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Detection of phase locking from non-stationary time series

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Abstract. – The detection of the interrelation between non-stationary time series is a key problem unsolved in the analysis of practical data. Here we present a novel approach to detect the phase locking of time series in non-stationary status, which does not need the reconstruction of the attractor but gets information from the consistence of local rotation speeds. This method can be conveniently used for practical situations, such as the epileptic seizure.

In the analysis of time series, an important task is to measure the relationship between different time series. From this one can predict what will happen in a near future, such as in epileptic seizure. It is found that, in the case of epileptic seizure, the EEG data from different parts of the brain are synchronized if the status of the patient is in epileptic seizure [1–3]. Hence the detection of synchronization and how the system changes to synchronization are very important in practice. A common feature in these systems is that the status of the systems is non-stationary and their parameters, such as coupling, are changing with time. As the equations of these practical systems are unknown and the relationship between two time series is not fixed but changing with time, how to detect their relationship is an open question.

For a system of stationary status, one can use the attractor-reconstructing technique to recover the topological structure of the underlying attractor as a time series containing all the necessary information for reconstructing the attractor. By the time-delay technique [4] one can first get the trajectory from a scalar time series and then calculate its transverse Lyapunov exponents. The coupled systems are synchronized or generalized synchronization if the largest transverse Lyapunov exponent is negative [5–7]. There are also other statistical methods [8–12] to judge the synchrony.

For a system of non-stationary status [1–3], strictly speaking, one cannot reconstruct its attractor as there is no stationary attractor. But if the system does not change very fast, one can treat the local part of time series as an approximate stationary state and reconstruct its local attractor, and then use the reconstructed trajectory to estimate the interdependence [13–15].

However, when the underlying system is high-dimensional, the needed data for attractor reconstruction is very large. In this case, the reconstructing technique may not work when there is lack of enough data of approximate stationary status.

Moreover, when the local part of time series cannot be treated as an approximate stationary state because of the fast varying parameters, the above approaches cannot be used. Hence the question we want to address in this work is, in general, how to detect the relationship between two time series in the case of non-stationary status. Notice that one fundamental feature of a dynamical system is the behavior of oscillation or rotation. The rotation reflects the instantaneous angle velocity of the phase point moving along the trajectory in a 2-dimensional projection. For an arbitrary time series $x(t)$, one can get the corresponding $y(t)$ by Hilbert transform [16] and then define a local rotation speed $w(t)$ from $\tan(\phi) = y(t)/x(t)$. Generally speaking, the local rotation speed will ignore the detailed information of $x(t)$ but keep the most basic information. We find that if two time series come from identical or non-identical systems, the synchronization between them can be conveniently estimated by an approach based on the consistence of local rotation speeds, which does not need to reconstruct the attractor. In practical situations, the interesting time series are from the same kind of oscillators, such as EEG data from neurons, so here we will only consider the cases of coupled identical and non-identical oscillators.

A common feature in practical situations is that there is always noise in time series. Hence we suppose that the time series considered in this paper come from the following coupled systems:

$$\frac{\mathrm{d}\boldsymbol{x}_{1,2}}{\mathrm{d}t} = \boldsymbol{F}(p_{1,2}, \boldsymbol{x}_{1,2}) + \boldsymbol{k}(\boldsymbol{x}_{2,1} - \boldsymbol{x}_{1,2}) + D\xi_{1,2}(t), \tag{1}$$

where $\boldsymbol{x}$ is in the phase space of dimension m, $\boldsymbol{F}$ is a function which may show chaos, $p_{1,2}$ are the mismatched parameters denoting the identical situation for $p_1 = p_2$ and non-identical situation for $p_1 \neq p_2$, $\boldsymbol{k}$ is the coupling matrix, $\xi_{1,2}$ are independent Gaussian white noises with average zero and standard deviation unity, and D is the noise strength. Rosenblum *et al.* show that for symmetrically coupled non-identical Rössler oscillators, with the increase of coupling, the systems can go through phase synchronization (PS) to Lag synchronization (LS) and finally to complete synchronization (CS) [17]. More generally, the final state of the coupled non-identical systems will be generalized synchronization (GS). Here CS means the two time series are identical [18], PS means the phase difference of the coupled oscillators is bounded, whereas their amplitudes remain chaotic and uncorrelated [19, 20], and GS means there is a functional relation $\boldsymbol{x}_2 = \Phi(\boldsymbol{x}_1)$ [8–12]. PS is usually studied in the non-identical coupled dynamical systems or in a single system with external force. Tass *et al.* have used the PS approach to detect $n : m$ phase locking [21] in MEG and EMG data. More generally, the relation $n : m$ will change during the different stages of the patient. The $n : m$ locking can be obtained by trial and error. This approach sometimes needs to try all the possible $n : m$, so that it will not be confused with the case of no PS locking, which is time consuming. Notice that $n : m$ locking in fact describes the proportional relationship between the corresponding phases. This proportional relationship reflects the consistence of local rotation speed. So we can use the detection of the consistence of local rotation speed to replace the detection of $n : m$ locking. That is, we transform the $n : m$ phase locking to a $1 : 1$ consistence locking, which makes the detection become much easier. In this novel approach, the local rotation speeds will change from non-consistence to consistence during the process of approach to phase locking.

Suppose we take a time series $x(t)$ from $\boldsymbol{x}_1$ in eq. (1), where $t \in (0, T)$. By Hilbert transform one can get its imaginary part $\tilde{x}(t) = \pi^{-1}\mathrm{P.V.}\int_{-\infty}^{\infty} \frac{x(t')}{t-t'}\mathrm{d}t'$, where P.V. means that the integral is taken in the sense of the Cauchy principal value, and hence obtain the phase

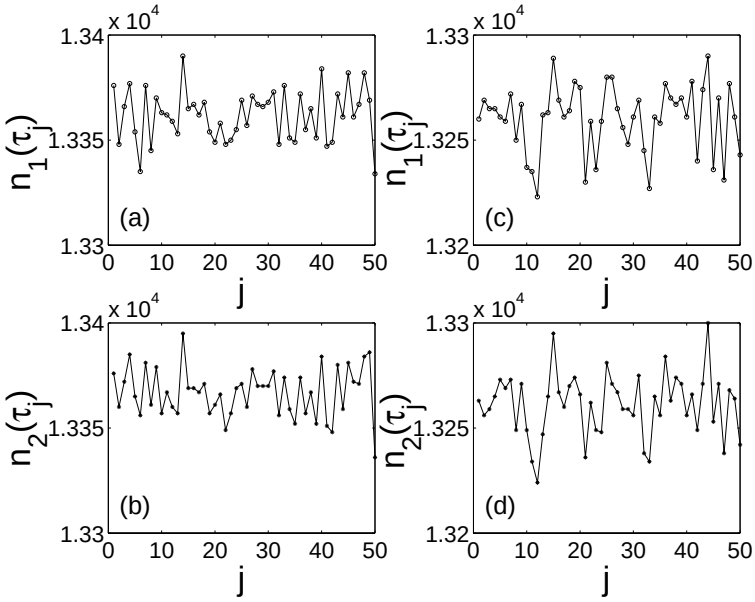

Fig. 1 – The numbers of rotation from time series $z_1(t)$ and $z_2(t)$ in a fixed time interval $\tau_j = 10^4$, $j = 1, 2, \cdots$ with coupling strength $k = 4.0$ and noise strength $D = 0.01$, where n_1 is from time series z_1, n_2 from z_2, (a) and (b) denote the case of coupled identical oscillators with $\sigma_1 = \sigma_2 = 10.0$, and (c) and (d) the case of coupled non-identical oscillators with $\sigma_1 = 10.0$ and $\sigma_2 = 11.0$.

$\tan(\phi_1) = \tilde{x}(t)/x(t)$. The rotation velocity is $w_1(t) = \mathrm{d}\phi_1/\mathrm{d}t$. Doing the same, one can get $w_2(t) = \mathrm{d}\phi_2/\mathrm{d}t$ for time series $y(t)$ from $\boldsymbol{x}_2$ in eq. (1). Dividing the whole evolution time T into N segments, each segment has a time interval $\tau = T/N$. In the interval τ_j, $j = 1, 2, \cdots N$, we have $\Delta\phi_1^j = \int_{(j-1)\tau}^{j\tau} w_1(t)\mathrm{d}t$ and $\Delta\phi_2^j = \int_{(j-1)\tau}^{j\tau} w_2(t)\mathrm{d}t$. In phase locking, their behaviors should be in-step. For describing this kind of in-step behavior, we introduce $\delta\phi_{1,2}^j = \Delta\phi_{1,2}^j - \Delta\phi_{1,2}^{j-1}$. Phase locking means that $\delta\phi_1^j$ and $\delta\phi_2^j$ will be in-step. So a convenient approach to measure the phase locking relationship between two time series is by the in-step relation between $\delta\phi_1^j$ and $\delta\phi_2^j$. In the following we use an example to explain this in detail.

Consider two coupled Lorenz systems

$$\begin{aligned}
\dot{x}_{1,2} &= \sigma(y_{1,2} - x_{1,2}) + k(x_{2,1} - x_{1,2}) + D\xi_{1,2}^x(t), \\
\dot{y}_{1,2} &= \gamma x_{1,2} - y_{1,2} - x_{1,2}z_{1,2} + D\xi_{1,2}^y(t), \\
\dot{z}_{1,2} &= -bz_{1,2} + x_{1,2}y_{1,2} + D\xi_{1,2}^z(t),
\end{aligned} \tag{2}$$

where $\gamma = 28.0$, $b = 8/3$, $\xi_{1,2}^x$, $\xi_{1,2}^y$, and $\xi_{1,2}^z$ are independent Gaussian white noises with average zero and standard deviation unity. We first consider the case of identical oscillators with $\sigma_1 = \sigma_2 = 10.0$. Generally speaking, the behavior of a dynamical system is some kind of oscillation. Suppose we take two time series $z_1(t)$ and $z_2(t)$. By the Hilbert transform we get their instantaneous phase $\phi_{1,2}$ and rotation velocity $w_{1,2}(t)$. Then the corresponding circles of rotation in a time interval τ_j is $n_{1,2} = \frac{1}{2\pi}\int_{t_j}^{t_j+\tau_j} w_{1,2}(t)\mathrm{d}t$, where t_j is the initial point of the interval τ_j. Figures 1(a) and (b) show the results for $k = 4.0$ and noise strength $D = 0.01$. Similarly, we get the numbers of rotation for the case of non-identical oscillators with $\sigma_1 = 10.0$, $\sigma_2 = 11.0$, $k = 4.0$ and $D = 0.01$, see figs. 1(c) and (d).

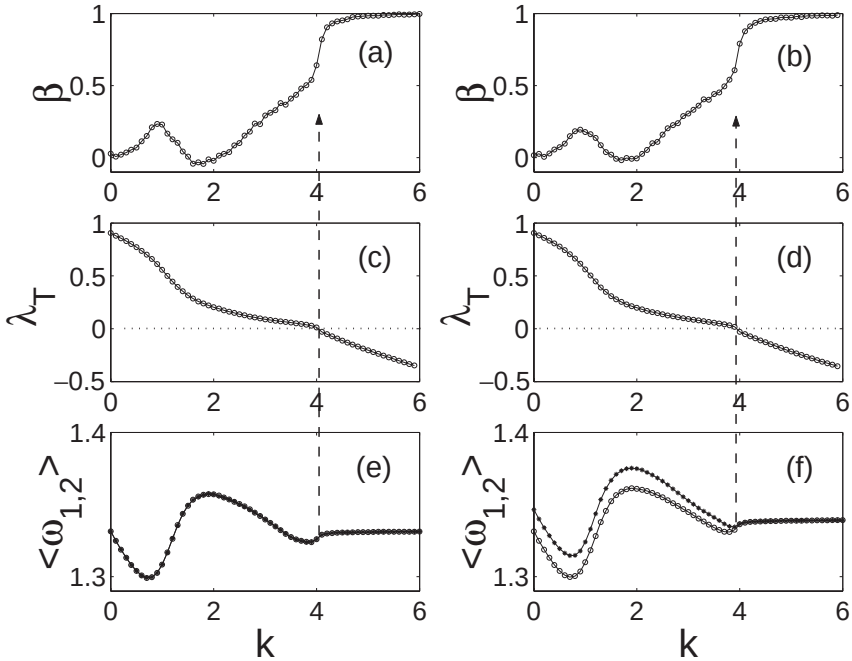

Fig. 2 – Identical case (a), (c) and (e) with parameters $D = 0.01$, $\tau = 10^3$, $N = 10^4$, $\sigma_1 = \sigma_2 = 10.0$; non-identical case (b), (d) and (f) with parameters $D = 0.01$, $\tau = 10^3$, $N = 10^4$, $\sigma_1 = 10.0$ and $\sigma_2 = 11.0$. In (e) and (f), circles denote $\langle w_1 \rangle$ and stars $\langle w_2 \rangle$.

Comparing fig. 1(a) with (b) and (c) with (d), respectively, it is easy to find that, with the increase of the time interval j, the changing tendency of n_1 is in-step with that of n_2. That is, in most of time, both n_1 and n_2 increase or decrease at the same time. Therefore, we can introduce an in-step parameter β to measure the degree of consistence between n_1 and n_2. Similar to the phase definition of a discrete system [22], we think that if both n_1 and n_2 increase or decrease at the same time, this has positive contribution to β; otherwise, this has negative contribution to β if n_1 increases and n_2 decreases or n_1 decreases and n_2 increases. This requirement does not concern the detailed information of numbers of rotation of n_1 and n_2, such as how large are n_1 and n_2, but only concern the changing tendency of n_1 and n_2 which is a coarse-graining of n_1 and n_2. Hence we come to the frame of symbolic dynamics [11]. Let us introduce a variable $S_1(\tau_j)$, and let $S_1(\tau_j) = 1$ if $n_1(\tau_j) > n_1(\tau_{j-1})$ and -1 otherwise. Similarly, we can define $S_2(\tau_j)$ for $n_2(\tau_j)$. Then the degree of consistence between n_1 and n_2 becomes the correlation between $S_1(\tau_j)$ and $S_2(\tau_j)$. Based on this analysis we have

$$\beta \equiv \langle S_1(\tau_j) S_2(\tau_j) \rangle, \tag{3}$$

where the angular bracket denotes an average over τ_j. β will be zero for non-step n_1 and n_2 and close to unity after GS or CS. Hence β is the quantity that describes the consistence between local rotation speeds of two time series [23]. Equation (3) can be also understood as follows: From the definition of $n_{1,2}(\tau_j)$ we know that they are nothing but instantaneous frequencies $w_{1,2}(\tau_j)$. Hence, their symbolic dynamics is just a mapping of instantaneous angular accelerations: $S_i = 1$ if $\mathrm{d}w_i/\mathrm{d}t > 0$ and $S_i = -1$ if $\mathrm{d}w_i/\mathrm{d}t < 0$. In this respect their quantity is almost equivalent to the cross-correlation of angular accelerations. Calculating β for different couplings k, we can observe how β changes with couplings. Figures 2(a) and (b)

show the results for the identical and non-identical oscillators, respectively. It is easy to find that, after $k \approx 4.0$, the value of β approaches unity. So the synchronization point is approximately $k_c \approx 4.0$. For several practical purposes, especially in neuroscience, it is important whether de-synchronization can be detected too. Our numerical simulation confirms that the β curves in figs. 2(a) and (b) are kept the same when we decrease k from 6 to 0, indicating this approach can be also used to detect de-synchronization.

In the symmetrically coupled non-identical Lorenz systems, it is found that the PS point is very close to the GS point [24]. For confirming it, let us calculate the largest transverse Lyapunov exponent λ_T from the model equations. Figures 2(c) and (d) show how λ_T changes with the coupling strength k for the identical and non-identical situations, respectively. As we know, the GS point is the zero-crossing point of λ_T [5–7]. Comparing fig. 2(a) with (c) and (b) with (d), respectively, one can see that the zero-crossing points in λ_T are approximately the transition points in β, see the dashed-lined arrows. Furthermore, figs. 2(e) and (f) show the relation between the average rotation speed $\langle w_{1,2} \rangle$ and the coupling k, where $\langle w_1 \rangle$ is from z_1 and $\langle w_2 \rangle$ from z_2. Obviously, $\langle w_{1,2} \rangle$ changes with k before k_c and then keeps constant after k_c. From fig. 2(e) one can see that $\langle w_1 \rangle$ and $\langle w_2 \rangle$ overlap for the whole k. And from fig. 2(f) one can see that the difference between $\langle w_1 \rangle$ and $\langle w_2 \rangle$ gradually decreases to zero when the coupled systems approach GS, indicating the PS transition is shadowed by the transverse Lyapunov exponent crossing zero.

Another point we want to mention here is the different changing tendency in β and λ_T. Obviously, λ_T in figs. 2(c) and (d) monotonously decreases while β in figs. 2(a) and (b) monotonously increases only after $k \approx 2.0$. The bell-shape of β before $k \approx 2.0$ comes from the fact that one of the two zero Lyapunov exponents of no coupling will become positive first and then become negative [25], which is consistent with the change of $w_{1,2}$ in figs. 2(e) and (f). That is, λ_T is only used to judge the synchronization. For describing a system, one has to consider the whole spectrum of Lyapunov exponents.

For a practical time series, the coupling may change during the evolution process. So the whole time series is not for a fixed coupling strength, such as in the EEG data. In this case, we need to use the sliding-window technique with the shorter length of time interval. Suppose we have two time series with the same length T, which are used to calculate each β in figs. 2(a) and (b). That is $T = N\tau$. Considering the parameter is varying during the evolution time T, now we divide the time series into $m_1 m_2$ segments, where m_1 and m_2 are positive integers. And then we calculate the β for each segment which is much shorter than T. For making sure we have enough $S_{1,2}(\tau_j)$ in each segment, we use $\tau^s = \tau/m_1$ as the new time interval and $N_s = N/m_2$ as the new average number. Usually, N_s should be around 100 and τ_s should contain at least 10 cycles. Hence eq. (3) becomes

$$\beta_s = \frac{1}{N_s} \sum_{j=1}^{N_s} S_1(\tau_j^s) S_2(\tau_j^s), \tag{4}$$

which shows that one β in (3) becomes $m_1 m_2 \beta_s$ in (4). That is how β_s reflects the local consistence of time series.

For checking eq. (4) by numerical simulation, let us assume that the coupling strength, k, increases uniformly from zero to 6.0 during the evolution process. Hence, later data in the time series has larger coupling strength. Figure 3(a) shows how the coupling strength k of the time series changes with time. Calculating β_s with $m_1 = 10$ and $m_2 = 100$ for each segment $N_s \tau^s$, we get the results as shown in figs. 3(b) and (c) for the identical and non-identical cases, respectively. Considering the fact that, in a practical situation, the coupling parameter k may sometimes change very fast, we increase the varying speed of k to 10 times that in fig. 3(a) and

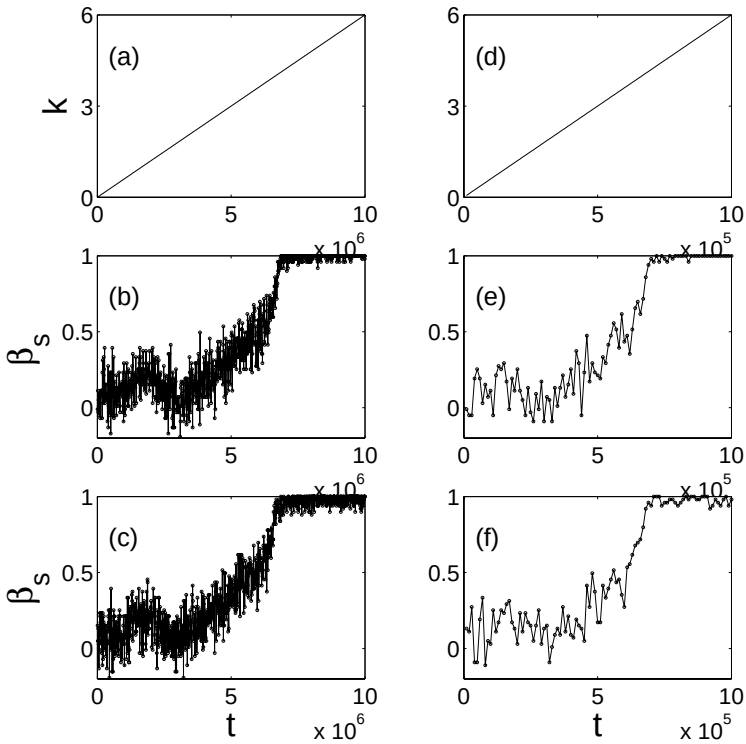

Fig. 3 – Case of varying coupling with $D = 0.01$, $m_1 = 10$, $m_2 = 100$, and $N_s = \tau^s = 100$, where (a) denotes how the coupling strength k changes with time. (b) Identical case with $\sigma_1 = \sigma_2 = 10.0$. (c) Non-identical case with $\sigma_1 = 10.0$ and $\sigma_2 = 11.0$. And (d), (e), and (f) denote the results corresponding to (a), (b), and (c), respectively, when the varying speed of k is 10 times faster than that in (a).

recalculate the corresponding β_s. We find that the evolution of β_s is similar to the cases in figs. 3(b) and (c), indicating our approach also works for the situation of fast varying parameters. Figures 3(d), (e), and (f) show the corresponding results. These results show that β_s will change with time for the case of varying coupling and monotonously increase when the coupling strength k of time series is approaching k_c. It may be useful in the prediction of epileptic seizure. Imagine β_s is over 0.5 in some time and still keeps rising, it is most possible that there is a seizure coming. Comparing figs. 3(b) and (c) with figs. 2(a) and (b), one can see that they are very similar except the fluctuations in β_s. So β_s can reflect the change of coupling.

We have observed similar behaviors from different variables, such as x from the first system and z from the second system, and also in other systems, such as coupled Rössler oscillators. We believe our approach works for the cases of general time series, especially for the cases of non-stationary status. As we know, most of the techniques for analyzing time series assume, in their application, that the dynamical parameters are unchanged. For the case with non-stationary parameters, the interpretation of the results of this analysis is problematic. So our method presents a way to go a further step toward the insight of understanding the non-stationary time series.

An advantage of our method is the effectiveness and convenience. As we know, the time-delay technique needs to get embedding dimension and delay-time first, which makes the result

partially depend on the chosen delay-time and embedding dimension, and hence have some uncertainty. Our method gets information directly from the rotation of time series. Comparing the method with time-delay technique, our approach is much easier, faster, and less uncertain.

In conclusions, we have given a convenient method to estimate the phase locking relationship between two measured time series. This method is based on the consistence of local rotation speeds and can be used for the case of non-stationary status.

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