A LETTERS JOURNAL EXPLORING
THE FRONTIERS OF PHYSICS

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EUROPHYSICS LETTERS 15 July 2004

Europhys. Lett., **67** (2), pp. 273–279 (2004)

DOI: 10.1209/epl/i2004-10065-1

Electric instability in superconductor-normal conductor ring

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(received 26 February 2004; accepted in final form 11 May 2004)

PACS. 74.45.+c – Proximity effects; Andreev effect; SN and SNS junctions.
PACS. 85.25.Dq – Superconducting quantum interference devices (SQUIDs).

Abstract. – Non-linear electrodynamics of a ring-shaped Andreev interferometer (superconductor/normal conductor/superconductor hybrid structure) coupled to a circuit of the dissipative current is investigated. The current-voltage characteristics (CVC) is demonstrated to be a series of loops with several branches intersecting at the CVC origin. The sensitivity of the transport current J_d (or the applied voltage V) to a change of the external magnetic flux can be the same as the sensitivity of conventional SQUIDs. Spontaneous arising of coupled non-linear oscillations of the transport current, the Josephson current and the magnetic flux in Andreev interferometers are also predicted and investigated. The frequency of these oscillations can be varied in a wide range, while the maximal frequency can reach $\omega_\mathrm{max} \sim 10^{12}\,\mathrm{s}^{-1}$.

Recently, much attention has been paid to the charge transport in mesoscopic systems which combine normal conductor (N) and superconductor (S) elements (for review papers, see, *e.g.*, [1, 2] and references therein). In such hybrid structures the superconducting correlations penetrate into the normal conductor changing its transport properties. This quantum effect is most pronounced in S-N-S structures ("Andreev interferometer") in which the quantum interference gives rise to a high sensitivity of the superconducting correlations to the phase difference between the superconductors φ. It results, in particular, in oscillations of the Josephson current $J_\mathrm{s} = J_\mathrm{s}(\varphi)$ with a change of φ [2].

If the normal region of an Andreev interferometer is connected to a voltage biased normal reservoir, the injection current $J_\mathrm{d} = G(\varphi)V$ is also phase-dependent ($G(\varphi)$ is the conductance of the system, V is the bias voltage) [1].

In a ring-shaped geometry of the S-N-S structure, the phase difference between the superconductors is controlled by the magnetic flux $\Phi = HS_\mathrm{r}$ (H is the magnetic field threading the ring, S_r is the area of the ring): $\varphi = -2\pi\Phi/\Phi_0$, where the flux quantum $\Phi_0 = \pi\hbar c/e$ (c is the light velocity, e is the electron charge). It allows to control the Josephson current $J_\mathrm{s}(\varphi)$ and the dissipative current $J_\mathrm{d}(\varphi)$ with a change of the external magnetic field H_ext. On the other hand, the Josephson current (which depends on the magnetic flux threading the ring) creates its proper magnetic field which modifies this flux. It results in hysteresis loops in the dependence of Φ on H_ext as shown in fig. 1 (see, *e.g.*, [3]).

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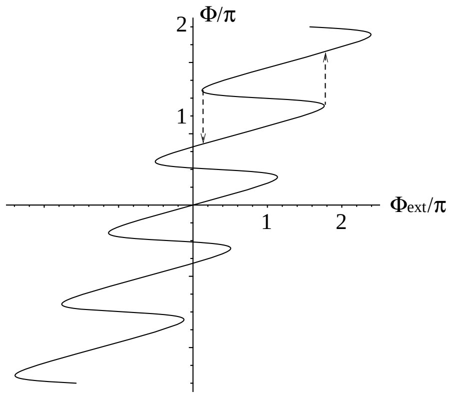

Fig. 1 – Dependence of the magnetic flux threading the ring, Φ, on the applied flux $\Phi_{\mathrm{ext}} = H_{\mathrm{ext}} S_{\mathrm{r}}$.

An injection of the dissipative current J_{d} in the normal segment of the S-N-S ring creates a new situation in which the Josephson current and the dissipative current are coupled due to their influence on the magnetic flux inside the S-N-S ring that, in its turn, affects both currents themselves.

The aim of this paper is to show that the inductive interaction between the dissipative and Josephson currents results in a complicated loop-shaped form of the current-voltage characteristics (CVC) of S-N-S structures schematically presented in fig. 2. In the general case, several branches of the CVC (which correspond to different values of the magnetic flux Φ inside the ring) intersect at the origin of CVC ($J_{\mathrm{d}} = 0$, $V = 0$), as is shown in the insets of figs. 4 and 5 below. Stable non-linear periodic-in-time oscillations of the dissipative current J_{d}, the Josephson current J_{s} and the magnetic flux Φ inside the ring (and hence the phase difference φ) are predicted and investigated. The form of the loop-shaped CVC and the frequency of these oscillations are shown to be extremely sensitive to the applied external magnetic field H_{ext} and the bias voltage V.

Loop-shaped current-voltage characteristics. – Injection of a dissipative current, $J_{\mathrm{d}} = G(\varphi)V$, in the normal part of an S-N-S system affects the Josephson current and adds an additional anomalous current in the S-N-S structure [4]. In the limit of a low applied voltage $eV \ll \Delta$ (Δ is the superconductor energy gap) and a weak coupling to the normal reservoir

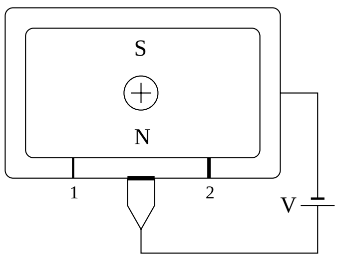

Fig. 2 – Superconductor/normal conductor/superconductor structure of the Andreev interferometer type to which a voltage drop V is applied. Thick lines indicate potential barriers at normal conductor/superconductor interfaces 1 and 2, and between the lead and the normal section of the interferometer.

$t_{\mathrm{r}} \ll 1$ (t_{r} is the transparency of the potential barrier between the normal segment and the biased reservoir), the non-equilibrium Josephson current is close to the equilibrium one, $J_{\mathrm{s}}(\phi)$, while the distribution of the injected current inside the ring is non-trivial, being dependent on many parameters [4]. In order to find the current-voltage characteristics in this limit, it is convenient to divide the magnetic flux induced by the currents into two parts: the magnetic flux of the Josephson current $\Phi_{\mathrm{s}} = (\mathcal{L}_{\mathrm{r}}/c)J_{\mathrm{s}}$ ($\mathcal{L}_{\mathrm{r}}$ is the self-inductance of the ring) which does not depend on the injection current, and the magnetic flux $\Phi_{\mathrm{d}} = (\mathcal{L}_{12}/c)J_{\mathrm{d}}$ which is controlled by the injected current J_{d}, where $\mathcal{L}_{12}$ is the mutual inductance of the ring and the dissipative current circuit. In the sample geometry of fig. 2, $\mathcal{L}_{12}$ depends on the distribution of the anomalous current inside the ring; below, I consider $\mathcal{L}_{12}$ as a phenomenological parameter. For the sample geometry of fig. 2 its value can be estimated as $\mathcal{L}_{12} < L$, $\mathcal{L}_{12} \sim L$ (L is the length of the ring).

Therefore, the total flux of the magnetic field, Φ, threading the ring can be written as $\Phi = \Phi_{\mathrm{ext}} + \Phi_{\mathrm{s}} + \Phi_{\mathrm{d}}$ (here $\Phi_{\mathrm{ext}} = H_{\mathrm{ext}} S_{\mathrm{r}}$). This equation and the relation $J_{\mathrm{d}} = G(\varphi)V$ define the following parametric form of the CVC:

$$\begin{aligned} J_{\mathrm{d}} &= J_{\mathrm{d}}(\varphi) \equiv \frac{1}{\mathcal{L}_{12}}\left(\frac{c\Phi_0}{2\pi}(\varphi_{\mathrm{ext}} - \varphi) - \mathcal{L}_{\mathrm{r}} J_{\mathrm{s}}(\varphi)\right), \\ V &= \frac{J_{\mathrm{d}}(\varphi)}{G(\varphi)}, \quad \varphi_{\mathrm{ext}} \equiv -2\pi\Phi_{\mathrm{ext}}/\Phi_0 \,. \end{aligned} \tag{1}$$

One of the distinguishing features of the CVC of the system under consideration is its "manifold degeneracy": several branches of the CVC (which correspond to different values of the magnetic flux Φ) can intersect at its origin ($V = 0$, $J_{\mathrm{d}} = 0$). As can be seen from eq. (1) and fig. 1, the number of the branches passing the CVC origin is equal to the number of the intersections of the vertical line $\Phi_{\mathrm{ext}} = \mathrm{const}$ and the curve $\Phi = \Phi(\Phi_{\mathrm{ext}})$.

Other key features of the CVC can be found if one considers the differential resistance $\mathrm{d}V/\mathrm{d}J_{\mathrm{d}}$ which is readily obtained from eq. (1):

$$\frac{\mathrm{d}V}{\mathrm{d}J_{\mathrm{d}}} = \frac{1}{G(\varphi)}\left(1 + \frac{2\pi}{c\Phi_0}\frac{\mathcal{L}_{12}J_{\mathrm{d}}}{G(\varphi)}\frac{G'(\varphi)}{A(\varphi)}\right)_{\varphi=\overline{\varphi}(J_{\mathrm{d}})} . \tag{2}$$

Here the prime means the derivative with respect to φ; the function $A(\varphi)$ [5] is

$$A(\varphi) = 1 + \frac{2\pi\mathcal{L}_{\mathrm{r}}}{c\Phi_0} J_{\mathrm{s}}'(\varphi). \tag{3}$$

All quantities in eq. (2) are taken at $\varphi = \overline{\varphi}(J_{\mathrm{d}})$, which is a solution of the first equation in eq. (1). Using eq. (1) one finds

$$\mathrm{d}\overline{\varphi}/\mathrm{d}J_{\mathrm{d}} = -\frac{2\pi\mathcal{L}_{12}}{c\Phi_0}\frac{1}{A(\overline{\varphi})} \,. \tag{4}$$

If $\mathcal{L}_{\mathrm{r}} > \mathcal{L}_{\mathrm{cr}}^{(1)} \equiv c\Phi_0/(2\pi \max\{|J_{\mathrm{s}}'|\})$, there are values of φ at which $A(\varphi) = 0$. Therefore, in this case, as follows from eq. (2) the CVC $J_{\mathrm{d}}(V)$ inevitably has points at which the differential resistance $\mathrm{d}V/\mathrm{d}J_{\mathrm{d}}$ tends to infinity changing its sign there. On the other hand, the derivative G' also changes its sign with a change of φ that can provide points at which $\mathrm{d}V/\mathrm{d}J_{\mathrm{d}}$ goes to zero changing its sign at them. The consecutive order of these peculiarities with an increase of J_{d} (that is the form of the CVC) depends on the relative positions of the maxima and minima of $G(\varphi)$ and $J_{\mathrm{s}}(\varphi)$.

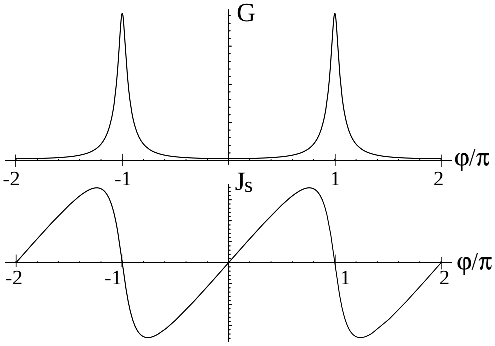

Fig. 3 – Typical dependences of the Josephson current and the conductance on the superconductor phase difference φ for the case of a low transparency ($t_\mathrm{r} \ll 1$) of the potential barrier between the normal section of the sample and the lead.

For the case of a low transparency of the potential barrier $t_\mathrm{r} \ll 1$, the conductance $G(\varphi)$ has maxima at odd numbers of π [1] while the Josephson current $J_\mathrm{s} = 0$ at odd and even numbers of π [2], as schematically shown in fig. 3. This general information together with eqs. (2)-(4) would suffice to find the CVC to be loop-shaped if $\mathcal{L}_\mathrm{r} > \mathcal{L}_\mathrm{cr}^{(1)}$. In order to see it, let us start from the magnitude of the external magnetic field which corresponds to $\overline{\varphi} = 0$. At $\overline{\varphi} = 0$, one has $G' = 0$, $J' > 0$ and hence $\mathrm{d}V/\mathrm{d}J_\mathrm{d} > 0$ (see eq. (2)). As at this point $\mathrm{d}\overline{\varphi}/\mathrm{d}J_\mathrm{d} < 0$, the phase difference $\overline{\varphi}$ decreases with an increase of the current J_d (see eq. (4)). Therefore, with an increase of the current G' becomes negative and after passing the minimum of $J_\mathrm{s}(\overline{\varphi})$, $A(\overline{\varphi})$ starts to decrease because $J' < 0$ now (see fig. 3); when A is small enough (but positive), the differential resistance becomes $\mathrm{d}V/\mathrm{d}J_\mathrm{d} = 0$. With a further increase of the current, $A(\overline{\varphi}) \to +0$ while G' remains negative, and hence $\mathrm{d}V/\mathrm{d}J_\mathrm{d} \to -\infty$. When $A(\overline{\varphi}) = -0$, the differential resistance $\mathrm{d}V/\mathrm{d}J_\mathrm{d} = +\infty$. In order to follow this second branch of the CVC one should decrease the current because one has $\mathrm{d}\overline{\varphi}/\mathrm{d}J_\mathrm{d} > 0$ now (see eq. (4)). Pursuing such a reasoning, one easily finds the current-voltage characteristic to be a series of loops which touch the lines $J_\mathrm{d} = G_\mathrm{min}V$ and $J_\mathrm{d} = G_\mathrm{max}V$ (in our case, $G_\mathrm{min} = G(0)$, $G_\mathrm{max} = G(\pi)$); the number of loops intersecting at the origin of the CVC increases with an increase of the ring inductance.

Examples of such current-voltage characteristics are presented in fig. 4 for the case that the normal segment of the Andreev interferometer is diffusive, and in fig. 5 for the case that it is ballistic. In both cases, potential barriers are present at the N/S interfaces.

For numerical calculations of the CVC in the diffusive case I have used the following formulae for the conductance [6] and the Josephson current [2], respectively:

$$G(\varphi) = \frac{G_\mathrm{N}}{\tilde{r}_\mathrm{b}\sqrt{1 - r_\mathrm{b}\sin^2\varphi/2}}, \qquad J_\mathrm{s}(\varphi) = J_\mathrm{c}\sin\varphi, \tag{5}$$

where G_N is the conductance of the normal metal in the ring, $\tilde{r}_\mathrm{b} = (G_\mathrm{b1} + G_\mathrm{b2})/G_\mathrm{b3}$ and $r_\mathrm{b} = 4G_\mathrm{b1}G_\mathrm{b2}/(G_\mathrm{b1} + G_\mathrm{b2})^2$, G_b1 and G_b2 are the N/S barrier conductances, G_b3 is the conductance of the barrier between the normal section of the ring and the lead, J_c is the Josephson critical current.

For numerical calculations of the CVC in the ballistic case I have used the following

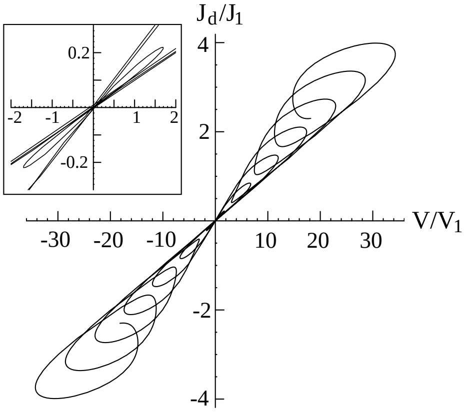

Fig. 4

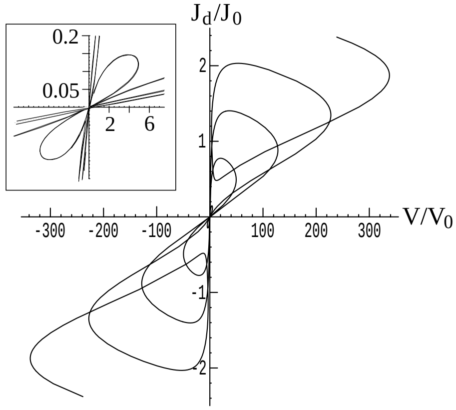

Fig. 5

Fig. 4 – Current-voltage characteristics of a diffusive S-N-S ring for the external flux $\Phi_{\mathrm{ext}}/\Phi_0$ equal to odd numbers of π; the ring self-inductance is $\mathcal{L}_{\mathrm{r}} = 2.5c\Phi_0/J_{\mathrm{c}}$; $\tilde{r}_{\mathrm{b}} = 10$, $r_{\mathrm{b}} = 3/4$; $J_1 = (\mathcal{L}_{\mathrm{r}}/\mathcal{L}_{12})J_{\mathrm{c}}$, $V_1 = J_1/G_{\mathrm{N}} \sim (\mathcal{L}_{\mathrm{r}}/\mathcal{L}_{12})\hbar v_{\mathrm{F}}/(eL_{\mathrm{N}})$. The inset shows a zoomed vicinity of the CVC origin.

Fig. 5 – Current-voltage characteristics of a ballistic S-N-S ring for the external flux $\Phi_{\mathrm{ext}}/\Phi_0$ equal to odd numbers of π; the barrier transparencies are $t_{\mathrm{r}} = 0.1$, $t_{\mathrm{N}}^{(1)} = 0.2$, $t_{\mathrm{N}}^{(2)} = 0.25$; the ring self-inductance is $\mathcal{L}_{\mathrm{r}} = 2.5c\Phi_0/J_{\mathrm{c}}^{(\mathrm{b})}$; $J_0 = (\mathcal{L}_{\mathrm{r}}/\mathcal{L}_{12})J_{\mathrm{c}}^{(\mathrm{b})}$, $V_0 = J_0/G_0 = (\mathcal{L}_{\mathrm{r}}/\mathcal{L}_{12})\hbar v_{\mathrm{F}}/(eL_{\mathrm{N}})$. The inset shows a zoomed vicinity of the CVC origin.

formulae for the conductance $G(\varphi)$ [7] and the Josephson current $J_{\mathrm{s}}(\varphi)$:

$$G(\varphi) \;=\; G_0 \frac{t_{\mathrm{r}}^2}{\sqrt{\left(1 + \left|r_{\mathrm{A}}^{(1)} r_{\mathrm{A}}^{(2)}\right| \cos\varphi + t_{\mathrm{r}}^2/2\right)^2 - \left|r_{\mathrm{N}}^{(1)} r_{\mathrm{N}}^{(2)}\right|^2}}\,, \tag{6}$$

$$J_{\mathrm{s}}(\varphi) \;=\; J_{\mathrm{c}}^{(\mathrm{b})} \left|r_{\mathrm{A}}^{(1)} r_{\mathrm{A}}^{(2)}\right| \sin\varphi \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi \sin\varphi_-(\phi)} \arctan \frac{e^{-2t_{\mathrm{r}}} \sin\varphi_-(\phi)}{1 - e^{-2t_{\mathrm{r}}} \cos\varphi_-(\phi)}\,. \tag{7}$$

Here $G_0 = N_\perp e^2/(\pi\hbar)$ and $J_{\mathrm{c}}^{(\mathrm{b})} = 2ev_{\mathrm{F}}^{\|} N_\perp/(\pi L_{\mathrm{N}})$; $N_\perp = S_{\mathrm{N}}/\lambda_{\mathrm{F}}^2$, S_{N} and L_{N} are the cross-section area and the length of the normal section of the ring, respectively, p_{F} is the Fermi momentum; $v_{\mathrm{F}}^{\|} = N_\perp^{-1} \sum_{\vec{n}}^{N_\perp} v_{\vec{n}} \sim v_{\mathrm{F}}$ and $v_{\vec{n}}$ is the electron velocity in the $\vec{n}$-th transverse mode; $r_{\mathrm{N}}^{(1,2)}$ are the normal reflection amplitudes at N/S interfaces 1 and 2 while $r_{\mathrm{A}}^2 + r_{\mathrm{N}}^2 = 1$, (see [8]);

$$\cos\varphi_-(\phi) = \left|r_{\mathrm{N}}^{(1)} r_{\mathrm{N}}^{(2)}\right| \cos\phi - \left|r_{\mathrm{A}}^{(1)} r_{\mathrm{A}}^{(2)}\right| \cos\varphi, \qquad \sin\varphi_-(\phi) = \sqrt{1 - \cos^2\varphi_-(\phi)}\,.$$

Expanding eq. (7) in $\exp[\varphi_-]$ and taking the limit $t_{\mathrm{r}} \to 0$, $t_{\mathrm{N}}^{(1,2)} \to 0$, one reduces it to the well-known expression for the Josephson current in a 3D SNS junction [9].

Electromagnetic self-oscillations. – A change of the magnetic flux threading the S-N-S ring results in the following current flowing in the ring:

$$J_{\mathrm{r}} = J_{\mathrm{s}}(\varphi) - \frac{1}{cR} \frac{\mathrm{d}\Phi}{\mathrm{d}t}\,, \tag{8}$$

where R is the resistance of the normal segment of the S-N-S ring [10]. Using the relation $\varphi = -2\pi\Phi/\Phi_0$, one sees eq. (8) to be the equation of the ac Josephson effect.

Using eq. (8), one gets the following set of equations that describes the time evolution of the transport current and the superconductor phase difference:

$$\begin{aligned}\frac{\mathcal{L}_{11}}{c^2}\frac{\mathrm{d}J_\mathrm{d}}{\mathrm{d}t}+\frac{\mathcal{L}_{12}\Phi_0}{2\pi c^3 R}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+\frac{\mathcal{L}_{12}J_\mathrm{s}'(\varphi)}{c^2}\frac{\mathrm{d}\varphi}{\mathrm{d}t}+\frac{J_\mathrm{d}}{G(\varphi)} &= V,\\ \frac{\Phi_0\mathcal{L}_\mathrm{r}}{2\pi cR}\frac{\mathrm{d}\varphi}{\mathrm{d}t}+\frac{c\Phi_0}{2\pi}\varphi+\mathcal{L}_\mathrm{r}J_\mathrm{s}(\varphi)+\mathcal{L}_{12}J_\mathrm{d} &= -c\Phi_\mathrm{ext}\,.\end{aligned}\tag{9}$$

Static solutions of eq. (9) correspond to the CVC of the ring (eq. (1)).

In order to investigate the time evolution of the system, it is convenient to eliminate J_d from the set of equations (9) and get the following equation for $\varphi(t)$:

$$\frac{\mathcal{L}_\mathrm{eff}}{Rc^2}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+\gamma(\varphi)\frac{\mathrm{d}\varphi}{\mathrm{d}t}+F(\varphi)=0,\tag{10}$$

where $\mathcal{L}_\mathrm{eff}=(\mathcal{L}_\mathrm{r}\mathcal{L}_{11}-\mathcal{L}_{12}^2)/\mathcal{L}_{11}$. Equation (10) is the equation for a non-linear oscillator under the "friction"

$$\gamma(\varphi)=1+\frac{2\pi\mathcal{L}_\mathrm{eff}}{c\Phi_0}J_\mathrm{s}'(\varphi)+\frac{\mathcal{L}_\mathrm{r}}{\mathcal{L}_{11}}\frac{1}{RG(\varphi)}\tag{11}$$

and the "force"

$$F(\varphi)=\frac{c^2}{\mathcal{L}_{11}G(\varphi)}\left(\varphi-\varphi_\mathrm{ext}+\frac{2\pi\mathcal{L}_\mathrm{r}}{c\Phi_0}J_\mathrm{s}(\varphi)+\frac{2\pi\mathcal{L}_{12}}{c\Phi_0}G(\varphi)V\right).\tag{12}$$

For low values of $\mathcal{L}_{11}$ the "friction" $\gamma>0$ at any value of φ and hence, starting from any initial state, the system approaches one of its static states determined by eq. (1). With an increase of $\mathcal{L}_{11}$, the "friction" $\gamma(\varphi)$ becomes negative in a certain range of φ and the static state can become unstable. Investigations of the stability of the static solutions of eq. (10) $\varphi=\varphi_\mathrm{st}$ show that the critical value of $\mathcal{L}_{11}$ is determined by the condition $\gamma(\varphi_\mathrm{st})=0$, that is

$$\mathcal{L}_\mathrm{cr}=\frac{\mathcal{L}_\mathrm{r}}{BRG(\varphi_\mathrm{st})}\,,\qquad B=-\left(1+\frac{2\pi\mathcal{L}_\mathrm{eff}}{c\Phi_0}J_\mathrm{s}'(\varphi_\mathrm{st})\right)>0.\tag{13}$$

The Poincaré method [11] shows that in the plane $(\varphi,\dot\varphi)$, a stable limit cycle arises if

$$L_{11}>\mathcal{L}_\mathrm{cr}\,,\qquad b\equiv 1+\frac{2\pi\mathcal{L}_\mathrm{r}}{c\Phi_0}J_\mathrm{s}'(\varphi_\mathrm{st})+\frac{2\pi\mathcal{L}_{12}}{c\Phi_0}G'(\varphi_\mathrm{st})>0.$$

These inequalities (together with the inequality in eq. (13)) can be satisfied only if $J_\mathrm{s}'(\varphi_\mathrm{st})<0$ and $G'(\varphi_\mathrm{st})>0$. From here and eqs. (2), (3) it follows that the stable limit cycle (that is non-linear periodic time-oscillations of $\varphi(t)$ and $\dot\varphi(t)$) can only be on those branches of the CVC with the negative differential resistance $\mathrm{d}V/\mathrm{d}J_\mathrm{d}<0$ that are close to the J_d-axis in fig. 1 (and on the loops shown in the insets of figs. 4 and 5). If $0<\mathcal{L}_{11}-\mathcal{L}_\mathrm{cr}\ll\mathcal{L}_\mathrm{cr}$, the frequency of these oscillations is

$$\omega_0=\sqrt{\frac{bc^4R}{(\mathcal{L}_{11}\mathcal{L}_\mathrm{r}-\mathcal{L}_{12}^2)G(\varphi_\mathrm{st})}}\,.$$

This frequency can be variated in a wide range, and estimations show that the maximal frequency is $\omega_\mathrm{max}\sim10^{12}\,\mathrm{s}^{-1}$.

In conclusion, for both the diffusive and ballistic cases I have shown that the current-voltage characteristics of an Andreev ring-shaped interferometer, $J_\mathrm{d}(V)$, is a series of loops with several branches intersecting at its origin (hence the CVC has sections with a negative differential resistance $\mathrm{d}V/\mathrm{d}J_\mathrm{d}$) if the ring inductance is large enough. These properties of the CVC are robust as the negative differential resistance and the intersection of several branches at the CVC origin appear at any value of the mutual inductance between the ring and the dissipative transport circuit, $\mathcal{L}_{12}$, as soon as the self-inductance of the ring $\mathcal{L}_\mathrm{r}$ is large enough to provide $A < 0$ (see eqs. (2), (3)). This inequality ($A < 0$) is the same as the inequality needed for functioning of conventional SQUIDs. What is more, changing the ring self-inductance $\mathcal{L}_\mathrm{r}$ and the external magnetic flux Φ_ext, one can get CVC loops (and therefore the hysteresis loops of the CVC) as small as wanted, this especially concerns the smallest loop around the CVC origin (see insets in figs. 4, 5). Therefore the sensitivity of the transport current J_d (or the applied voltage V) to a change of the external magnetic flux can be as high as the sensitivity of conventional SQUIDs. Spontaneous arising of coupled non-linear oscillations of the transport current $J_\mathrm{d}(t)$, the Josephson current $J_\mathrm{s}(t)$ and the flux $\Phi(t)$ in Andreev interferometers have also been predicted and investigated. The frequency of the oscillations ω can be varied in a wide range, and the maximal frequency can reach $\omega_\mathrm{max} \sim 10^{12}\,\mathrm{s}^{-1}$.

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The author is grateful to A. F. VOLKOV for useful discussions.

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