the question: what causes the growing amplitude of the two-point correlation function? Already, with the current galaxy catalogues one is able to show that the amplitude of the two-point correlation function depends on the luminosity of the galaxies. If one *a priori* assumes that the galaxy distribution is a fractal, these results may be interpreted as large fluctuations, which indeed are common in fractals. However, these fluctuations must conspire in all the samples considered here, to give the observed result of a constant amplitude of the two-point correlation function. Fluctuations in the morphological properties of the large-scale distribution of galaxies have been detected out to a scale of  $200h^{-1}$ Mpc [15]. However, these fluctuations are barely visible with two-point measures, and they are still compatible with fluctuations expected from a  $\Lambda$ CDM model. I did not comment on the topic, whether one already does see a turnover to homogeneity from current galaxy surveys [15], and whether  $\xi(r)$  or  $\Gamma(r)$  shows the more extended scaling regime. For a discussion see [5, 23]. I expect that the completed Sloan Digital Sky Survey will offer conclusive evidence for these points.

\* \* \*

I would like to thank R. DURRER and F. SYLOS LABINI for organising the workshop "Facts and Fiction in Cosmology" in April 2001 in Sils Maria. The discussions during this workshop generated the impulse to re-investigate these galaxy catalogues. For comments on an earlier version of the manuscript I am grateful to A. DOMÍNGUEZ, J. GAITE, M. JOYCE, M. MONTUORI and, especially, to C. BEISBART. I acknowledge support from the Sonderforschungsbereich 375 für Astroteilchenphysik der DFG.

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