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# Gravitational localization of matters in 6D 

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#### Abstract

We present a new 3-brane solution to Einstein's equations in (1+5)-spacetime with a negative bulk cosmological constant. This solution has decreasing scale factor approaching a finite non-zero value at radial infinity. It is shown that the zero modes of all local fields are localized on the brane only through the gravitational interaction.


It is believed that the idea of extra dimensions would be one of the most attractive ideas concerning unification of gauge fields with general relativity. Large extra dimensions offer an opportunity for new solutions to old problems (smallness of cosmological constant, the origin of the hierarchy problem, the nature of flavor, etc.). This idea has been much investigated since the appearance of papers [1-9]. In such theories, our world can be associated with a 3 -brane, embedded in a higher-dimensional spacetime with non-compact extra dimensions and non-factorizable geometry. In this scenario it is assumed that all the matter fields are constrained to live on the 3-brane and the corrections to four-dimensional Newton's gravity law from bulk gravitons are small for macroscopic scales. But this model still needs some natural mechanism of localization of known particles on the brane. The question of matter localization on the brane has been investigated in various papers [10-23]. In our opinion, the localizing force must be universal for all types of 4 -dimensional matter fields. The gravity is known to be the unique interaction which has universal coupling with all matter fields. So if extended extra dimensions exist, it is natural to assume that trapping of matter on the brane has a gravitational nature. It is of interest that recently in [24] a model in the brane world was found where all the local bulk fields (ranging from the spin-0 scalar field to the spin-2 gravitational field) are localized on the 3 -brane only by the universal interaction, i.e., the gravity. The solution is found in $(1+5)$-spacetime for the positive bulk cosmological constant $\Lambda>0$ and has increasing scale factor $\phi(r)$ asymptotically approaching a finite value at radial infinity. Recently, in [25] this solution was extended to the case of a general ( $p-1$ )-brane model with codimension $n$ in general $D=p+n$ spacetime dimensions.

[^0]In this letter we introduce the new 3 -brane solution in $(1+5)$-spacetime for the negative cosmological constant $\Lambda<0$, which has decreasing scale factor $\phi(r)$ asymptotically approaching a finite non-zero value at radial infinity. We explicitly show that the zero modes of all local fields (spin-0 scalar field, spin-(1/2) spinor field, spin-1 gauge field, spin- $(3 / 2)$ gravitino field and spin-2 gravitational field) as well as of totally antisymmetric tensor fields are localized on the 3 -brane by the gravity.

Let us begin with the details of our solution. In 6D the Einstein equations with a bulk cosmological constant $\Lambda$ and stress-energy tensor $T_{A B}$ have the following form:

$$
\begin{equation*}
R_{A B}-\frac{1}{2} g_{A B} R=\frac{1}{M^{4}}\left(\Lambda g_{A B}+T_{A B}\right), \tag{1}
\end{equation*}
$$

where $R_{A B}, R$ and $M$ are, respectively, the Ricci tensor, the scalar curvature and the fundamental scale. All of these physical quantities refer to $(1+5)$ space with signature $(+-\cdots-)$, capital Latin indices run over $A, B, \ldots=0,1,2,3,5,6$. Suppose that eqs. (1) admit a solution that is consistent with four-dimensional Poincaré invariance. Introducing for the extra dimensions the polar coordinates $(r, \theta)$, where $0 \leq r<+\infty, 0 \leq \theta<2 \pi$, we can choose the six-dimensional metric satisfying this ansatz in the form [22,24]

$$
\begin{equation*}
\mathrm{d} s^{2}=\phi^{2}(r) \eta_{\alpha \beta}\left(x^{\nu}\right) \mathrm{d} x^{\alpha} \mathrm{d} x^{\beta}-g(r)\left(\mathrm{d} r^{2}+r^{2} \mathrm{~d} \theta^{2}\right) \tag{2}
\end{equation*}
$$

where small Greek indices $\alpha, \beta, \ldots=0,1,2,3$ numerate coordinates and physical quantities in four-dimensional space, the functions $\phi(r)$ and $g(r)$ depend only on $r$ and are cylindrically symmetric in the extra-space, the metric signature is $(+-\cdots-)$. The function $g(r)$ must be positive to fix the signature of the metric (2).

The source of the brane is described by a stress-energy tensor $T_{A B}$ also cylindrically symmetric in the extra-space. We choose its non-zero components in the form

$$
\begin{equation*}
T_{\alpha \beta}=-g_{\alpha \beta} F_{0}(r), \quad T_{i j}=-g_{i j} F(r), \tag{3}
\end{equation*}
$$

where we have introduced two source functions $F_{0}$ and $F$, which depend only on the radial coordinate $r$.

By using cylindrically symmetric metric ansatz (2) and stress-energy tensor (3), the Einstein equations become

$$
\begin{gather*}
3 \frac{\phi^{\prime \prime}}{\phi}+3 \frac{\phi^{\prime 2}}{\phi^{2}}+3 \frac{\phi^{\prime}}{r \phi}+\frac{g^{\prime \prime}}{2 g}-\frac{g^{\prime 2}}{2 g^{2}}+\frac{g^{\prime}}{2 r g}=\frac{g}{M^{4}}\left(F_{0}-\Lambda\right)+\frac{g}{\phi^{2}} \frac{\Lambda_{\mathrm{phys}}}{M_{\mathrm{P}}^{2}},  \tag{4}\\
6 \frac{\phi^{\prime 2}}{\phi^{2}}+2 \frac{g^{\prime} \phi^{\prime}}{g \phi}+4 \frac{\phi^{\prime}}{r \phi}=\frac{g}{M^{4}}(F-\Lambda)+2 \frac{g}{\phi^{2}} \frac{\Lambda_{\mathrm{phys}}}{M_{\mathrm{P}}^{2}}  \tag{5}\\
4 \frac{\phi^{\prime \prime}}{\phi}+6 \frac{\phi^{\prime 2}}{\phi^{2}}-2 \frac{g^{\prime} \phi^{\prime}}{g \phi}=\frac{g}{M^{4}}(F-\Lambda)+2 \frac{g}{\phi^{2}} \frac{\Lambda_{\mathrm{phys}}}{M_{\mathrm{P}}^{2}} \tag{6}
\end{gather*}
$$

where the prime denotes differentiation $\mathrm{d} / \mathrm{d} r$. The constant $\Lambda_{\text {phys }}$ represents the physical four-dimensional cosmological constant, where $R_{\alpha \beta}^{(4)}-\frac{1}{2} g_{\alpha \beta} R^{(4)}=\frac{\Lambda_{\mathrm{phys}}}{M_{\mathrm{P}}^{2}} g_{\alpha \beta}$. In this equation $R_{\alpha \beta}^{(4)}, R^{(4)}$ and $M_{\mathrm{P}}$ are four-dimensional physical quantities: Ricci tensor, scalar curvature and Planck scale.

In the case $\Lambda_{\text {phys }}=0$, from eqs. (4), (5) and (6) we can find

$$
\begin{align*}
& F^{\prime}+4 \frac{\phi^{\prime}}{\phi}\left(F-F_{0}\right)=0,  \tag{7}\\
& g=\frac{\delta \phi^{\prime}}{r}, \quad \delta=\text { const },  \tag{8}\\
& r \frac{\phi^{\prime \prime}}{\phi}+3 r \frac{\phi^{\prime 2}}{\phi^{2}}+\frac{\phi^{\prime}}{\phi}=\frac{r g}{2 M^{4}}(F-\Lambda), \tag{9}
\end{align*}
$$

where $\delta$ denotes the integration constant. Equation (7) represents the connection between source functions; it is simply a consequence of the conservation of the stress-energy tensor and can be also independently derived directly from $D_{A} T_{B}^{A}=0$.

Suppose that the "width" of brane sitting in the origin $r=0$ is equal to $\varepsilon$. It is worthwhile to mention that the brane is assumed to have the non-vanishing "brane width" since the "brane width" $\varepsilon$ appears in the later arguments of localization of the bulk fields and plays a role as a short-distance cutoff [25]. Outside a core of radius $\varepsilon$, we take the source functions in the form $F(r)=f \phi^{-2}$, where $f$ is some constant. Taking the first integral of the last equation [21], we get

$$
\begin{equation*}
r \phi^{\prime}=-\frac{\delta \Lambda}{10 M^{4}}\left(\phi^{2}-\frac{5 f}{3 \Lambda}\right)+\frac{C}{\phi^{3}}, \tag{10}
\end{equation*}
$$

where $C$ is the integration constant. Setting $C=0$, introducing the positive parameters

$$
\begin{equation*}
a=\frac{\delta \Lambda}{10 M^{4}}>0, \quad d^{2}=\frac{5 f}{3 \Lambda}>0, \tag{11}
\end{equation*}
$$

and imposing boundary condition at the infinity of transverse space,

$$
\begin{equation*}
\left.\phi(r)\right|_{r=+\infty}=\text { const }>0, \tag{12}
\end{equation*}
$$

we can easily find two solutions of eq. (10) in the following cases:
i) $\Lambda>0, f>0, \delta>0$

$$
\begin{equation*}
\phi(r)=d \frac{r^{b}-c}{r^{b}+c}, \quad g(r)=2 \delta d b \frac{c r^{b-2}}{\left(r^{b}+c\right)^{2}} ; \tag{13}
\end{equation*}
$$

ii) $\Lambda<0, f<0, \delta<0$

$$
\begin{equation*}
\phi(r)=d \frac{r^{b}+c}{r^{b}-c}, \quad g(r)=2|\delta| d b \frac{c r^{b-2}}{\left(r^{b}-c\right)^{2}} \tag{14}
\end{equation*}
$$

where $c>0$ is a positive integration constant and we have introduced the parameter $b=2 a d$. The first solution (13) is the same found in [24]. This solution exists in the case of positive bulk cosmological constant $\Lambda>0$. The scale factor $\phi(r)$ of this solution is an increasing function asymptotically approaching a finite value at radial infinity. The second solution (14) is the new one. It exists for the negative cosmological constant $\Lambda<0$, and its scale factor $\phi(r)$ is a decreasing function asymptotically approaching a finite non-zero value at radial infinity. In this letter we consider the second solution (the first one has already been examined in [24]).

To avoid singularities on the brane (sitting at the origin $r=0$ ), we take the boundary conditions for the solution in the form

$$
\begin{equation*}
\left.\phi(r)\right|_{r=\varepsilon}=1,\left.\quad \phi(r)\right|_{r=+\infty}=d \tag{15}
\end{equation*}
$$

In (15), $\varepsilon$ denotes the "brane width", which takes a fixed value. These boundary conditions allow us to express the integration constant $c$ in terms of the "brane width": $c=\frac{1-d}{1+d} \varepsilon^{b}$, where $0<d<1$.

Now we turn our attention to the problem of the localization of the bulk fields on the brane in the background geometry (14). Of course, in due analysis, we will neglect the back-reaction on the geometry induced by the existence of the bulk fields, and from now on, without loss of generality, we shall take a flat metric on the brane.

We start with a massless, spin- 0 , real scalar coupled to gravity:

$$
\begin{equation*}
S_{0}=-\frac{1}{2} \int \mathrm{~d}^{6} x \sqrt{-{ }^{6} g} g^{A B} \partial_{A} \Phi \partial_{B} \Phi . \tag{16}
\end{equation*}
$$

The corresponding equation of motion has the form

$$
\begin{equation*}
\frac{1}{\sqrt{-{ }^{6} g}} \partial_{M}\left(\sqrt{-{ }^{6} g} g^{M N} \partial_{N} \Phi\right)=0 . \tag{17}
\end{equation*}
$$

It turns out that in the background metric (14) the zero-mode solution of (17) is $\Phi_{0}\left(x^{M}\right)=$ $v\left(x^{\mu}\right) \rho_{0}$, where $\rho_{0}=$ const, and $v\left(x^{\mu}\right)$ satisfies the Klein-Gordon equation on the brane $\eta^{\mu \nu} \partial_{\mu} \partial_{\nu} v\left(x^{\mu}\right)=0$. Substituting this solution into the starting action (16), the action can be cast to

$$
\begin{align*}
S_{0} & =-\frac{1}{2} \rho_{0}^{2} \int_{0}^{2 \pi} \mathrm{~d} \theta \int_{\varepsilon}^{+\infty} \mathrm{d} r \phi^{2} g r \int \mathrm{~d}^{4} x \sqrt{-\eta} \eta^{\mu \nu} \partial_{\mu} v\left(x^{\alpha}\right) \partial_{\nu} v\left(x^{\alpha}\right)+\cdots=  \tag{18}\\
& =-\frac{\pi|\delta| \rho_{0}^{2}}{3}\left(1-d^{3}\right) \int \mathrm{d}^{4} x \sqrt{-\eta} \eta^{\mu \nu} \partial_{\mu} v\left(x^{\alpha}\right) \partial_{\nu} v\left(x^{\alpha}\right)+\cdots . \tag{19}
\end{align*}
$$

The integral over $r$ in (18) is finite, so the 4-dimensional scalar field is localized on the brane.
For the spin-(1/2) fermion the starting action is the Dirac action given by

$$
\begin{equation*}
S_{\frac{1}{2}}=\int \mathrm{d}^{6} x \sqrt{-^{6} g} \bar{\Psi} i \Gamma^{M} D_{M} \Psi \tag{20}
\end{equation*}
$$

from which the equation of motion is given by

$$
\begin{equation*}
\Gamma^{M} D_{M} \Psi=\left(\Gamma^{\mu} D_{\mu}+\Gamma^{r} D_{r}+\Gamma^{\theta} D_{\theta}\right) \Psi=0 . \tag{21}
\end{equation*}
$$

We decompose the 6 -dimensional spinor into the form $\Psi\left(x^{M}\right)=\psi\left(x^{\mu}\right) A(r) \sum e^{i l \theta}$, and require that the four-dimensional part satisfies the massless equation of motion $\gamma^{\mu} \partial_{\mu} \psi\left(x^{\beta}\right)=0$ and the chiral condition $\gamma^{r} \psi\left(x^{\mu}\right)=\psi\left(x^{\mu}\right)$. As a result, we obtain the following equation for the $s$-wave mode:

$$
\begin{equation*}
\left[\partial_{r}+2 \frac{\phi^{\prime}}{\phi}+\frac{1}{2} \frac{\partial_{r}\left(r g^{\frac{1}{2}}\right)}{r g^{\frac{1}{2}}}\right] A(r)=0 \tag{22}
\end{equation*}
$$

The solution to this equation reads: $A(r)=A_{0} \phi^{-2} g^{-\frac{1}{4}} r^{-\frac{1}{2}}$, with $A_{0}$ being an integration constant. Substituting this solution into the Dirac action (20) and integrating over $\theta$ we get

$$
\begin{equation*}
S_{\frac{1}{2}}=2 \pi A_{0}^{2} \sqrt{\frac{2|\delta| b c}{d}} \int_{\varepsilon}^{+\infty} \frac{r^{\frac{1}{2} b} \mathrm{~d} r}{r\left(r^{b}+c\right)} \int \mathrm{d}^{4} x \sqrt{-\eta} \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi+\cdots . \tag{23}
\end{equation*}
$$

As long as $b>0$ and the "brane width" $\varepsilon$ is non-vanishing the integral over $r$ is obviously finite. Indeed, the integrand in (23) scales as $r^{-\frac{1}{2} b-1}$ at radial infinity and is the smooth function between $r=\varepsilon$ and $r=+\infty$, so this integral over $r$ is finite even if the analytic expression is not available. So the spin-(1/2) fermion is localized on the brane only by the gravitational interaction.

Now let us consider the action of $U(1)$ vector field:

$$
\begin{equation*}
S_{1}=-\frac{1}{4} \int \mathrm{~d}^{6} x \sqrt{-{ }^{6} g} g^{A B} g^{M N} F_{A M} F_{B N}, \tag{24}
\end{equation*}
$$

where $F_{M N}=\partial_{M} A_{N}-\partial_{N} A_{M}$ as usual. From this action the equation of motion is given by

$$
\begin{equation*}
\frac{1}{\sqrt{-{ }^{6} g}} \partial_{M}\left(\sqrt{-{ }^{6} g} g^{M N} g^{R S} F_{N S}\right)=0 \tag{25}
\end{equation*}
$$

By choosing the gauge condition $A_{\theta}=0$ and decomposing the vector field as $A_{\mu}\left(x^{M}\right)=$ $a_{\mu}\left(x^{\mu}\right) \sum_{l, m} \sigma_{m}(r) e^{i l \theta}, A_{r}\left(x^{M}\right)=a_{r}\left(x^{\mu}\right) \sum_{l, m} \sigma_{m}(r) e^{i l \theta}$, it is straightforward to see that there is the $s$-wave $(l=0)$ constant solution $\sigma_{m}(r)=\sigma_{0}=$ const and $a_{r}=$ const. In deriving this solution we have used $\partial_{\mu} a^{\mu}=\partial^{\mu} f_{\mu \nu}=0$ with the definition of $f_{\mu \nu}=\partial_{\mu} a_{\nu}-\partial_{\nu} a_{\mu}$. As in the previous cases, let us substitute this constant solution into the action (24). It turns out that the action is reduced to

$$
\begin{align*}
S_{1} & =-\frac{\pi}{2} \sigma_{0}^{2} \int_{\varepsilon}^{+\infty} \mathrm{d} r g r \int \mathrm{~d}^{4} x \sqrt{-\eta} \eta^{\alpha \beta} \eta^{\mu \nu} f_{\alpha \mu} f_{\beta \nu}+\cdots=  \tag{26}\\
& =-\frac{\pi|\delta|}{2} \sigma_{0}^{2}(1-d) \int \mathrm{d}^{4} x \sqrt{-\eta} \eta^{\alpha \beta} \eta^{\mu \nu} f_{\alpha \mu} f_{\beta \nu}+\cdots . \tag{27}
\end{align*}
$$

As we can see from (27), the integral over $r$ in (26) is finite. Thus, the vector field is localized on the brane.

Next we consider the spin-(3/2) field (the gravitino). We begin with the action of the Rarita-Schwinger gravitino,

$$
\begin{equation*}
S_{\frac{3}{2}}=\int \mathrm{d}^{6} x \sqrt{-6} g \bar{\Psi}_{A} i \Gamma^{[A} \Gamma^{B} \Gamma^{C]} D_{B} \Psi_{C} \tag{28}
\end{equation*}
$$

from which the equation of motion is given by

$$
\begin{equation*}
\Gamma^{[A} \Gamma^{B} \Gamma^{C]} D_{B} \Psi_{C}=0 \tag{29}
\end{equation*}
$$

After taking the gauge condition $\Psi_{\theta}=0$ we look for the solutions of the form $\Psi_{\mu}\left(x^{A}\right)=$ $\psi_{\mu}\left(x^{\nu}\right) u(r) \sum e^{i l \theta}, \Psi_{r}\left(x^{A}\right)=\psi_{r}\left(x^{\nu}\right) u(r) \sum e^{i l \theta}$, where $\psi_{\mu}\left(x^{\nu}\right)$ satisfies the following equations: $\gamma^{\nu} \psi_{\nu}=\partial^{\mu} \psi_{\mu}=\gamma^{[\nu} \gamma^{\rho} \gamma^{\tau]} \partial_{\rho} \psi_{\tau}=0, \gamma^{r} \psi_{\nu}=\psi_{\nu}$. For the $s$-wave solution and $\psi_{r}\left(x^{\nu}\right)=0$, the equation of motion (29) gets the following form:

$$
\begin{equation*}
\left[\partial_{r}+\frac{3}{2} \frac{\phi^{\prime}}{\phi}+\frac{1}{2} \frac{\partial_{r}\left(r g^{\frac{1}{2}}\right)}{r g^{\frac{1}{2}}}\right] u(r)=0 \tag{30}
\end{equation*}
$$

The solution to this equation is $u(r)=u_{0} \phi^{-\frac{3}{2}} g^{-\frac{1}{4}} r^{-\frac{1}{2}}$, with $u_{0}$ being an integration constant. Substituting this solution into the action (28) and integrating over $\theta$ we get

$$
\begin{equation*}
S_{\frac{3}{2}}=2 \pi u_{0}^{2} \sqrt{\frac{2|\delta| b c_{2}}{d}} \int_{\varepsilon}^{+\infty} \frac{r^{\frac{1}{2} b}\left(r^{b}-c_{2}\right)}{r\left(r^{b}+c_{2}\right)^{2}} \mathrm{~d} r \int \mathrm{~d}^{4} x \sqrt{-\eta} \bar{\psi}_{\mu} i \gamma^{[\mu} \gamma^{\nu} \gamma^{\rho]} \partial_{\nu} \psi_{\rho}+\cdots \tag{31}
\end{equation*}
$$

As in the case of spin-(1/2) fermion (23), the integrand in (31) scales as $r^{-\frac{1}{2} b-1}$ at radial infinity and is a smooth function between $r=\varepsilon$ and $r=+\infty$, so the integral over $r$ is finite as long as the "brane width" $\varepsilon$ is non-zero. This means that the spin-(3/2) field is localized on the brane.

Now let us examine the spin-2 gravitational field. In this case we consider the spin-2 metric fluctuations $H_{\mu \nu}$ :

$$
\begin{equation*}
\mathrm{d} s^{2}=\left\{\phi^{2}(r) \eta_{\alpha \beta}\left(x^{\nu}\right)+H_{\alpha \beta}\right\} \mathrm{d} x^{\alpha} \mathrm{d} x^{\beta}-g(r)\left(\mathrm{d} r^{2}+r^{2} \mathrm{~d} \theta^{2}\right) . \tag{32}
\end{equation*}
$$

The corresponding equation of motion for the fluctuations has the following form:

$$
\begin{equation*}
\frac{1}{\sqrt{-{ }^{6} g}} \partial_{A}\left(\sqrt{-^{6} g} g^{A B} \partial_{B} H_{\mu \nu}\right)=0 . \tag{33}
\end{equation*}
$$

From the 4 -dimensional point of view, fluctuations are described by a tensor field $H_{\mu \nu}$ which is transverse and traceless: $\partial_{\mu} H_{\nu}^{\mu}=0, H_{\mu}^{\mu}=0$. We look for solutions of the form $H_{\alpha \beta}=$ $h_{\alpha \beta}\left(x^{\nu}\right) \sum_{m l} \tau_{m}(r) \exp [i \theta l]$, where $\partial^{2} h_{\mu \nu}\left(x^{\alpha}\right)=m_{0}^{2} h_{\mu \nu}\left(x^{\alpha}\right)$. It is easy to show that the equation of motion (33) has the zero-mass $\left(m_{0}=0\right)$ and $s$-wave $(l=0)$ constant solution $\tau_{0}=$ const. Substitution of this zero mode into the Einstein-Hilbert action leads to

$$
\begin{align*}
S_{2} & \sim 2 \pi \tau_{0}^{2} \int_{\varepsilon}^{+\infty} \mathrm{d} r \phi^{2} g r \int \mathrm{~d}^{4} x\left[\partial^{\rho} h^{\alpha \beta} \partial_{\rho} h_{\alpha \beta}+\cdots\right]=  \tag{34}\\
& =\frac{2}{3} \pi|\delta| \tau_{0}^{2}\left(1-d^{3}\right) \int \mathrm{d}^{4} x\left[\partial^{\rho} h^{\alpha \beta} \partial_{\rho} h_{\alpha \beta}+\cdots\right] . \tag{35}
\end{align*}
$$

The integral over $r$ is finite, so the bulk graviton is trapped on the brane. With our background metric the general expression for the four-dimensional Planck scale $M_{\mathrm{P}}$, expressed in terms of $M$, is $M_{\mathrm{P}}^{2}=2 \pi M^{4} \int_{\varepsilon}^{+\infty} \mathrm{d} r \phi^{2} g r=2 \pi \delta M^{4} \int_{\varepsilon}^{+\infty} \mathrm{d} r \phi^{2} \phi^{\prime}=\frac{2}{3} \pi|\delta| M^{4}\left(1-d^{3}\right)$. The inequality $M \ll M_{\mathrm{P}}$ is possible by adjusting $M^{4}$ and the product $|\delta|\left(1-d^{3}\right)$, and thus could lead to a solution of the gauge hierarchy problem.

Finally, let us consider the totally antisymmetric tensor fields. The action of $k$-rank totally antisymmetric tensor field $A_{k}$ is of the form

$$
\begin{equation*}
S_{k}=-\frac{1}{2} \int F_{k+1} \wedge * F_{k+1} \tag{36}
\end{equation*}
$$

where $F_{k+1}=d A_{k}$. The corresponding equation of motion is given by

$$
\begin{equation*}
d \wedge * F_{k+1}=0 \tag{37}
\end{equation*}
$$

It is easy to show that $A_{\mu_{1} \mu_{2} \ldots \mu_{k}}=a_{\mu_{1} \mu_{2} \ldots \mu_{k}}\left(x^{\nu}\right) u_{0}$ with $u_{0}=$ const is a solution to the equation of motion (37) if $d \wedge * f=0$, where $f=d a$. Substituting this solution in the action (36) leads to the expression

$$
\begin{equation*}
S_{k} \sim \int_{\varepsilon}^{+\infty} \mathrm{d} r \phi^{2-2 k} g r \int f_{k+1} \wedge * f_{k+1}+\cdots=\frac{|\delta|\left(1-d^{3-2 k}\right)}{(3-2 k)} \int f_{k+1} \wedge * f_{k+1}+\cdots \tag{38}
\end{equation*}
$$

As we can see from (38), the integral over $r$ is finite, so the totally antisymmetric tensor fields are also localized on the brane by the gravitational interaction.

In conclusion, in this letter we have presented a new 3-brane solution with a decreasing scale factor $\phi(r)$. This solution is found for the negative bulk cosmological constant $\Lambda<0$.

In addition, we have presented a complete analysis of localization of the zero-mode bulk fields on a brane via the gravitational interaction without including an additional interaction. The technical reason of the localization of zero-mode bulk fields lies not only in the fact that the scale factor $\phi(r)$ is a smooth function without singularities from the edge of the brane to radial infinity and approaches a definite non-zero value at infinity. The main reason is connected with the properties of the function $g(r)$. On the edge of the brane $(r=\varepsilon)$ it has a value $g(\varepsilon)=a \delta\left(1-d^{2}\right) \varepsilon^{-2}$ and fast enough tends to zero as one moves off the brane, so in our model the wave functions of the zero-mode solutions of the bulk fields along the extra dimensions are peaked at the location of the brane.

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