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The Pioneer riddle, the quantum vacuum and the variation of the light velocity

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Abstract. – It is shown that the same phenomenological Newtonian model recently proposed, which accounts for the cosmological evolution of the fine-structure constant, suggests furthermore an explanation of the unmodelled acceleration $a_{\mathrm{P}} \simeq 8.5 \times 10^{-10}\,\mathrm{m/s^2}$ of the Pioneer 10/11 spaceships reported by Anderson *et al.* in 1998. In the view presented here, the permittivity and permeability of the empty space are decreasing adiabatically, and the light is accelerating therefore, as a consequence of the progressive attenuation of the quantum vacuum due to the combined effect of its gravitational interaction with all the expanding universe and the fourth Heisenberg relation. It is argued that the spaceships might not have any extra acceleration (but would follow instead the unchanged Newton laws), the observed effect being due to an adiabatic acceleration of the light equal to a_{P}, which has the same observational radio signature as the anomalous acceleration of the Pioneers.

Introduction and purpose. – In a previous paper [1], an explanation was proposed for the cosmological variation of the fine-structure constant observed by Webb *et al.* [2], which is based on the gravitational interaction of the quantum vacuum with all the universe. As was argued there, the quantum vacuum must thin or attenuate adiabatically along the history of the universe with the consequent decrease of its permittivity and its permeability. This causes, in turn, a time-dependent increase of the fine-structure constant, which agrees well with the observations by Webb *et al.* The reader is referred to [1] for all the necessary details. This letter shows that the same model offers, besides, an explanation for the anomalous Pioneer's acceleration.

The anomalous Pioneer's acceleration. – A tiny but significant anomaly in the motion of the Pioneer 10/11 spaceships was reported by Anderson *et al.* [3] in 1998: the solar attraction seems to be slightly larger than what predicted by Newton's laws. The Nasa analysis of the data from the two missions showed in the motion of both spacecrafts an extra unmodelled constant acceleration towards the Sun, equal to $a_{\mathrm{P}} \simeq 8.5 \times 10^{-10}\,\mathrm{m/s^2}$. The data from the Galileo and Ulysses spaceships confirmed the effect. Surprisingly, no similar extra acceleration was found in the case of the planets, as would be required by the equivalence principle if the effect were due to gravitational forces. Anderson *et al.* concluded: "it is interesting to

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speculate on the possibility that the origin of the anomalous signal is new physics". In spite of a thorough search, no reason could be found as yet for that extra acceleration (see [4] for a detailed review of the problem and of the observational techniques involved).

In the explanation suggested here, there is indeed a genuine extra blueshift of the radio waves from the Pioneers (*i.e.* the Nasa team observed a real existing effect). The spaceships, however, followed the exact trajectories predicted by Newton unchanged law of Gravitation, without any extra pull from the Sun, the observed effect being not due to any kind of unknown acceleration of the ships, but to an acceleration of the light. Indeed, the model proposed in [1] to explain the cosmological evolution of the fine-structure constant predicts an adiabatic acceleration of light which, at the present time, would be of the same order as $H_0 c = 6.9 \times 10^{-10}\,\mathrm{m/s^2}$ (H_0 being the Hubble parameter), if two coefficients related to the renormalization effects of the quantum vacuum are of order one. Such acceleration would be due to an adiabatic decrease of the permittivity and the permeability of empty space, consequence of the decrease of the quantum vacuum density, produced by the combined effects of the fourth Heisenberg relation and the universe expansion on the quantum vacuum. It will be shown moreover that an adiabatic acceleration of the light has the same observational signature as a blueshift of the radio waves due to an acceleration a_{P} of the Pioneers towards the Sun.

Summary of the model. – The model used in [1] is based on the effect of the gravitational potential Φ on the density of the quantum vacuum, which is treated phenomenologically as a transparent optical medium (note that as Φ is the potential due to all the universe, this model is close in spirit to the Mach principle). As the virtual particles in the vacuum have a gravitational potential energy $E\Phi/c^2$, E being the non-gravitational energy, the fourth Heisenberg relation implies that their average lifetime depends on Φ, and, consequently, the very density of the vacuum as well. More precisely, their average lifetime in a gravitational potential is $\tau_\Phi = \tau_0/(1+\Phi/c^2)$, τ_0 being its value with $\Phi = 0$. As shown in [1], a consequence is that, since the relative permittivity and permeability of empty space must depend on the gravitational potential $\Phi(\boldsymbol{r},t)$, they vary in spacetime, their expressions at first order being

$$\epsilon_{\mathrm{r}}(\boldsymbol{r},t) = 1 - \beta\big[\Phi(\boldsymbol{r},t) - \Phi_{\mathrm{E}}\big]/c^2, \qquad \mu_{\mathrm{r}}(\boldsymbol{r},t) = 1 - \gamma\big[\Phi(\boldsymbol{r},t) - \Phi_{\mathrm{E}}\big]/c^2, \tag{1}$$

where Φ_{E} is the gravitational potential today and at a reference terrestrial laboratory, and β and γ are certain coefficients, which must be positive since the quantum vacuum is dielectric but paramagnetic (its effect on the magnetic field is due to the magnetic moments of the virtual pairs). This implies that, at first order, the light velocity at a generic spacetime point must be equal to

$$c(\boldsymbol{r},t) = c\big[1 + (\beta+\gamma)\big(\Phi(\boldsymbol{r},t) - \Phi_{\mathrm{E}}\big)/\big(2c^2\big)\big], \tag{2}$$

c being its value now at Earth (the constant in the tables), with corresponding variations for the observed electron charge and the fine-structure constant. It follows from (2) that $c(\boldsymbol{r},t)$ is smaller where Φ is more negative (or less positive), *i.e.* it decreases when approaching massive objects, but increases monotonously in time since the galaxies are separating because of the expansion. Note that the two kinds of variation of $\Phi(\boldsymbol{r},t)$ due to the changes of $\boldsymbol{r}$ and t, respectively, have different effects. The first causes the light to behave as in an ordinary optical medium, in such a way that the frequency is constant during the propagation, while the wavelength and the light velocity change according to the value of a refractive index as in an inhomogeneous transparent optical medium. It will be shown here that the second causes an adiabatic increase of the light velocity and of the frequency, the wavelength remaining constant. The former is describable with a refractive index $n(\boldsymbol{r},t)$ $[= \{1 + (\beta+\gamma)(\Phi(\boldsymbol{r},t) - \Phi_{\mathrm{E}})/(2c^2)\}^{-1}]$. The latter arises because the expansion implies, as noted before, that Φ is increasing and ϵ_{r}, μ_{r} are decreasing, with the corresponding acceleration of light. It must be emphasized that while the former is either positive or negative, according to how much matter is around, the latter is secular and consists in a monotonous adiabatic increase in the light velocity and the frequency, as is shown in the following, which is however negligible in terrestrial laboratory experiments. This is important since one or the other of the two variations can be neglected in some interesting cases.

A variation of c should not be a matter of concern. Einstein himself made the following enlightening comment in 1912: "the constancy of the velocity of light can be maintained only insofar as one restricts oneself to spatio-temporal regions of constant gravitational potential. This is where, in my opinion, the limit of validity of the principle of the constancy of the velocity of light —though not of the principle of relativity— and therewith the limit of validity of our current theory of relativity lies" [5]. Note that Einstein states clearly i) that a variation of c does not imply necessarily a violation of the principle of relativity if c depends on the potential, as it happens in eq. (2) where $c(\boldsymbol{r}, t) = c[\Phi(\boldsymbol{r}, t)]$, and ii) that we must distinguish carefully between the principle of relativity and any particular theory of relativity.

The attenuation of the quantum vacuum and the time variation of the light velocity. – Note that $\Phi(\boldsymbol{r}, t)$ and Φ_{E} are the sum of the space-averaged potential of all the universe $\Phi_{\mathrm{av}}(t)$ plus the contributions of the nearby inhomogeneities Φ_{inh}, in the case of a terrestrial laboratory the Earth, the Sun and the Milky Way. The former is time dependent because of the general expansion, while the latter is constant at Earth since these three objects are not expanding. This means that at the Earth surface, the effect of the inhomogeneities cancels in the differences in (1). For a spaceship travelling through the solar system, however, the variation of the potential of the Sun and Earth must be included, the second being negligible, but remember that the space change of Φ does not produce any change in the frequency. Let us assume now that all the matter (ordinary plus dark) and dark energy are uniformly distributed. Since the distances between the galaxies are increasing, $\Phi_{\mathrm{av}}(t)$ becomes less negative (or more positive) as time goes on, a consequence being that the quantum vacuum thins down, its density being a decreasing function of time. Hence the decrease of the permittivity and permeability of the empty space and the increase of the light velocity. The consequences of these ideas agree well with the observations (see fig. 1 in [1]).

It must be stressed that, as was argued in [1], eqs. (1)-(3) are not *ad hoc* assumptions but unavoidable consequences of the fourth Heisenberg relation. We can average eq. (1), writing instead

$$\epsilon_{\mathrm{r}} = 1 - \beta\big[\Phi_{\mathrm{av}}(t) - \Phi_{\mathrm{av}}(t_0)\big]/c^2, \qquad \mu_{\mathrm{r}} = 1 - \gamma\big[\Phi_{\mathrm{av}}(t) - \Phi_{\mathrm{av}}(t_0)\big]/c^2, \tag{3}$$

where $\Phi_{\mathrm{av}}(t)$ is the space-averaged gravitational potential of all the universe at time t and t_0 is the present time (*i.e.* the age of the universe).

Let Φ_0 be the gravitational potential produced by the critical density distributed up to the distance of R_{U} ($\Phi_0 = -\int_0^{R_{\mathrm{U}}} G\rho_{\mathrm{cr}} 4\pi r\, \mathrm{d}r \simeq -0.3c^2$, if $R_{\mathrm{U}} \approx 3000$ Mpc) and let Ω_{M}, Ω_Λ be the corresponding present-time relative densities of matter (ordinary plus dark) and dark energy corresponding to the cosmological constant Λ. Because of the expansion of the universe, the gravitational potentials due to matter and dark energy equivalent to the cosmological constant vary in time as the inverse of the scale factor $a(t)$ and as its square $a^2(t)$, respectively. It turns out therefore that

$$\Phi_{\mathrm{av}}(t) - \Phi_{\mathrm{av}}(t_0) = \Phi_0 F(t), \quad \text{with} \quad F(t) = \Omega_{\mathrm{M}}[1/a(t) - 1] - 2\Omega_\Lambda\big[a^2(t) - 1\big], \tag{4}$$

where $F(t_0) = 0$, $\dot{F}(t_0) = -(1 + 3\Omega_\Lambda)H_0$. Let us assume a universe with flat sections t = constant (*i.e.* $k = 0$), with $\Omega_{\mathrm{M}} = 0.3$, $\Omega_\Lambda = 0.7$ and Hubble parameter to $H_0 = 71\,\mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1} = 2.3\times 10^{-18}\,\mathrm{s}^{-1}$. To find the evolution of the average quantities,

$\Phi_{\rm av}(t)-\Phi_{\rm av}(t_0)$ must be substituted for $\Phi(\boldsymbol{r},t)-\Phi_{\rm E}$ in (2), which gives for the time evolutions of the fine-structure constant and the light velocity, at first order in the potential,

$$\alpha(t)=\alpha\big[1+(3\beta-\gamma)F(t)\Phi_0/\big(2c^2\big)\big],\qquad c(t)=c\big[1+(\beta+\gamma)F(t)\Phi_0/\big(2c^2\big)\big], \tag{5}$$

$c(t)$, $\alpha(t)$ being the time evolutions and $\alpha=\alpha(t_0)$, $c=c(t_0)$, the present-time values, *i.e.* the constants that appear in the tables. It was shown in ref. [1] that the first of eq. (5) gives a good agreement with the observations by Webb *et al.* if $\xi=(3\beta-\gamma)/2=1.3\times10^{-5}$ if $(\Omega_{\rm M},\Omega_\Lambda)=(0.3,0.7)$ (respectively $(1,0)$) (see fig. 1 in [1]).

The adiabatic acceleration of light. – It follows from the second eq. (5) that the velocity of light increases in time, the present value of the acceleration $a=\dot{c}(t_0)$ being equal to

$$a=-H_0c(\beta+\gamma)(1+3\Omega_\Lambda)\Phi_0/\big(2c^2\big). \tag{6}$$

Note that, as $H_0c=6.9\times10^{-10}\,\mathrm{m/s^2}$, a is of the same order as the Pioneer acceleration $a_{\rm P}$ if β and γ are close to 1 and $\Omega_\Lambda=0.7$. It was shown in [1] that the observed cosmological variation of α can be explained with a value for $\xi=(3\beta-\gamma)/2$ of the order of 10^{-5}. We will show now that this same model suggests an explanation of the anomalous Pioneer acceleration as an effect of the quantum vacuum if $(\beta+\gamma)/2$ has a value of the order of one, which we assume as a working hypothesis to be the case.

The adiabatic acceleration of light implies a blueshift. – It will be shown now that the frequency ω_0 of a monochromatic light wave with such an adiabatic acceleration a increases so that its time derivative $\dot\omega$ satisfies

$$\dot\omega/\omega_0=a/c. \tag{7}$$

Furthermore, an adiabatic acceleration of light has the same radio signature as a blueshift of the emitter, albeit a peculiar blueshift with no change of the wavelength (*i.e.* all the increase in velocity is used to increase the frequency).

Equations (3)-(4) tell that the time derivatives of the permittivity $\epsilon=\epsilon_{\rm r}\epsilon_0$ and permeability $\mu=\mu_{\rm r}\mu_0$ of the empty space at the present time t_0 are equal to

$$\dot\epsilon=\epsilon_0\beta\big(1+3\Omega_\Lambda\big)H_0\big(\Phi_0/c^2\big),\qquad \dot\mu=\mu_0\gamma\big(1+3\Omega_\Lambda\big)H_0\big(\Phi_0/c^2\big). \tag{8}$$

These two derivatives are negative and very small. To study the propagation of light in a medium whose permittivity and permeability decrease adiabatically, we must take the Maxwell equations and deduce the wave equations for the electric field $\boldsymbol{E}$ and the magnetic intensity $\boldsymbol{H}$, which are $\nabla^2\boldsymbol{E}-\partial_t(\mu\partial_t(\epsilon\boldsymbol{E}))=0$, $\nabla^2\boldsymbol{H}-\partial_t(\epsilon\partial_t(\mu\boldsymbol{H}))=0$, or, more explicitly,

$$\nabla^2\boldsymbol{E}-\partial_t^2\boldsymbol{E}/c^2(t)-\big(\dot\mu/\mu_0+2\dot\epsilon/\epsilon_0\big)\partial_t\boldsymbol{E}/c^2(t)-\dot\epsilon\dot\mu\boldsymbol{E}/\big(\epsilon_0\mu_0c^2(t)\big)=0, \tag{9}$$

$$\nabla^2\boldsymbol{H}-\partial_t^2\boldsymbol{H}/c^2(t)-\big(2\dot\mu/\mu_0+\dot\epsilon/\epsilon_0\big)\partial_t\boldsymbol{H}/c^2(t)-\dot\epsilon\dot\mu\boldsymbol{H}/\big(\epsilon_0\mu_0c^2(t)\big)=0, \tag{10}$$

since at the present time $\epsilon_{\rm r}=1$, $\mu_{\rm r}=1$. Because of (8), $\dot\epsilon/\epsilon_0$ and $\dot\mu/\mu_0$ are of order $H_0=2.3\times10^{-18}\,\mathrm{s^{-1}}$, so that the third and the fourth terms on the LHS of (9) and (10) can be neglected for frequencies $\omega\gg H_0$, in other words for any practical purpose. We are left with two classical wave equations with time-dependent light velocity $c(t)$:

$$\nabla^2\boldsymbol{E}-\partial_t^2\boldsymbol{E}/c^2(t)=0,\qquad \nabla^2\boldsymbol{H}-\partial_t^2\boldsymbol{H}/c^2(t)=0. \tag{11}$$

In order to find the behavior of a monochromatic light beam according to these two wave equations, we start with the first one and take $\boldsymbol{E}=\boldsymbol{E}_0\exp[-i(\kappa z-(\omega_0+\dot\omega t/2)t)]$, where the frequency is the time derivative of the phase of $\boldsymbol{E}$, *i.e.* $\omega_0+\dot\omega t$. Neglecting the second time derivatives and working at first order in $\dot\omega$ (with $\dot\omega t\ll\omega_0$, $\dot\omega\ll\omega_0^2$), substitution in (9) gives

$\kappa^2 = [(\omega_0+\dot\omega t)^2-i\dot\omega]/c^2(t)$. It follows that $\kappa = k+i\zeta = \pm(\omega_0/c(t))[1+4\dot\omega t/\omega_0](\cos\varphi+i\sin\varphi)$, with $\varphi = -\dot\omega/2\omega_0^2$, so that $k = \pm(\omega_0/c)(1+\dot\omega t/\omega_0)/(1+at/c)$ which implies $k = \pm\omega_0/c$ and eq. (7), $\dot\omega/\omega_0 = a/c$, as stated before. Also, $\zeta = -\dot\omega/2\omega_0 c = a/2c^2$. The wave amplitude decreases in the direction of propagation as $e^{-z/\ell}$ with $\ell = 2c^2/a$, but as a is of order H_0c, ℓ is of order of 5000 Mpc, so that this attenuation can be neglected. It is easy to show that to take $k+\dot kt$ for the wave vector leads to $\dot k = 0$. These results are valid both for the solutions of (9) and (10).

This shows that the electromagnetic waves verify eq. (7), so that k, and therefore the wavelength λ, remain constant while the frequency increases with the same relative rate as the light velocity. Note an important point: in a measurement of the frequency of radiowaves, a blueshift is found (unrelated to the velocity of the emitter), but optical observations of the wavelength fail to find any effect.

Non-mechanical and non-gravitational explanation of the Pioneer acceleration. – In this model, the Pioneer effect is neither gravitational nor mechanical (it is not produced by any force) but electromagnetic. What Anderson *et al.* observed was a blueshift in the radiowaves from the two Pioneers. More precisely, a drift of the Doppler residuals corresponding to a positive-constant time derivative of the frequency received from the spaceships such that

$$\dot\omega/\omega_0 = a_{\mathrm{P}}/c, \tag{12}$$

with $a_{\mathrm{P}} \approx 8.5\times10^{-10}$ m/s. Obviously, the simplest interpretation of this observation is that there is an unexpected acceleration of the ship towards the Sun, due to an extra force alien to Newton law of Gravitation. However, as we have seen in the previous section (eq. (7)), this could be also the signature of the acceleration of light. Indeed if $(\beta+\gamma)/2$ is close to 1, then eq. (6) implies that the acceleration of light is close to $H_0c \approx 6.9\times10^{-10}\,\mathrm{m/s^2} = 0.8a_{\mathrm{P}}$ (in the case $\Omega_{\mathrm{M}} = 0.3$, $\Omega_\Lambda = 0.7$).

This would explain why the effect is not seen in the planets. Indeed, the cartography of the solar system, being based on radar-ranging methods that measure the delay of round trips of electromagnetic waves, is quite independent of an eventual acceleration of light equal to a_{P}, too small to have any detectable influence. To be more precise, let us consider two radar-ranging observations in which the flight time of light is measured. If the second observation is made one year after the first, the relative difference between the two results would be about 1 year $\times\, a_{\mathrm{P}}/c \simeq \mathrm{O}(10^{-10})$, which has a completely negligible effect on the measurement. For instance, the difference and the sum of Mars and Earth orbit radii are known since the radar-ranging studies of the Viking missions with precisions of 100 m and 150 m, respectively [3]. If the same observations had been repeated a year later, the changes of these lengths due to an acceleration a_{P} of the light would have been close to 10^{-8} m and 1.5×10^{-8} m, respectively, quite unobservable. This gives a simple explanation of the riddle that the Pioneer effect is observed in the spaceships but not in the planets.

Comparison with the experiments. – One could fear at first sight that this effect could be in conflict with the various experimental tests that put stringent bounds to any departure from special relativity or the equivalence principle [6], the most important being here the Eötvös, the Hughes-Drever and the gravitational redshift experiments. Let us see which are the bounds that they impose on β and γ. It has been acknowledged that a variation of e could lead to a violation of the equivalence principle, since a small part of the mass of a body would change in a way that depends on its chemical composition [7,8]. Indeed, according to von Weizsäcker semiempirical mass formula, there is a Coulomb contribution to the mass of a nucleus m given by $m_{\mathrm{C}} = 3e^2Z(Z-1)/20\pi\epsilon_0 r_0A^{1/3}$, with $r_0 \simeq 1.5$ fm, plus the electromagnetic mass of each of the protons. The ratio $u = m_{\mathrm{C}}/m$ is of the order 10^{-3} and increases with Z. In this model, the difference of the values of the mass of a body at two points would be therefore $\Delta m = \Delta m_{\mathrm{C}} = 2\beta u m \Delta\Phi/c^2$, u being the average value of m_{C}/m of its nuclei, which depends on its chemical composition. Note that $|\nabla\Phi|/c^2 \simeq 10^{-16}$ at Earth, the field of our planet being the dominant part. In an Eötvös experiment, the contribution of the effect proposed in this model to the Eötvös ratio η would satisfy $\eta \lesssim 2\beta u |\nabla\Phi \cdot \boldsymbol{h}|/c^2$, where $\boldsymbol{h}$ is a vector between the two positions of the balance (the time variation of Φ can be neglected here). Assuming that $h < 1\,\mathrm{m}$, one has $\eta \lesssim \beta \times 10^{-18}$, while the best bound in this type of experiments is $\eta < 10^{-12}$ (obtained by Roll, Krotkov and Dicke and Braginski), from which $\beta < 10^{+6}$. No problem for this model.

The Hughes-Drever experiments [9, 10] were devised as tests of the Mach principle. By observing the Zeeman effect in nuclei, they establish the bound $\Delta m/m < 10^{-23}$, Δm being the anisotropic part of the mass of a nucleus. Although the mass is technically isotropic in this model (it is always a scalar), a certain anisotropy arises in the sense that the electromagnetic mass of a nucleus changes differently along the diverse directions around a point. The above-given expression for the electromagnetic mass must be used then, assuming a displacement of the order of the diameter of a nucleus, and taking the potential of the Earth (again the main contribution). One finds thus easily that the effect proposed in this model gives a contribution $\Delta m/m \lesssim \beta \times 10^{-32}$ to the difference of the relative changes of the electromagnetic mass of a nucleus between two directions in a terrestrial laboratory. The corresponding restriction for the model is $\beta < 10^{+9}$. No problem again.

The gravitational redshift of the frequency is given as $\Delta\omega/\omega = -(1+\delta)\Delta\Phi/c^2$ with $\delta = 0$ (a non-zero value would indicate a violation of the equivalence principle). Several experimental tests set bounds for δ, the best being $|\delta| \lesssim 2 \times 10^{-4}$ [6]. It was obtained by Vessot and Levine [11, 12] with the 1420 MHz line of the hyperfine spectrum of hydrogen (*i.e.* measuring frequencies), between a terrestrial laboratory and a rocket travelling upwards until a height of 10000 km. The frequency of that line here at Earth is $\omega_{\mathrm{E}} = 8\alpha^4 g_{\mathrm{p}} m^2 c^2/3M\hbar$, m and M being the electron and proton masses and $g_{\mathrm{p}} = 2.79$ [13]. Assuming that the rest energy of the electron is of electromagnetic origin [14], then $m \propto e^2/c^2$, so that $\Delta m/m = (\beta - \gamma)\Delta\Phi/c^2$, and $\omega_{\mathrm{E}} \propto e^4\alpha^4/c^2$. Hence the line would be emitted at spacetime point $(\boldsymbol{r}, t)$ with the frequency $\omega(\boldsymbol{r}, t) = \omega_{\mathrm{E}}[1 + 4\Delta e/e + 4\Delta\alpha/\alpha - 2\Delta c/c]$, so that $\Delta\omega/\omega = (\omega(\boldsymbol{r}, t) - \omega_{\mathrm{E}})/\omega_{\mathrm{E}} = [4\beta + 4\xi - (\beta + \gamma)]\Delta\Phi/c^2 = 6\xi\Delta\Phi/c^2$. Note that the effect of the acceleration a is negligible here, since the change of Φ during the short time flight of the ray ($\leq 0.033\,\mathrm{s}$) is very small. In other words, the line is produced with a slightly different frequency $\omega(\boldsymbol{r}, t)$, and travels after until the receiver with constant frequency, as in an optical medium. To be specific, if $\Phi(\boldsymbol{r}, t) < \Phi_{\mathrm{E}}$, then the line is seen at Earth with frequency $\omega(\boldsymbol{r}, t)$ ($< \omega_{\mathrm{E}}$) and conversely. The effect described in this model would produce, therefore, a shift corresponding to $\delta = 6\xi \simeq 8 \times 10^5$, below the Vessot-Levine bound (near borderline at most). Conclusion: there is no conflict between this model and the experimental tests of special relativity and of the equivalence principle.

Summary and conclusion. – Using a Newtonian approximation, the model presented in ref. [1] offers a unified picture that accounts both for the cosmological variation of the fine-structure constant, observed by Webb *et al.* [2], and for the anomalous Pioneer acceleration, observed by Anderson *et al.* [3]. More precisely, it explains these two phenomena as due to the progressive attenuation of the quantum vacuum because of the fourth Heisenberg relation combined with its gravitational interaction with all the expanding universe. In the first case, the fine-structure constant increases in time because a thinner vacuum implies a lesser renormalization of the electron charge and an acceleration of light, the resulting value of α being an increasing function of time. In the second, because an acceleration of light has the same radio signature as a blueshift of the frequency. In both cases the effect depends on the coefficients β and γ in eqs. (1) which express the permittivity and the permeability of the quantum vacuum as functions of the gravitational potential at first order. It must be stressed that this model does not conflict with the experimental tests of special relativity or of the equivalence principle. Indeed, Einstein second postulate of special relativity would be still valid as an extremely good approximation, its practical value being unaffected. The effect was not observed in the planets because they were not submitted to the same kind of observation as the Pioneers. The Pioneer acceleration would be thus a manifestation of the universal expansion (see [15] for another model with this in common). The experimental test of this idea is surely difficult. One way would be to repeat the measurements of c during several years, another to measure the frequency of the radiowaves emitted by a very stable source (not necessarily in a spaceship) during a sufficiently long time.

Note that this model is free of *ad hoc* assumptions and does predict, using well-known basic laws of physics, i) that the time dependence of α is given by the function $F(t)$ in eqs. (4), which agrees with the observations (see fig. 1 in [1]); ii) that a blueshift must be seen in the radio signal of any spaceship moving away from the Sun, quite similar to the shift due to an extra acceleration of the ship towards the Sun but unrelated to its motion. The change in the light velocity during one year if its acceleration is a_{P} would be just about 2.7 cm/s, only after 37 years the change would amount to 1 m/s.

To summarize, *this letter proposes as a possibility worth of consideration that the Pioneers did not suffer any extra acceleration but, quite on the contrary, that they followed the standard Newton laws, the observed and unmodelled acceleration a_{P} being an observational effect of an acceleration of light a equal to a_{P}, due to its interaction with the quantum vacuum.*

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