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Impulse distributions in dense granular flows: Signatures of large-scale spatial structures

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Abstract. – We study the impulse distribution in a model of gravity-driven granular flow and show that changes in the distribution are correlated with the appearance of clusters of "frequently-colliding" particles. Our simulations show increasingly large linear clusters as the flow velocity decreases. The dissipative nature of the collisions leads to an increase of small-impulse events as the size of these clusters grow and can explain the observed changes in the impulse distribution as the flow rate decreases.

Introduction. – Granular materials exhibit a wide spectrum of behaviour ranging from gaseous to liquid to solid. Remarkably, all of these phases of granular matter respond to external stimuli in a manner notably different from ordinary fluids and solids [1, 2]. Spatial inhomogeneities are thought to play a crucial role in determining the macroscopic properties of these systems. In static granular piles, the inhomogeneous stress distribution is strikingly demonstrated by the appearance of force chains [3, 4]. Experiments have also shown that the force distribution $P(f)$ at the boundaries of static piles is exponential at large forces and exhibits a plateau or peak at small forces [5]. The nature of inhomogeneities and the associated force distribution in flowing granular media is still a matter of some debate. Transient "clusters" have been identified experimentally in granular surface flows [6] and shear flows [7]. Both simulation [8, 9] and experiment [10, 11] have demonstrated that there are measurable differences between $P(f)$ in flowing and jammed granular flows. Computer simulations of granular chute flow [8] have shown that the flowing medium is characterized by an excess of small forces, and the number of small forces decreases as the flow velocity is decreased, an observation in agreement with simulations of supercooled liquids and foams [12]. These studies have suggested that $P(f)$ can be used as a static measure distinguishing between flowing and jammed regimes in both thermal and athermal systems. Recent experiments, however, have indicated that the complete picture is more complex. Experiments on dense gravity-driven, collisional granular flows done in a two-dimensional hopper geometry [10] measure an analogous quantity to $P(f)$, the impulse distribution $P(I)$. The observed $P(I)$ in these experiments shows a trend opposite to that observed in these previous works. In the present work we seek to shed some light on this interesting puzzle by investigating the properties of a simple model of the experimental system of ref. [10]: a two-dimensional gravity-driven system of monodispersed hard disks in a hopper geometry undergoing instantaneous inelastic collisions.

The monodispersed hard-disk system is a good idealization of the experimental system because: a) the grains used in the experimental system were also monodispersed and indeed, large crystalline domains were visible as they are in images of our simulated system and b) measurements of the time trace of impulses delivered to the wall in the experiment reveal that the flow is purely collisional for all measured flow rates, thus instantaneous inelastic collisions are a reasonable approximation.

Our study of this simple model provides some insight into the nature of the experimentally measured trend in the impulse distribution as well as the intriguing connection between this trend and the presence of dynamical heterogeneities. The main results of our analysis are a) a clear evidence of an *increasing* proportion of collisions with *small impulses* as the flow velocity is decreased; b) the formation of clusters of disks which collide "frequently" and are reminiscent of the "collapse strings" observed in freely cooling granular matter [13] and c) a clear correlation between the growth of these clusters and the increase of small-impulse events.

Simulations. – The grain dynamics used in the simulations are exactly as in ref. [14]. Specifically, i) at each interparticle collision, momentum is conserved and the energy loss is proportional to $(1-\mu^2)$, where μ is the coefficient of restitution; ii) to ensure that the pressure is independent of the height the side walls must absorb some vertical momentum, therefore we impose the condition that collisions with the walls are inelastic in the tangential direction; iii) since we wish to observe the system over many events, particles exiting the system at the bottom must be replaced at the top to create uniform, sustained flow. The flow velocity is controlled similarly to the experiments, by adjusting the width of the hopper opening. The introduction of a probability of reflection p at the bottom of the hopper [14] reduces the time needed to reach the steady-state flow and provides another parameter, besides the hopper opening, with which to tune the flow velocity. The reflection parameter was held constant throughout our simulations. Typically, our simulations were done on systems of 500 disks, with $\mu = 0.9$ and $p = 0.4$. The simulation was run for 2×10^6 events for each flow velocity, with 1.5×10^5 discarded initially to allow the system time to reach steady state, ensured by the time independence of the measured impulse distributions.

Simulation results. – As in the experiment of ref. [10] we measure the distribution of impulses transferred at each collision, $P(I)$. However, while in the experiment measurements were made at the wall of the hopper, we measure the magnitude of the momentum transfer at all collision events, both in the bulk and at the boundaries of the system [15]. We can separately analyze data from events located within the bulk, at the walls or at a specific point on the wall to mimic the experimental measurement. For most of our discussion we will focus on observations of $P(I)$ for the bulk of the material.

As in ref. [10], we observe an exponential form of $P(I)$ at impulses larger than the average impulse (fig. 1a) for the full range of flow rates. After scaling the impulse by the average impulse, $\langle I \rangle$, the curves collapse onto each other for large impulses. The small-impulse behaviour of the distribution, however, changes markedly with changes in the flow velocity. As the flow velocity decreases, the height of the distribution at impulses much smaller than the average impulse, $\langle I \rangle$, for example at $I_{\min} = 0.0075\langle I \rangle$, increases and $P(I)$ begins to develop a plateau at small I. In particular, the ratio of $P(I)$ at the peak impulse to $P(I)$ at $I_{\min}$ decreases by almost two orders of magnitude as the flow rate is decreased by a factor of three (see inset to fig. 1a).

The change in $P(I)$ at small I, observed in our simulations, is in the same direction as that observed in the experiments in the two-dimensional hopper flow. The experiments, however, do

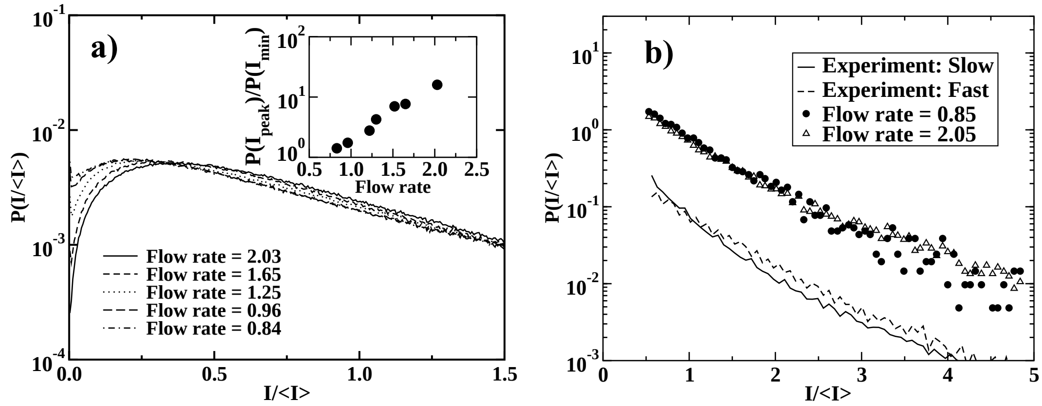

Fig. 1 – a) Simulation results for impulse distribution scaled to the average impulse, $\langle I \rangle$, with flow velocity decreasing from the bottom curve to the top curve. The inset shows the ratio of $P(I_{\rm peak})$ to $P(I_{\rm min})$ as a function of the flow velocity. b) Comparison of simulation results with experimental data obtained from [16]. Note that the data obtained from the simulation have been displaced with respect to the experimental data for clarity. Without this displacement the lines would lie on top of each other.

not observe the peak and its disappearance. Keeping in mind that the experiment had a finite impulse threshold due to the resolution of the transducer and measured impulse distributions at the wall, we carried out measurements which mimicked these conditions and found that the resulting impulse distributions were in semi-quantitative agreement with the experiments (fig. 1b). These findings indicate that the details of the measurement can influence the exact form of $P(I)$ but the overall effect of increasing small-impulse events with decreasing flow rates is more robust [17]. In addition, these results provide some evidence that the model is capturing the essential features of the experiment.

If the hard disks behaved as completely uncorrelated particles, the impulse distribution would be a convolution of the individual momentum (velocity) distributions. Since there is an average flow velocity, this would give rise to a peak in the distribution and the large-impulse behaviour would reflect the form of the velocity distribution at large velocities. These observations imply that the exponential tail in $P(I)$ arises from uncorrelated particles and is a consequence of the shape of the velocity distribution [18]. In contrast, the changes in shape of $P(I)$ at small values of I are difficult to justify from the perspective of uncorrelated particles and indicate increasing correlations among the disks as the flow velocity decreases.

Spatial structures. – To explore the nature of correlations and possible connections between the changes in $P(I)$ and the appearance of spatial inhomogeneities, we considered a question first asked in studies of inelastic collapse in freely cooling granular gases [20], "*How many collisions does a given grain undergo in a fixed number of events?*" We can construct images of our simulated system at regular intervals and colour individual disks according to the number of collisions they have experienced in the most recent interval (fig. 2a, b). The number of collisions divided by the total time of the interval yields a collision frequency; based on review of the images we chose a threshold frequency of 1750 (in inverse simulation time units) and defined all particles with frequencies above this threshold as *frequently colliding*.

As we decrease the flow velocity, the frequently colliding particles form increasingly larger linear clusters (compare fig. 2a, where $v_{\rm f} = 2.03$ in our units or 35.6 cm/s and fig. 2b, where

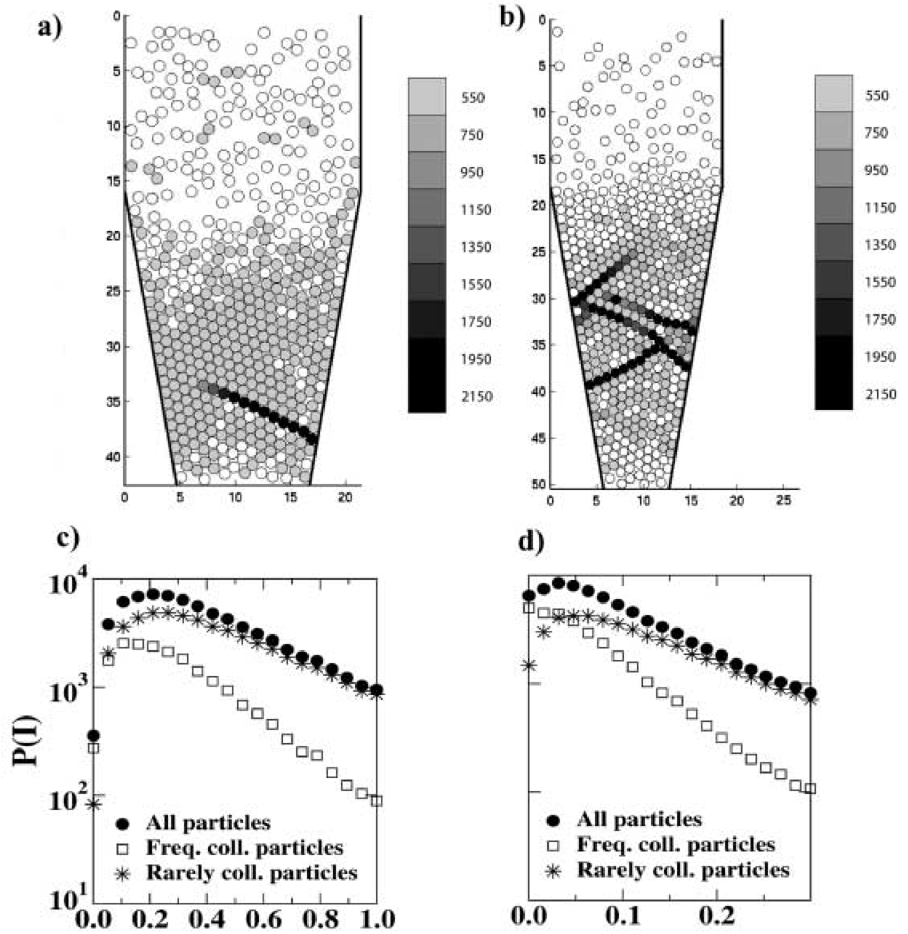

Fig. 2 – a), b) Sample image of simulation for $v_{\mathrm{f}} = 2.03$ (a) and for $v_{\mathrm{f}} = 1.2$ (b). Frequencies listed beside the scale are in units of inverse simulation time. c), d) Impulse distributions for all particles, frequently colliding particles and rarely colliding particles for $v_{\mathrm{f}} = 2.03$ (c) and for $v_{\mathrm{f}} = 0.96$ (d). Note that these distributions are unnormalized.

$v_{\mathrm{f}} = 1.2$ or 21.1 cm/s [21]). These 1D structures observed in our simulation are reminiscent of the transient solid chains postulated by the hydrodynamic model of ref. [22]. Comparison of the impulse distribution of the frequently colliding particles and the impulse distribution of the remaining "rarely colliding" particles (fig. 2c, d) reveals that it is the contribution from the frequently colliding particles which causes the height of the total impulse distribution at I_{min} to increase in the manner seen in fig. 1a. The large-impulse behaviour appears to be dominated by the rarely colliding particles. These observations are relatively insensitive to changes in the threshold frequency provided it is larger than a minimum value.

Growing clusters of frequently colliding particles lead to inelastic collapse in freely cooling hard-disk systems and can be avoided by making collisions elastic if the relative velocity of the colliding particles becomes smaller than a cutoff [23]. Since we observe growing number of collisions with small values of the relative velocity, we have verified that our results are insensitive to such a cutoff velocity as long as the cutoff is small compared to the average flow velocity.

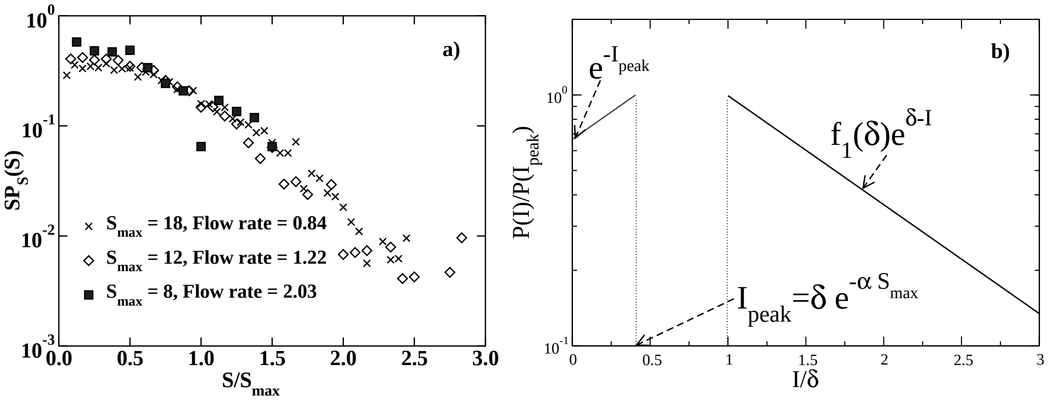

Fig. 3 – a) Simulation results for $SP_S(S)$ for varying flow rates. b) Illustration of analytic results, cf. text for discussion.

Calculation of impulse distribution. – The observed impulse distribution can be approximately calculated by considering the simulation system as being comprised of two types of particles, as shown in fig. 2: i) the linear chains of frequently colliding particles and ii) the remainder of the system. The linear chains are undergoing many inelastic collisions which act to make the velocities of individual grains within the chain similar. The velocities of the rarely colliding particles, on the other hand, are essentially uncorrelated. Thus, we can further idealize to a system of one-dimensional clusters of particles moving with the same velocity in a bath of uncorrelated hard disks.

Consider a one-dimensional cluster of particles which are all moving with the same velocity. Now if another particle, travelling at some speed v relative to the cluster, collides inelastically with one end of the chain, the impulse associated with that collision will be $(1-\epsilon)v$, where ϵ is related to the coefficient of restitution by $\epsilon = \frac{(1-\mu)}{2}$ [24]. If the chain were only comprised of one grain, the impulse distribution $P(I|v,1)$ (given an initial incoming speed v) would be a single spike of height 1 located at $(1-\epsilon)v$. For two particles, $P(I|v,2)$ would have two spikes of height 1 at $(1-\epsilon)^2 v$ and $(1-\epsilon)v$. Continuing this argument for clusters containing S particles,

$$P(I|v,S) = \sum\nolimits_{i=1}^{S} \delta\big(I - (1-\epsilon)^i v\big). \tag{1}$$

It is clear that as S becomes very large, the leftmost limit of $P(I|v,S)$ approaches zero. Given a distribution of speeds $P_v(v)$ for the incident particle, and a distribution of cluster sizes $P_S(S)$, the total impulse distribution $P(I)$ is

$$\begin{aligned} P(I) &= \int \mathrm{d}S\, \mathrm{d}v P_v(v) P_S(S) P(I|v,S) \\ &= \int \mathrm{d}S P_S(S) \sum\nolimits_{i=1}^{S} e^{\alpha i} P_v(e^{\alpha i} I), \end{aligned} \tag{2}$$

where $\alpha = -\ln(1-\epsilon)$. If the cluster size distribution falls off sharply, very few terms from eq. (2) will contribute and the shape of $P(I)$ will essentially reflect the shape of $P_v(v)$. If, however, the cluster size distribution becomes broad, the small-impulse end of $P(I)$ flattens out due to the superposition of a large number of terms from eq. (2).

The cluster size distribution observed in our simulations is consistent with the form $P_S(S) = \exp[-S/S_{\max}]/S$ with $S_{\max}$ increasing as the flow velocity decreases (cf. fig. 3).

Note that a change in the threshold frequency will produce a change in $S_{\max}$ for a given flow velocity but will not alter the shape of $P_S(S)$ nor the trend of increasing $S_{\max}$ with decreasing flow velocity. The shape of $P(I)$ seen in fig. 2 can be qualitatively explained by eq. (2) and the increasing value of $S_{\max}$.

The asymptotic forms of $P(I)$ predicted by eq. (2) can be explicitly calculated by adopting an exponential form for the velocity distribution, $P_v(v) \simeq e^{-a|v-\delta|}$, where δ represents a characteristic velocity of the linear chains relative to the average flow velocity. We assume that the dominant effect of the flow velocity is a change in $P_S(S)$ and that a and δ are weak functions of the flow velocity. The calculated asymptotic form for $I > \delta$ is $P(I) \simeq f_1(\delta, S_{\max}) e^{-(I-\delta)} e^{\delta e^{-\alpha S_{\max}}} P(I = 0)$, where $f_1 \simeq 1.0$ is a slowly varying function of δ and $S_{\max}$. For $I < \delta e^{-\alpha S_{\max}}$, $P(I) = P(I = 0) e^{aI}$. The limiting value of $P(I)$ as $I \to 0$ is $\simeq \int_1^{S_{\max}} (\mathrm{d}s/s)(e^{\alpha s} - 1)$ and increases with $S_{\max}$. In the intermediate regime, $\delta e^{-\alpha S_{\max}} < I < \delta$, $P(I)$ does not have a simple form but it can be shown that it is a slowly varying function in this regime, increasing logarithmically but decreasing quadratically as $I \to \delta$. The peak of $P(I)$, therefore, occurs at $I \simeq \delta e^{-\alpha S_{\max}}$ and $P(I_{\text{peak}})/P(I = 0) \simeq e^{a\delta e^{-\alpha S_{\max}}}$. This ratio is $\simeq e^{a\delta}$ for small values of $S_{\max}$ and approaches unity as $S_{\max} \to \infty$. The peak in $P(I)$ also shifts to smaller values of I/δ and the intermediate region of slow variation increases as $\delta(1 - e^{-\alpha S_{\max}})$. All of these features are in qualitative agreement with the results of our simulations. In fig. 3b, we have shown this analytic form of $P(I)$. This figure shows that the calculated function captures the interesting variation of the observed $P(I)$. However, the variations are much weaker than that observed in the simulations and the behaviour at large flow velocities is not captured by the analytic form. This is not surprising, since modeling the system as a collection of linear chains in a bath of uncorrelated particles and parametrizing the flow by $S_{\max}$ should breakdown at large flow velocities where the clusters become ill-defined.

The picture which seems to be emerging from our simulations is that of increasingly larger scale dynamical heterogeneities developing as the flow rate decreases in our simulated system. Since the clusters are essentially the same as the ones identified in freely cooling granular matter, their origin lies in the dissipative nature of the medium. Dissipation is also crucial in determining the nature of the changes in $P(I)$: small-impulse events occur in long chains because of the inelastic nature of the collisions. The clusters are dynamic in nature and do not appear to be correlated with a specific local ordering: measurements of the distribution of bond orientational order [25] of the frequently colliding grains and the entire system reveal that both sets of particles have the same distribution. The connection between previous work concerning $P(f)$ in soft systems and our results for $P(I)$ in the hard-disk system remains to be examined. In addition, a possible relationship of the clusters that we have identified to incipient force chains can be explored, for example, through the calculation of stress correlations in the flowing medium.

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