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Bouncing gel balls: Impact of soft gels onto rigid surface

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Abstract. – After being thrown onto a solid substrate, very soft spherical gels bounce repeatedly. Separate rheological measurements suggest that these balls can be treated as nearly elastic. The Hertz contact deformation expected in the static (elastic) limit was observed only at very small impact velocities. For larger velocities, the gel ball deformed into flattened forms like a pancake. We measured the size of the gel balls at the maximal deformation and the contact time as a function of velocities for samples different in the original spherical radius and the Young modulus. The experimental results revealed a number of scaling relations. To interpret these relations, we developed scaling arguments to propose a physical picture.

Introduction. – Impact of spherical objects is concerned with a wide range of subjects from daily life to solid mechanics [1–3]. Accordingly, it has been studied for a long time in different contexts and still continues to be an important research subject. The Hertz contact theory [4] is a classic example *in the static limit* within the linear elastic theory. For larger impact velocities, internal vibrations [5] and dissipative processes such as solid viscosity [6], plastic deformation [2, 7] should be considered. Impact of solid spheres in a granular medium has been found to be an important fundamental process in granular flow, and has attracted a considerable attention [7–9]. Impacts of balls of soft matter could also be an interesting problem: unique phenomena are expected due to a variety of constitutive equations and due to large deformations. For example, impact or dynamics of liquid balls has attracted a wide audience [10–17].

In this paper, we study another example from soft matter: impacts of soft gel balls vertically impinging onto a solid substrate. There the Hertz contact deformation is observed only for small velocities. For larger velocities, gel balls are flattened globally during the impact on

TABLE I – *The composition of four samples of acrylamide gel and their Young moduli E.*

Sample	Water	AA	BIS	E (Pa)
BIS4	100 cc	10 g	0.04 g	1.24×10^4
BIS10	100 cc	10 g	0.1 g	2.71×10^4
BIS15	100 cc	10 g	0.15 g	3.87×10^4
BIS20	100 cc	10 g	0.20 g	4.56×10^4

the substrate, *i.e.*, expanded into the lateral direction. We present experimental data on the lateral dimension of balls at the maximal deformation and the contact time as a function of impact velocities to show the existence of a number of scaling relations. These relations are interpreted via a naive physical picture.

Sample gels. – We used a series of acrylamide gels with the same polymer concentration but with different cross-link densities; acrylamide monomer (AA, $M_\mathrm{w} = 71.08$) constitutes sub-chains while methylenebisacrylamide (BIS, $M_\mathrm{w} = 154.17$) cross-links. The amount of each reagent for preparing acrylamide gels is shown in table I. Ammonium persulphate (1 wt % of AA) and tetramethylethlylenediamine (0.25 vol % of water) were added to initiate and to accelerate the radical polymerization of AA and BIS. The pre-gel solutions were sealed into a spherical mold ($R = 14$ mm or 31.5 mm) consisting of two hemispherical shells. Gelation reaction continued for 24 hours at 30 °C.

Rheological characterization was carried out on cylindrical gels with a rheometer (REOGEL, UBM Co.) in an oscillatory compression mode. As a result, we found that the mechanical responses of our samples can be all regarded as *nearly elastic*. The values of $\tan\delta$ are less than 0.02 and the real part of the complex modulus E' is almost constant in the range of strain frequency between 0.1 Hz and 100 Hz for all samples except BIS4 (here, δ is the phase difference between the stress and the strain). Even for this softest sample, $\tan\delta$ increases by a fairly small amount with f (for example, $\tan\delta \simeq 0.02$ at $f = 0.01$ Hz, $\tan\delta \simeq 0.05$ at $f = 50$ Hz and $\tan\delta \simeq 0.1$ at $f = 100$ Hz). Table I also shows the modulus E' at $f = 10$ Hz that is regarded as *the static Young's modulus* E in the following.

Impact experiment. – Gel balls freely fall on a fixed aluminum plate of 20 mm thickness. Before the impact, the gel ball is pinned at a height L from the plate by a tube sucking air weakly. The gel ball begins to fall by switching-off the sucking, and then impinging onto the aluminium substrate. We coated the surface of balls in white with aluminum oxide powder to avoid sticking of the gel balls to the substrate (this is especially important for low impact velocities). The impact velocity V can be determined from the relation $V = \sqrt{2gL}$. The impact processes are recorded by a high-speed CCD video camera (Motion Coder Analyzer SR: Kodak Co.) with recording rates of 1000 FPS (samples of $R = 31.5$ mm) or 2000 FPS (samples of $R = 14$ mm).

Experimental results. – We found experimentally that modes of deformation are rather different depending on the velocities. The form of maximal deformations can be categorized in three classes: 1) the Hertz type for very small impact velocities (fig. 1a); 2) quasi-ellipsoid for intermediate velocities (fig. 1b); 3) pancake for large velocities (fig. 1c).

For small impact velocities the Hertz-type deformation is theoretically expected: in the static limit, because of the nearly pure elasticity of gel balls, the localized Hertz deformation should be observed. However, this theory should break down for large impact velocities where a ball deforms non-locally. Indeed, the Hertz regime can be observed only for very small

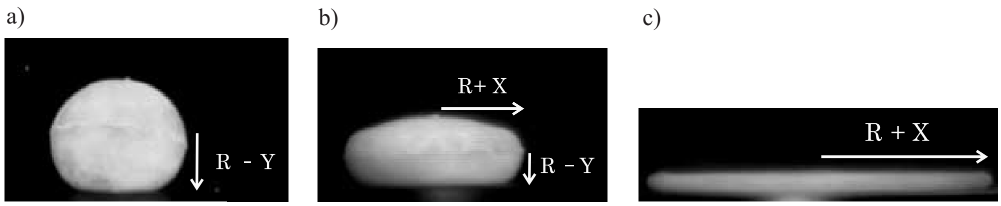

Fig. 1 – Maximal deformations of gel balls (R = 31.5 mm). a) Hertz regime for very low impact velocities (BIS10, $V \lesssim 0.4$ m/s). b) Quasi-ellipsoid regime for intermediate velocities (BIS4, V = 2.0 m/s). c) Pancake regime for high velocities (BIS4, V = 7.0 m/s).

velocities: the impact in fig. 1a was achieved by dropping the gel ball from a low height L less than 1 cm (without using the sucking system) where a precise determination of V is difficult ($V \lesssim 0.4$ m/s). (We note here that all the data points used in the following plots are limited to the case where velocity determination can be done rather precisely.) For moderate impact velocities deformation is no longer localized around the contact area and the ball takes a flattened shape which is quasi-ellipsoidal (fig. 1b). For large impact velocities the ball is strongly deformed to take a pancake shape (fig. 1c).

We measured the horizontal radius $R+X$ (see fig. 1) at the maximal deformation and the contact time at various conditions. As shown below, we found that these experimental data can be well characterized by the length scale R, the sound velocity V_c and the corresponding time scale τ_c:

$$V_\mathrm{c} = \sqrt{E/\rho}, \tag{1}$$

$$\tau_\mathrm{c} = R/V_\mathrm{c}, \tag{2}$$

where ρ is the density of gel balls ($\rho \simeq 1.05 \times 10^3\,\mathrm{kg/m^3}$). Characteristic scales V_c and τ_c come out naturally from theoretical considerations presented below.

Figure 2 shows the maximal deformation X/R as a function of reduced velocities V/V_c. As mentioned above, owing to the characteristic scales R and V_c, the data from the four samples for each size (R = 14 mm or 31.5 mm) collapse well onto a single behavior which can

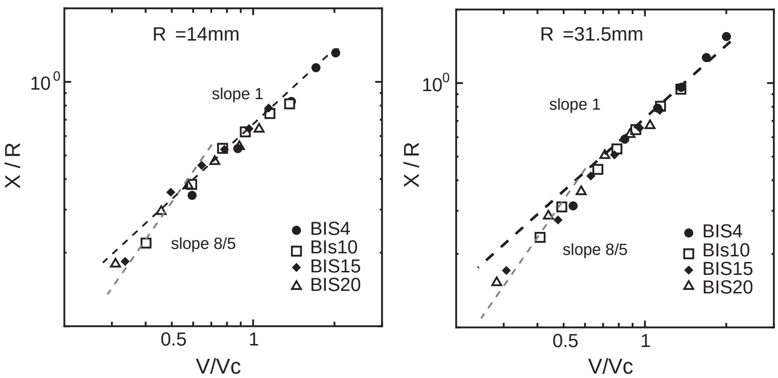

Fig. 2 – Maximal deformation X/R as a function of impact velocities V/V_c for (a) smaller samples (R = 14 mm) and (b) larger samples (R = 31.5 mm). Two dashed lines in the plots correspond to lines with slopes 1 and 8/5, respectively.

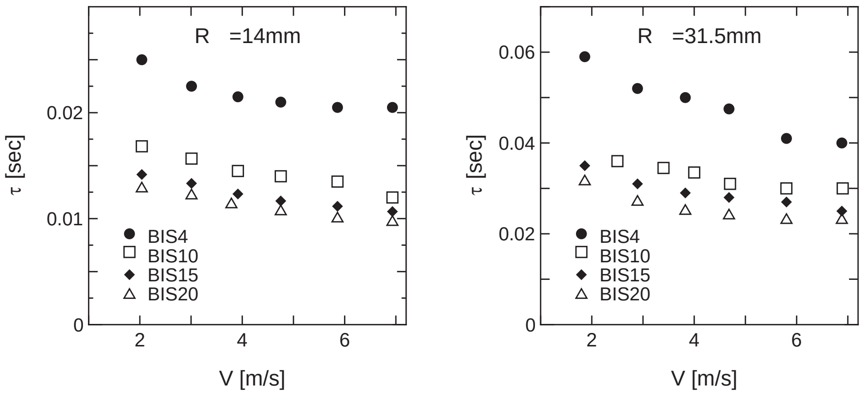

Fig. 3 – Contact τ as a function of velocities V for gel balls of radius $R = 14\,\mathrm{mm}$ (a) and $R = 31.5\,\mathrm{mm}$ (b).

be divided into two regimes as indicated by two dashed lines with different slopes (8/5 and 1). For large velocities ($V \gtrsim V_\mathrm{c}$) it scales as $X \sim V$, while for small velocities ($V \gtrsim V_\mathrm{c}$) $X \sim V^n$ with n being a certain value larger than unity. A theoretical value of the exponent, $n = 8/5$, for the small-velocity region seems consistent with the data (see below).

Figure 3 shows a contact time τ as a function of velocities. The two plots collapse well onto a single master curve in fig. 4, thanks to the characteristic scales τ_c and V_c as mentioned above. The contact time τ decreases with increase in velocities, following a scaling law ($\tau \sim V^{-1/5}$), and seems to approach a constant value (figs. 3 and 4). The plateau value is of the order of the millisecond. This value increases with the size R and decreases with the modulus E. Accordingly, the data from different sizes and moduli collapse by the characteristic scale τ_c.

As seen above, both X and τ can be divided into two regimes by a characteristic velocity of the order of m/s. This velocity scale increases with a modulus E (this can be confirmed due to the data collapse via renormalization of velocity by V_c).

Theoretical considerations. – As mentioned above and as seen from the snapshots at the maximal deformation, modes of deformation seem to change with increase in impact velocity:

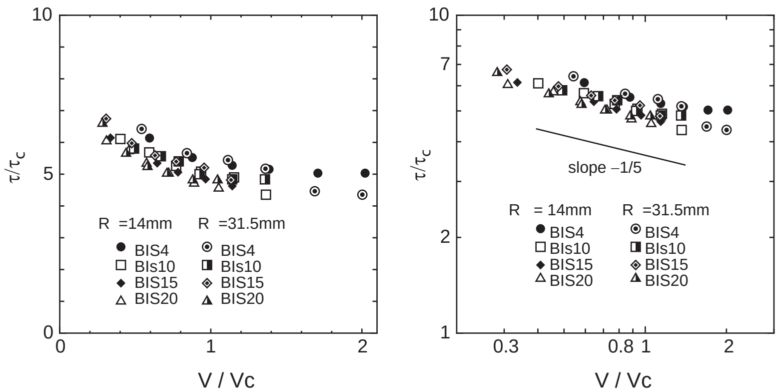

Fig. 4 – Reduced plot of a contact time τ *vs.* impact velocity V. The right plot suggests a scaling relation $\tau \sim V^{-1/5}$.

from 1) Hertz type, to 2) ellipsoid, and then to 3) pancake (see fig. 1). We shall develop a theory based on these observed forms of deformation. We note in advance that experimental results can be understood by combining only the two limiting regimes (Hertz and pancake). However, we still present the result based on the ellipsoid approximation as reference, which is consistent with the experimental results in the turn-over regime. We start from scaling arguments for the Hertz regime only to reproduce the essential properties of the well-known complete analytical solution. The arguments are then extended to the other regimes.

In the Hertz regime at small impact velocities, the elastic energy stored at the maximal deformation can be estimated dimensionally as $U_{\mathrm{H}} \sim E(Y/a)^2a^3$ with E being the elastic modulus of the ball (Y is defined as in fig. 1 and a is the radius of the contact area); a strain field of the order of Y/a is localized in a volume of the order of a^3 (the deformation field relaxes out at a distance of the order of a from the contact surface because the strain field is governed by the Laplace equation). Due to the geometrical relation $Y \sim a^2/R$ with R being the radius of the ball, the energy reduces to the well-known form

$$U_{\mathrm{H}} \sim E\sqrt{R}Y^{5/2}. \tag{3}$$

Assumption of the energy conservation between the initial time and the maximally deformed moment $MV^2 \sim U_{\mathrm{H}}$ ($M \sim \rho R^3$ is the mass of the ball) leads us to an estimate for the contact time $\tau \sim Y/V$, which results in the well-known relation

$$\tau/\tau_{\mathrm{c}} \sim \left(V/V_{\mathrm{c}}\right)^{-1/5}, \tag{4}$$

where the characteristic scales V_{c} and τ_{c} given in eqs. (1) and (2) have naturally come out as announced. This implies a moderate increase of the contact time with decrease in velocity. To obtain a non-trivial X-V relation, we assume that in our "Hertz regime" the shape is *a sphere cut out by a plane* (see below) and require the incompressibility condition: the part of the sphere cut out by the substrate is compensated by an increase in radius, *i.e.*

$$a^2Y \sim R^2X. \tag{5}$$

Combined with the previous relation of the energy conservation, we have the relation between impact velocity and deformation in the horizontal direction as mentioned above:

$$X/R \sim \left(V/V_{\mathrm{c}}\right)^{8/5}. \tag{6}$$

In the ellipsoid regime at intermediate velocities, the maximal elastic energy is given by $U_{\mathrm{E}} \sim E(Y/R)^2R^3$; a strain field of the order of $Y/R \sim X/R$ is distributed within the whole volume of the order of R^3. Note here the relation, $Y = 2X$, which expresses the condition of volume conservation for small deformation ($X/R < 1$). Thus, we have

$$U_{\mathrm{E}} \sim ERY^2. \tag{7}$$

The energy conservation, $MV^2 \sim U_{\mathrm{E}}$, allows us to obtain an estimate for the contact time and a velocity-radius relation,

$$\tau \sim \tau_{\mathrm{c}}, \tag{8}$$

$$X/R \sim V/V_{\mathrm{c}}. \tag{9}$$

In the pancake regime at high velocities, where $X > R > Y$, the maximal energy becomes $U_{\mathrm{P}} \sim E((R+X)/R)^2R^3$ (the ideal rubber deformation energy), or a linear spring energy

$$U_{\mathrm{P}} \sim ERX^2.$$

Noting that, in this case, the contact time might well be estimated not by Y/V but by X/V due to an analogy with a spring system (physically, we can imagine that after a strong impact the vertical velocity is immediately redirected toward the horizontal direction), we obtain the same scaling relations with the ellipsoid case (eqs. (8) and (9)).

The transition between the Hertz and the ellipsoid regimes is given by the condition $U_{\mathrm{E}} \sim U_{\mathrm{H}}$, which implies $Y \lesssim R$. In terms of velocity this is expressed as $V \sim V_{\mathrm{c}}$ (compare eqs. (4) and (8)). On the other hand, the transition between the ellipsoid and the pancake regimes should be marked by $X > Y$, or $V > V_{\mathrm{c}}$. Since the transition from Hertz to ellipsoid regimes and that from ellipsoid to pancake regimes are predicted to occur at the same velocity at the level of scaling laws, the ellipsoid regime is not expected to manifest itself clearly.

We summarize the above theoretical predictions which agree with experimental observations: 1) X scales as $X/R \sim (V/V_{\mathrm{c}})^{8/5}$ for small velocities but as $X/R \sim V/V_{\mathrm{c}}$ for large velocities. 2) The contact time τ is constant for large impact velocities but below a certain velocity it deviates from the plateau value and increases with velocity decrease following a scaling law, $\tau \sim V^{-1/5}$. The typical plateau value τ_{c} for our samples is of the order of a few ms. It increases with R while it decreases with E. 3) Both X and τ can be divided in the two regimes by a transition velocity V_{c}. The typical value V_{c} is of the order of a few m/s and it increases with E. 4) Data should be well characterized by reducing size, time, and velocity variables by scales R, τ_{c}, and V_{c}, respectively.

In our Hertz regime, we assumed that the shape is a sphere cut out by a plane and, in addition, we required eq. (5), which is natural at least in the case of water drops where the surface energy is dominant [13,18]. As a result, we have a global strain X/R distributed over the whole volume R^3, in addition to the original local distribution Y/a over a^3. The former extra energy is far smaller than the latter with a small ratio $\simeq (a/R)^3$; this correction does not change the results of our previous analysis.

Conclusion. – Experimentally, the scaling relation $X \sim V$ for large velocities is shown and this relation seems to change into another scaling relation with a stronger power below $V \simeq V_{\mathrm{c}}$. Another scaling relation $\tau \sim V^{-1/5}$ for small velocities is also shown while, for larger velocities ($V \gtrsim V_{\mathrm{c}}$), τ seems to approach a plateau value. These behaviors of τ and X can be explained by the theory which starts from experimentally observed shapes of balls at the maximal deformation and employs the energy conservation. This theory also leads to the prediction $X \sim V^{8/5}$ for small velocities, which seems consistent with experimental data.

Discussion. – Experimentally, it is difficult to observe a wide plateau region of the contact time predicted by the pancake form and only an asymptotic behavior (or the ellipsoid behavior) to this limiting regime is observed. This is because of the fact that 1) it is practically difficult to achieve such high impact velocities by the present experimental setup, and 2) it is inherently impossible to exceed a certain high impact velocity above which the impact causes irreversible damages to gel balls.

The effect of gravity can be another source of increase in contact time with decrease in impact velocity as might be the case in some bouncing water drops [19]. In the present case, however, this possibility seems to be excluded; the ratio $MgY/U_{\mathrm{H}} \sim \rho g R (R/Y)^{3/2}/E$ suggests that the gravitational energy MgY becomes important only for $Y \lesssim 0.04R$, which is outside of our experimental region. Furthermore, if we define the length l_g by $\rho g l_g \sim E$, which is a counterpart of the capillary length, this is about 1 m and is well beyond the characteristic length scale R (of the order of the cm). This also suggests a weak gravity effect. These arguments, in turn, suggest that the gravity may play a role for very small deformations, which are not studied here.

The effect of viscosity is possibly more important and will be discussed in detail elsewhere. In fact, if we closely look at fig. 4, we find that the behavior of the softest sample with the highest dissipation seems to deviate slightly from the others (if we removed these points from these plots, the data collapse would be much better). This suggests a possibility that viscous effects become important already for the softest sample.

The characteristic velocity V_c that emerged as the result of the static energy evaluation turned out to coincide with the velocity of the shear wave propagation, at the level of scaling laws. To understand this meaning, a systematic study on the static large deformations of soft gel balls would be helpful, and thus such measurements are now under study.

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[19] Okumura K., Clanet C, Richard D. and Quéré D., *Europhys. Lett.*, **62** (2003) 237. In the case of water drop, we cannot consider the Hertz-type deformation as the source of the contact time increase with decrease in velocities because such deformation mode results in the constant contact time (note that there are no elastic contributions and the surface increase scales as X^2).