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Exact solution of the Hubbard model with boundary hoppings and fields

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Abstract. – The Hubbard model with boundary hopping integrals and fields is proposed and solved exactly by means of the Bethe ansatz method. We find that for certain values of the boundary hopping integrals and fields the ground state contains boundary bound states, these new types of states are represented as "boundary charge and spin strings". We show that the boundary bound states are realized at a strong boundary interaction for large values of the boundary hopping integrals or the boundary fields. The magnetic moment and charge of the boundary have been calculated numerically as a function of the magnetic field and band filling. It is found that the "boundary charge strings" solutions define the properties of the boundary.

Dedicated to Alexander Ovchinnikov's memory

Attempts to study effects due to the presence of the boundary conditions in many-body quantum systems in the framework of integrable models have a long successful history [1–3]. Quantum impurity models such as the Kondo model and the Anderson model also have deep connection with the boundary problem. Recent progress in the experimental study of low-dimensional materials, *e.g.* spin ladders, carbon nanotubes [4], quantum wires [5], has been an additional source of motivation to investigate one-dimensional exactly solvable models. The Hubbard model is marked among integrable models since many interesting statistical-mechanical systems can be described in the framework of this model. Most of the literature on the one-dimensional Hubbard model is based on the seminal paper of Lieb and Wu [6] (see the review [7] too); the authors solved the model and showed that the system at half-filling is an insulator for arbitrary repulsive interaction.

The purpose of this letter is to study new types of strings corresponding to boundary bound states in the framework of a new integrable version of the Hubbard model with boundary hoppings and fields. We identify boundary bound states with new solutions of the Bethe ansatz equations called "boundary charge and spin strings". Note that the boundary bound state is realized in the case of a strong boundary interaction [3]. The peculiar properties of the model proposed follow from the interaction of fermions with boundaries. The Hamiltonian contains the host and boundary terms $\mathcal{H} = \mathcal{H}_{\rm Hub} + \mathcal{H}_{\rm b}$,

$$\mathcal{H}_{\rm Hub} = -\sum_{j=2}^{L-2} \sum_{\sigma=\uparrow,\downarrow} \left(c_{j\sigma}^{\dagger} c_{j+1\sigma} + c_{j+1\sigma}^{\dagger} c_{j\sigma}\right) + U \sum_{j=2}^{L-1} n_{j\uparrow} n_{j\downarrow}\,, \tag{1}$$

$$\mathcal{H}_{\rm b} = -t_1 \sum_{\sigma=\uparrow,\downarrow} \left(c_{1\sigma}^{\dagger} c_{2\sigma} + c_{2\sigma}^{\dagger} c_{1\sigma}\right) + h_1 n_1 - t_L \sum_{\sigma=\uparrow,\downarrow} \left(c_{L\sigma}^{\dagger} c_{L-1\sigma} + c_{L-1\sigma}^{\dagger} c_{L\sigma}\right) + h_L n_L\,, \tag{2}$$

where $c_{j\sigma}^{\dagger}$ and $c_{j\sigma}$ are creation and annihilation operators of fermions with spin $\sigma = \uparrow,\downarrow$ at lattice site j, the particle number operator for electrons is defined by $n_{j\sigma} = c_{j\sigma}^{\dagger} c_{j\sigma}$ and $n_j = \sum_{\sigma=\uparrow,\downarrow} n_{j\sigma}$, L is the length of the chain. The hopping integral in (1) is equal to unity, U is the on-site Coulomb repulsion, $h_{1,L}$ and $t_{1,L}$ are the boundary fields and boundary hopping integrals, respectively. The kinetic terms of the Hamiltonian $\mathcal{H}$ are electron-hole invariant: indeed applying the combined electron-hole symmetry $c_{j\sigma}^{\dagger} \to (-1)^j c_{j\sigma}$ we obtain $\mathcal{H}(1, U, t_1, t_L, h_1, h_L) \to \mathcal{H}(1, U, t_1, t_L, -h_1, -h_L) + U(L-N) + h_1 + h_L$. Hence we can restrict our consideration to the case $N \le L$ for different boundary fields: h_1, h_L and $-h_1$, $-h_L$ correspond to $N \le L$ and $L \le N \le 2L$, respectively, N is the total number of electrons.

Below we present the exact solution of the model (1), (2) obtained by the Bethe ansatz. The eigenvector $|\psi\rangle$ with N particles is defined as $|\psi\rangle = \sum_{\{(x_j,\sigma_j)\}} \psi_{\sigma_1,\ldots,\sigma_N}(x_1,\ldots,x_N)|x_1\sigma_1,\ldots, x_N\sigma_N\rangle$, where the Bethe function takes a traditional form:

$$\psi_{\sigma_1,\ldots,\sigma_N}(x_{Q1},\ldots,x_{QN}) = \sum_P (-1)^P A_{\sigma_1,\ldots,\sigma_N}(x_{Q1},\ldots,x_{QN}) \exp\left[i \sum_{j=1}^{N} k_{Pj} x_{Qj}\right], \tag{3}$$

where the P summation extends over all the permutation of the momenta $\{k_j\}$, and $Q = \{Q_1,\ldots,Q_N\}$ is the permutation of the N particles such that the coordinates satisfy $1 \le x_{Q1} \le \ldots \le x_{QN} \le L$. The coefficients $A_{\sigma_1,\ldots,\sigma_N}(x_{Q1},\ldots,x_{QN})$ arising from different permutation Q are connected via the scattering matrices as follows: $A_{\ldots,\sigma_j,\sigma_i,\ldots}(\ldots,x_j,x_i,\ldots) = S_{ij}(k_i,k_j)$ $A_{\ldots\sigma_j,\sigma_i,\ldots}(\ldots,x_i,x_j,\ldots)$, $A_{\sigma_j\ldots}(-x_j,\ldots) = R_L(k_j) A_{\ldots\sigma_j}(x_j,\ldots)$, $A_{\ldots\sigma_j}(\ldots,-x_j) = R_R(k_j)\times$ $A_{\ldots\sigma_j}(\ldots,x_j)$, where the scattering matrices are given by

$$\begin{aligned} S_{ij}(k_i,k_j) &= \frac{\sin k_i - \sin k_j + iu\mathcal{P}_{ij}}{\sin k_i - \sin k_j + iu}\,, \\ R_L(k_j) &= \exp\left[2ik_j\right] \frac{(2-\tau_1)\cos k_j + h_1 - i\tau_1 \sin k_j}{(2-\tau_1)\cos k_j + h_1 + i\tau_1 \sin k_j}\,, \\ R_R(k_j) &= \exp\left[2ik_j L\right] \frac{(2-\tau_L)\cos k_j + h_L + i\tau_L \sin k_j}{(2-\tau_L)\cos k_j + h_L - i\tau_L \sin k_j}\,; \end{aligned}$$

here $u = \frac{U}{2}$, $\tau_{1,L} = t_{1,L}^2$ and $\mathcal{P}_{ij}$ is the usual spin permutation operator.

According to the model Hamiltonian (1), (2) the boundary interactions are one-particle ones, since two particles with different spins do not interact on lattice sites 1 and L, they interact for $2 \le j \le L-1$ only; as a result the Schrödinger equation for the two-particle wave function $\psi_{\sigma_1\sigma_2}(x_1,x_2)$ on the boundaries is reduced in fact to the following one-particle equations. For example, the Bethe function (3) is the solution of the equations $E_2\psi_{\sigma_1\sigma_2}(1,2) = -\psi_{\sigma_1\sigma_2}(1,3) - t_1\psi_{\sigma_1\sigma_2}(1,1) - t_1\psi_{\sigma_1\sigma_2}(2,2) + h_1\psi_{\sigma_1\sigma_2}(1,2)$, $E_2\psi_{\sigma_1\sigma_2}(1,1) = -t_1\psi_{\sigma_1\sigma_2}(2,1) - t_1\psi_{\sigma_1\sigma_2}(1,2) + 2h_1\psi_{\sigma_1\sigma_2}(1,1)$ for $E_2 = -2\cos k_1 - 2\cos k_2$ and an arbitrary two-particle S-matrix, since the corresponding equations for the one-particle Bethe function have the similar form $E_1\psi_\sigma(2) = -\psi_\sigma(3) - t_1\psi_\sigma(1)$, $E_1\psi_\sigma(1) = -t_1\psi_\sigma(2) + h_1\psi_\sigma(1)$, where $E_1 = -2\cos k_1$.

Since the boundary scattering matrices ($R_{R,L}$ matrices) do not depend on the spin variables, the model has been solved for arbitrary values of the boundary hoppings and fields. We diagonalize the Hamiltonian (1), (2) following standard Bethe ansatz techniques developed for open chain [1]. We introduce the spin rapidities $\lambda_\alpha (\alpha = 1, \ldots, M)$ and write the Bethe ansatz equations in the following form:

$$R_R(k_j)R_L(-k_j) = \prod_{\alpha=1}^{M} \frac{\sin k_j - \lambda_\alpha + \frac{i}{2}u}{\sin k_j - \lambda_\alpha - \frac{i}{2}u} \frac{\sin k_j + \lambda_\alpha + \frac{i}{2}u}{\sin k_j + \lambda_\alpha - \frac{i}{2}u}, \tag{4}$$

$$\prod_{j=1}^{N} \frac{\lambda_\alpha - \sin k_j + \frac{i}{2}u}{\lambda_\alpha - \sin k_j - \frac{i}{2}u} \frac{\lambda_\alpha + \sin k_j + \frac{i}{2}u}{\lambda_\alpha + \sin k_j - \frac{i}{2}u} = \prod_{\beta=1}^{M} \frac{\lambda_\alpha - \lambda_\beta + iu}{\lambda_\alpha - \lambda_\beta - iu} \frac{\lambda_\alpha + \lambda_\beta + iu}{\lambda_\alpha + \lambda_\beta - iu}, \tag{5}$$

where M is the number of down-spin electrons.

In the thermodynamic limit the ground state of the Hubbard model is made of the real solutions for momenta and spin rapidities k_j and λ_α [7]. Let us turn to the new solutions of eqs. (4), (5) called as boundary strings. Boundary excitations have their wave function (3) localized at the left or the right ends, and in the $L \to \infty$ limit the two ends may be considered separately. The basis of the "boundary charge string solutions" consists of two sets (denoted as $\alpha = 1, 2$) of the solutions of eqs. (4) for the charge rapidities that correspond to each boundary ($\beta = L, R$):

$$k_{\beta\pm}^{(\alpha)} = i\mathcal{K}_{\beta\pm}^{(\alpha)} + \epsilon_{\beta\pm}^{(\alpha)}, \quad \epsilon_{\beta\pm}^{(\alpha)} \sim \exp\Big[-\delta_{\beta\pm}^{(\alpha)} L\Big], \quad \delta_{\beta\pm}^{(\alpha)} > 0.$$

It is convenient to use the following parameterization for the boundary coupling constants $\coth \eta_\beta = \frac{\tau_\beta}{2-\tau_\beta}$ at $\tau_\beta > 1$ and $\sinh \Delta_\beta = \tilde{h}_\beta$, $\tilde{h}_\beta = 2h_\beta\sqrt{|\tau_\beta - 1|}$, then the "boundary charge solutions" are defined as $\mathcal{K}_{\beta\pm}^{(1)} = \pm(\eta_\beta + \Delta_\beta)$ at $\eta_\beta + \Delta_\beta < 0$, and $\mathcal{K}_{\beta\pm}^{(2)} = \pm(\eta_\beta - \Delta_\beta) - i\pi$ at $\eta_\beta - \Delta_\beta < 0$; for $\tau_\beta < 1$ $\tanh \eta_\beta = \frac{\tau_\beta}{2-\tau_\beta}$ the solutions have the form $\mathcal{K}_{\beta\pm}^{(1)} = -i\pi \pm (\eta_\beta + \Delta_\beta)$, at $\eta_\beta + \Delta_\beta < 0$, and $\mathcal{K}_{\beta\pm}^{(2)} = -i\pi \pm (\eta_\beta - \Delta_\beta)$, at $\eta_\beta - \Delta_\beta < 0$, where $\cosh \Delta_\beta = \tilde{h}_\beta$ at $\tilde{h}_\beta \geq 1$; $\mathcal{K}_{\beta\pm}^{(1)} = \pm(\eta_\beta + \Delta_\beta)$, $\eta_\beta + \Delta_\beta < 0$, and $\mathcal{K}_{\beta\pm}^{(2)} = \pm(\eta_\beta - \Delta_\beta)$, at $\eta_\beta - \Delta_\beta < 0$, where $\cosh \Delta_\beta = -\tilde{h}_\beta$ at $\tilde{h}_\beta \leq -1$.

New "boundary spin string solutions" of the Bethe ansatz equations (5) exist in a strong spin coupling regime for $u < 2|\Lambda_\beta^{(\alpha)}|$:

$$\lambda_{\beta n\pm}^{(\alpha)} = \pm i S_\beta^{(\alpha)} + i\frac{u}{2}(n + 1 - 2p) + \varepsilon_{\beta n\pm}^{(\alpha)} \quad \text{for } p = 1, 2, \ldots, n,$$

where $S_\beta^{(\alpha)} = |\Lambda_\beta^{(\alpha)}| - \frac{u}{2}$, $\Lambda_\beta^{(\alpha)} = \sinh(\mathcal{K}_{\beta+}^{(\alpha)})$, $\varepsilon_{\beta n\pm}^{(\alpha)} \sim \exp[-\gamma_{\beta n\pm}^{(\alpha)} L]$, $\gamma_{\beta n\pm}^{(\alpha)} > 0$. Note that the "boundary strings" can appear only in the case of strong boundary interactions. One branch of the solutions exists for $1 < \tau_\beta < 2$ and an arbitrary value of $\tilde{h}_\beta$, or $\tau_\beta < 1$ and $|\tilde{h}_\beta| > 1$, and two branches for $\tau_\beta > 2$. In the case of a weak interaction for $\tau_\beta < 1$ and $|\tilde{h}_\beta| < 1$ no such solutions exist. The "boundary charge string solutions" correspond to the poles in the physical sheet of the boundary scattering matrices. The boundary bound states describe coupled pairs of the particles, each branch of the charge and spin rapidities is double and n degenerated, respectively. The "boundary spin string solutions" meet the formation of the local moment of the boundary. Numerical calculations show that stable "boundary n-spin string solutions", that correspond to the minimum of the surface ground-state energy, are realized if $u_n < u \le u_{n-1}$; here $u_n = 2S_\beta^{(\alpha)}/n$. We will consider the case of a weak spin coupling regime $u > 2|\Lambda_\beta^{(\alpha)}|$, when the "boundary spin string solutions" do not exist.

Taking logarithms of the Bethe equations (4), (5) and afterward performing the thermodynamic limit, we are able to obtain the integral equations of the Fredholm type for the distribution functions $\tilde\rho(k)$ and $\tilde\sigma(\lambda)$ for the k and λ variables. The presence of new solutions in the Bethe equations, that account for the "boundary strings", deforms the distribution charge and spin rapidities and modifies their distribution functions by the terms $\frac{1}{L}\delta\rho_\beta^{(\alpha)}(k)$ and $\frac{1}{L}\delta\sigma_\beta^{(\alpha)}(\lambda)$ ($\tilde\rho(k) = \rho(k) + \frac{1}{L}\sum_{\alpha=1,2}\sum_{\beta=L,R}\delta\rho_\beta^{(\alpha)}(k)$ and $\tilde\sigma(\lambda) = \sigma(\lambda) + \frac{1}{L}\sum_{\alpha=1,2}\sum_{\beta=L,R}\delta\sigma_\beta^{(\alpha)}(\lambda)$). The distribution functions $\delta\rho_\beta^{(\alpha)}(k)$ and $\delta\sigma_\beta^{(\alpha)}(\lambda)$ satisfy the integral equations

$$
\begin{aligned}
&\delta\rho_\beta^{(\alpha)}(k) - \cos k\int_{-\Lambda}^{\Lambda} \mathrm{d}\lambda a_u(\sin k - \lambda)\delta\sigma_\beta^{(\alpha)}(\lambda) = 0,\\
&\delta\sigma_\beta^{(\alpha)}(\lambda) + \int_{-\Lambda}^{\Lambda}\mathrm{d}\lambda' a_{2u}(\lambda-\lambda')\delta\sigma_\beta^{(\alpha)}(\lambda') - \int_{-Q}^{Q}\mathrm{d}k a_u(\lambda - \sin k)\delta\rho_\beta^{(\alpha)}(k) =\\
&\qquad 2\Big[a_{u+2\Lambda_\beta^{(\alpha)}}(\lambda) + a_{u-2\Lambda_\beta^{(\alpha)}}(\lambda)\Big],
\end{aligned}\tag{6}
$$

where $a_u(\lambda) = \frac{u}{2\pi}\frac{1}{\lambda^2 + (u/2)^2}$. The driving terms in (6) are the result of the "boundary charge string solutions".

Equations for the $\rho(k)$ and $\sigma(\lambda)$ distribution functions have the following form:

$$
\begin{aligned}
&\rho(k) - \cos k\int_{-\Lambda}^{\Lambda}\mathrm{d}\lambda a_u(\sin k - \lambda)\sigma(\lambda) = \frac{1}{\pi} - \frac{1}{L}\left[\frac{1}{\pi} + \cos k a_u(\sin k) - g_L(k) - g_R(k)\right],\\
&\sigma(\lambda) + \int_{-\Lambda}^{\Lambda}\mathrm{d}\lambda' a_{2u}(\lambda-\lambda')\sigma(\lambda') - \int_{-Q}^{Q}\mathrm{d}k a_u(\lambda - \sin k)\rho(k) = \frac{1}{L}a_{2u}(\lambda),
\end{aligned}\tag{7}
$$

where $g_\beta(k) = \frac{\tau_\beta}{\pi}\frac{2-\tau_\beta + h_\beta\cos k}{\tau_\beta^2 + h_\beta^2 + 2h_\beta(2-\tau_\beta)\cos k + 4(1-\tau_\beta)\cos^2 k}$.

The k-Fermi level denotes as Q controls the band filling and the density of fermions $n = N/L$ is defined by $n = \widetilde{n} + \frac{1}{L}\delta n$, where $\widetilde{n} = \frac{1}{2}\int_{-Q}^{Q}\mathrm{d}k\tilde\rho(k)$, $\delta n = 2\sum_{\alpha=1,2}\sum_{\beta=L,R}\nu_\beta^{(\alpha)}$, $\nu_\beta^{(\alpha)}$ is the number of occupied boundary charge state. The density of the magnetization is defined by $m = \frac{1}{2}[\widetilde{n} - \int_{-\Lambda}^{\Lambda}\mathrm{d}\lambda\tilde\sigma(\lambda)]$. Let us study the behavior of the magnetic moment and charge of the boundary in the case of the formation of the "boundary charge string solutions" only, namely at a weak spin coupling regime $u > 2|\Lambda_\beta^{(\alpha)}|$, when the "boundary charge string solutions" are stable. The charge of the boundary is defined as $n_\beta = 2\sum_{\alpha=1,2}\nu_{\alpha\beta} + \frac{1}{2}\sum_{\alpha=1,2}\int_{-Q}^{Q}\mathrm{d}k\delta\rho_\beta^{(\alpha)}(k) + \frac{1}{2}\int_{-Q}^{Q}\mathrm{d}k\delta\rho_\beta(k)$, where the first term is the local charge of the boundary and $\rho(k) = \rho_0(k) + \frac{1}{L}\sum_{\beta=L,R}\delta\rho_\beta(k)$, $\sigma(\lambda) = \sigma_0(\lambda) + \frac{1}{L}\sum_{\beta=L,R}\delta\sigma_\beta(\lambda)$, $\rho_0(k)$ and $\sigma_0(\lambda)$ are the host distribution functions. The magnetic moment of the boundary is equal to the sum of the following terms: $m_\beta = \frac{1}{2}\sum_{\alpha=1,2}[\frac{1}{2}\int_{-Q}^{Q}\mathrm{d}k\delta\rho_\beta^{(\alpha)}(k) - \int_{-\Lambda}^{\Lambda}\mathrm{d}\lambda\delta\sigma_\beta^{(\alpha)}(\lambda)] + \frac{1}{2}[\frac{1}{2}\int_{-Q}^{Q}\mathrm{d}k\delta\rho_\beta(k) - \int_{-\Lambda}^{\Lambda}\mathrm{d}\lambda\delta\sigma_\beta(\lambda)]$ (local charge boundary states form the singlet (nonmagnetic) state).

Below we consider the host magnetization and the magnetic moment of the boundary in the weak magnetic field. In the small magnetic-field limit we have $\Lambda \to \infty$, with increasing magnetic field the value of Λ decreases monotonically. In the limit of a small magnetic field, for which $\Lambda \gg 1$ and $Q \ll \Lambda$, these equations can be solved by iteration in the standard way and

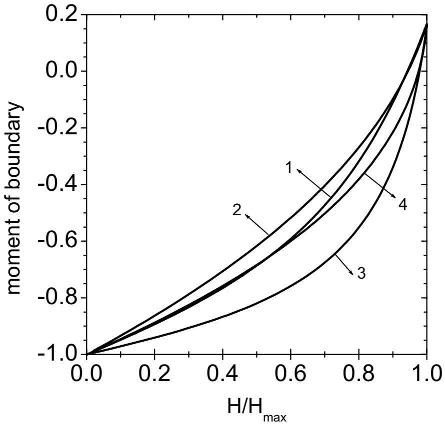

Fig. 1

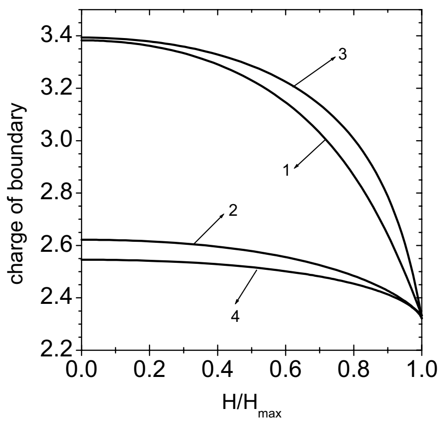

Fig. 2

Fig. 1 – Magnetic moment of the boundary as a function of magnetic field at $\tau_\beta = 1.5$, $h_\beta = -0.4$ for $n = 0.5$ and $u = 1$ ($H_{\max} = 1.236$, curve 1), $n = 0.5$ and $u = 4$ ($H_{\max} = 0.472$, curve 2), $n = 0.75$ and $u = 1$ ($H_{\max} = 2.195$, curve 3), $n = 0.75$ and $u = 4$ ($H_{\max} = 0.856$, curve 4).

Fig. 2 – Charge of the boundary as a function of magnetic field, similar to fig. 1.

we can calculate the value of Λ as a function of the magnetic field H: $\Lambda = -\frac{u}{\pi}\ln(\frac{H}{A}) + \frac{u}{4\pi\ln(\frac{H}{A})}$; here the scale $A = -\frac{1}{u}\sqrt{\frac{2}{\pi e}}\int_{-Q}^{Q}\mathrm{d}k\cos k\exp[\frac{\pi}{u}\sin k]\varepsilon_0(k)$ is defined by the dressed energy $\varepsilon_0(k)$, that is the solution of the relevant integral equations for the dressed energies at zero temperature and $H = 0$ in the Hubbard model: $\varepsilon_0(k) - \int_{-Q}^{Q}\mathrm{d}k'\cos k' R(\sin k - \sin k')\varepsilon_0(k') = \mu - 2\cos k$; here $R(\lambda) = \frac{1}{\pi}\int_0^\infty \cos(\lambda\omega)\frac{\mathrm{d}\omega}{1+\exp[u\omega]}$, μ is the chemical potential.

The leading contributions to the density of the magnetization of itinerant electrons and the magnetic moment of the boundary have the form

$$
\begin{aligned}
M_{\text{host}} &= \frac{2}{\pi}T_H \exp\left[-\frac{\pi\Lambda}{u}\right]\left(1 + \frac{u}{4\pi\Lambda}\right),\\
m_{\text{b}} &= \frac{4}{\pi}\sqrt{\frac{2}{\pi e}}\cos\left(\frac{\pi}{u}\Lambda_\beta^{(\alpha)}\right)\exp\left[-\frac{\pi\Lambda}{u}\right]\left(1 + \frac{u}{4\pi\Lambda}\right);
\end{aligned}
\tag{8}
$$

here the magnetic scale $T_H = \sqrt{\frac{2}{\pi e}}\int_{-Q}^{Q}\rho_0(k)\exp[\frac{\pi}{u}\sin k]\mathrm{d}k$, where $\rho_0(k)$ is the host distribution function at $H = 0$.

Note that the contribution to the magnetic moment and charge of the boundary defined by the distribution functions $\delta\rho_j^{(\alpha)}$ and $\delta\sigma_j^{(\alpha)}$ dominates. Numerical calculations of the charge and magnetic moment of the boundary as a function of the magnetic field and band filling for several values of the on-site interaction, band filling and magnetic field are shown in figs. 1-4.

The magnetic moment of the boundary increases linearly for small magnetic field and reaches the maximum value at a saturation field $H_{\max}$ (see fig. 1) (the field $H_{\max}$ corresponds the saturated magnetization of the host, when the ground state becomes ferromagnetic). The value of $H_{\max}$ depends on the on-site interaction and the density of electrons and increases with the band filling. Only magnetic fields below $H_{\max}$ are considered. The magnetic scale

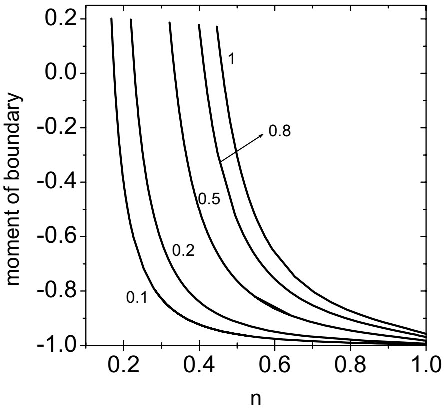

Fig. 3

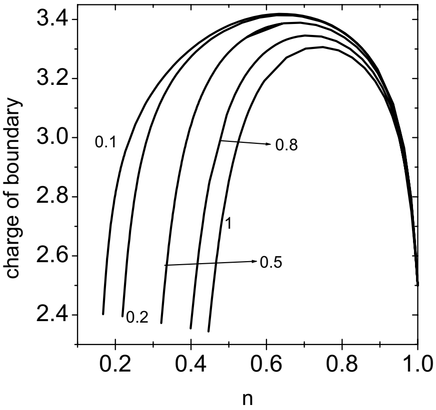

Fig. 4

Fig. 3 – Magnetic moment of the boundary as a function of band filling at $\tau_\beta = 1.5$, $h_\beta = -0.4$ and $u = 1$. The individual curves are labeled by the value of an external magnetic field.

Fig. 4 – Charge of the boundary as a function of band filling, similar to fig. 3.

is defined by the ration $\frac{m_{\rm b}}{M_{\rm host}} = 2\sqrt{\frac{2}{\pi e}}\cos(\frac{\pi}{u}\Lambda_j^{(\alpha)})/T_H$ that depends on the band filling. The value of T_H is proportional to the density of electrons for small band filling and increases with the band filling.

The curves have a universal form in dimensionless fields (see fig. 1). At $H = 0$ the magnetic moment of the boundary has a minimal value -1, a maximum value is reached in a saturation field, it has the same value that does not depend on the host parameters. The boundary magnetic moment increases with the magnetic field (fig. 1), whereas the charge of the boundary decreases with magnetic field and reaches an identical minimal value at $H_{\rm max}$ (fig. 2). A similar behavior takes place for the behavior of the magnetic moment of the boundary as a function of band filling (fig. 3). The magnetic moment of the boundary is a decreasing function of the band filling shifted in the magnetic field. The "charge boundary strings" induce an abnormal behavior of the charge of the boundary as a function of the band filling (fig. 4). The charge of the boundary has a minimal value at extreme electron densities and a maximum value in the middle of this interval. An external magnetic field shifts deforming the curves and does not change their behavior.

In summary, we have presented a soluble version of the Hubbard model with boundary hoppings and fields, leading to a nontrivial behavior. At strong boundary interaction new solutions of the Bethe equations, that correspond to the "charge and spin boundary rapidities", are obtained. We find the criteria of the realization of these solutions, the existence of "charge and spin boundary solutions" leads to local boundary charge and moment.

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