The attraction of Brazil nuts

To cite this article: D. A. Sanders *et al* 2006 *EPL* **73** 349

EUROPHYSICS LETTERS 1 February 2006

Europhys. Lett., **73** (3), pp. 349–355 (2006)
DOI: 10.1209/epl/i2005-10415-5

The attraction of Brazil nuts

D. A. Sanders, M. R. Swift, R. M. Bowley and P. J. King
School of Physics and Astronomy, University of Nottingham
Nottingham, NG7 2RD, UK

received 2 September 2005; accepted in final form 9 December 2005
published online 4 January 2006

PACS. 45.70.-n – Granular systems.
PACS. 45.70.Mg – Granular flow: mixing, segregation and stratification.

Abstract. – Simulations of intruder particles in a vertically vibrated granular bed suggest that neutrally-buoyant intruders are attracted to one another (*Phys. Rev. Lett.*, **93** (2004) 208002). The simulations, however, ignore important physical effects such as friction and convection which are known to influence intruder behaviour. Here, we present experimental evidence for this intruder-intruder interaction, obtained by monitoring the position of neutrally-buoyant metallic disks in a vibrated bed of glass spheres. An effective long-range attraction is shown to exist between a pair of intruders for a range of driving conditions. If further intruder particles are added, a tightly bound cluster of intruders can form. These results highlight the difficulty of retaining well-mixed granular beds under vertical vibration.

Granular materials are ubiquitous in nature [1] and widely used in industrial processes [2]. The simplest granular mixture is that of a single large intruder in a bed of identical host particles [3]. By varying the experimental conditions, two distinct types of behaviour have been observed in these systems. Over a range of excitations a large heavy intruder will rise to the top of a deep bed, the well-known Brazil nut effect (BNE) [4]. The rise of the intruder has been attributed to a range of processes, including ratcheting and convection [4, 5]. Recently, the presence of air has been shown to strongly influence the BNE in fine particulate beds [6]. A second type of intruder behaviour, the reverse BNE, has also been observed [7]. Here a large, light intruder sinks to the bottom of a deep bed provided the amplitude of oscillation is sufficiently large.

Closely related to the BNE is the study of multiple intruders in a vibrated bed [8]. Simulations [9] and kinetic theories [10] have been used to predict the conditions under which these simple granular mixtures will separate vertically. Some of these predictions have been confirmed experimentally [11]; however, there is still much ongoing debate about these phenomena [12].

In sufficiently fluidised beds the BNE has been shown to depend on the density of the intruder particles, such that intruders with similar density to the bed are not driven to the top or bottom [13], but fluctuate extensively around a mean height approximately half-way up the bed. Under these conditions the intruder is effectively "neutrally buoyant". Recently, simulations have been used to predict a third type of BNE for neutrally buoyant intruders [14]. Here, intruders in a shallow granular bed under vertical vibration tend to cluster together, leading to spontaneous horizontal segregation. Furthermore, the clustering of multiple intruders has been attributed to an effective attractive interaction between intruders, induced by asymmetry in the periodic motion of the bed. While these simulations aimed to capture the basic underlying dynamics of vibrated granular beds, they did not include all potentially relevant physical details such as particle friction and granular convection. It is not clear how robust the clustering behaviour is in real granular systems or, even if it exists at all.

In this letter we present experimental confirmation of a long-ranged intruder interaction and of the resulting clustering behaviour. First, we characterize the dynamics of a single neutrally buoyant intruder in a vertically vibrated, quasi–two-dimensional granular bed. Next we describe the behaviour of two neutrally buoyant intruders moving within the same bed. We measure experimentally the spatial correlation function of the two intruders, which shows a strong attractive interaction between them. To determine the range of this attraction, we compare our results with simulations in which the two intruders are non-interacting. We also show that the experimentally observed attractive interaction diminishes as the amplitude of vibration increases. To complete the picture we demonstrate that seven intruders placed in the bed exhibit strong clustering.

The nature of the intruder-intruder interaction investigated here is quite distinct from that reported by Duran and Jullien [15]. They proposed an attractive mechanism based on potential energy considerations and simulated the behaviour using steepest-decent techniques. The attraction they observed is extremely short-ranged and mediated by crystalline defects within a dense granular bed. In contrast, the attraction we have measured here occurs for neutrally buoyant intruders in fluidised granular beds.

Our granular system consisted of soda glass spheres (ballotini) with metallic (duralumin) disks as the intruders. These particles were vertically shaken in a narrow glass cell such that their motion was very close to two-dimensional. The glass spheres were sieved so that those with diameters in the range 2.0 to 2.2 mm were collected; any which appeared non-spherical were then removed. The average mass of each glass sphere was 11.4 mg and their average diameter was 2.05 mm. For particles of this size and mass, air effects are negligible. The effective 2D density of the glass spheres was $3.5\,\mathrm{mg\,mm^{-2}}$. The number of glass spheres used was 910, which corresponded to approximately eleven layers of glass spheres within our cell when it was at rest. The metallic disks used as intruder particles had diameter 6.0 mm and thickness 2.1 mm. To ensure that the disks were neutrally buoyant within the bed, holes were drilled in the centre of each disk to adjust their mass appropriately. Initially, we considered a range of intruder masses and chose one which best fitted our definition of neutral buoyancy. For our experiments we used copies of this intruder, each having a mass of 130 mg corresponding to a 2D density of $4.6\,\mathrm{mg\,mm^{-2}}$. The neutrally buoyant intruders had a 2D density slightly larger than that of the bed particles, presumably due to the slightly different frictional interactions of the particles and of the intruders with the front and back faces of the cell.

The front and back faces of the cell were built of two parallel sheets of soda glass, spaced 2.3 mm apart, such that the glass spheres and metallic disks could move vertically in a near rectangular cavity of width 175 mm and of height 77 mm. The base and side walls of the cell were also constructed of glass. The sides of the cavity were angled outward by approximately 1°–2° to reduce wall-driven convection [16]. The cell was vibrated vertically by attaching it to a rigid frame mounted between two electromagnetic transducers. This ensured that the motion was accurately one-dimensional and aligned to the vertical to within 0.2°. For all the results presented here our cell was vibrated sinusoidally at a frequency $f = 20\,\mathrm{Hz}$. The amplitude, A, was varied such that the dimensionless acceleration, $\Gamma = A(2\pi f)^2/g$, ranged from 2.0 to 7.0.

In order to track the motion of the intruder particles during vibration, the cell was filmed using a digital camera. Image analysis software was used to identify the position of the

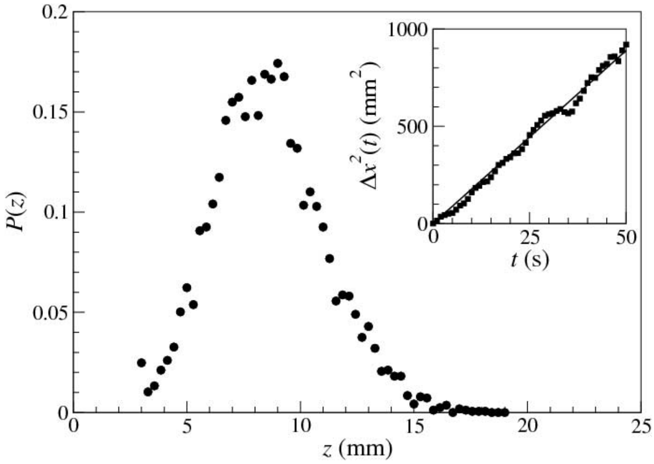

Fig. 1 – The probability density, $P(z)$, of the centre of a single intruder being at height z in a vibrated bed with $\Gamma = 5.0$. As the intruder has a diameter of 6.0 mm its lowest possible height is 3.0 mm from the base of the cell. The insert shows the mean-squared horizontal displacement $\Delta x^2(t)$ of an intruder particle *vs.* time, t, in seconds. The line is a linear least-squares fit through the origin.

intruders once every second. During preliminary experimental runs it became apparent that the glass spheres become significantly electrically charged after about 30 minutes of vibration. To minimise the influence of the build-up of static charge, the glass spheres were frequently removed from the cell and placed in a metal container, allowing them to discharge. The maximum time we allowed any experiment to run before discharging the grains was 15 minutes.

We shall first consider the behaviour of a single intruder within the bed. The value of Γ used was gradually increased from $\Gamma = 2.0$ to $\Gamma = 7.0$ and any changes in the behaviour of the granular bed were noted.

For $\Gamma \leq 2.0$ the intruder remained locked in position, moving with the glass spheres which were in a hexagonal close-packed arrangement. Only the top layer of glass spheres was mobile. As Γ was increased the bed became more agitated and vacancies appeared in the hexagonal lattice allowing the metallic disk to rise and fall. For $\Gamma \geq 3.5$ the whole bed of particles moved as if it were a dense fluid, enabling the intruder particle to move sideways as well as rising and falling in the bed. If $\Gamma \geq 6.5$ the intruder particle moved rapidly through the bed.

Within the intermediate range $4.0 \leq \Gamma \leq 6.0$, the intruder wandered randomly through the bed, suggesting that its horizontal motion was diffusive. However, once an intruder particle reached one of the side walls of the cell, it tended to remain close to that wall indefinitely. Consequently, in order to quantify the motion of the intruder through the bulk of the bed, it was necessary to minimise the influence of boundary effects on our measurements. To achieve this, we adopted the following experimental procedure: a single intruder was positioned near the centre of the bed and the cell vibrated; the apparatus was filmed, allowing the intruder's motion to be tracked; the filming was stopped when the intruder came within 4.0 mm of either side wall or when 15 minutes had elapsed. This procedure was repeated many times; for each value of Γ at least 4000 s of footage was recorded. From each recording i, we measured the horizontal, $x_i(t)$, and vertical position, $z_i(t)$, of the intruder relative to the cell, once every second.

Figure 1 shows the distribution of heights of the intruder, $P(z)$, within the bed for $\Gamma = 5.0$. The average height of the intruder was determined to be about 9.0 mm and the average depth of the vibrated bed was approximately 21 mm. From these estimates we see that the mean in-

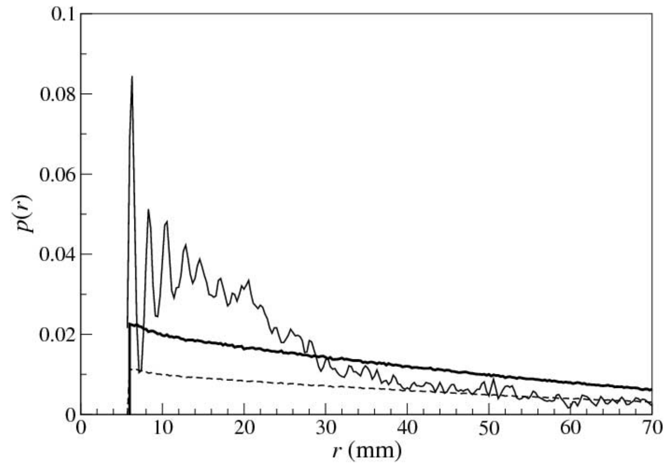

Fig. 2 – Intruder separation probability density, $p(r)$, *vs.* separation r in mm. The thin solid line shows the experimentally measured distribution for $\Gamma = 5.0$. The heavy solid line shows the simulated distribution for non-interacting intruders. The dashed line shows the simulated distribution scaled to match the experimental data at large separations.

truder height is about half-way up the bed. Note also that the distribution of heights is broadly peaked, the standard deviation being approximately 3.0 mm. This shows that our intruder particle is indeed neutrally buoyant, and moves extensively up and down within the bed.

In order to analyse the horizontal motion of the intruder, the data sets $x_i(t)$ were divided into sections 50 s long; for each value of Γ we had over 80 such sections which were treated as independent trajectories. From these trajectories we calculated the variance in the intruder's horizontal motion, $\Delta x^2(t)$, which is shown as a function of time in the inset to fig. 1. The variance grows linearly with t, indicating that the motion is diffusive. In this way we estimated D from the relation $\Delta x^2(t) = 2Dt$, and obtained $D = 8.0 \pm 2.0\,\mathrm{mm^2\,s^{-1}}$ for $\Gamma = 5.0$. The above procedure was repeated for $\Gamma = 4.0$ giving $D = 2.0 \pm 0.7\,\mathrm{mm^2\,s^{-1}}$ and $\Gamma = 6.0$ giving $D = 9.0 \pm 2.5\,\mathrm{mm^2\,s^{-1}}$. While attempts were made to ensure that the apparatus was level, it is impossible to completely remove any horizontal bias. We therefore determined the resulting drift velocity, $\overline{u}_x$.

We now introduced a second intruder into the cell and used the same methodology to characterise the behaviour. At the beginning of every run we placed the intruders near the centre of the cell, approximately 18 mm apart, roughly half-way up the bed. As the cell was vibrated the separation of the two intruders was measured every second and a normalised probability distribution constructed such that the probability of finding the two intruders at a separation between r and $r + \mathrm{d}r$ is $p(r)\mathrm{d}r$. Again, runs were stopped if either intruder came to within 4.0 mm of a side wall or if 15 minutes elapsed.

In fig. 2 we show $p(r)$ obtained for $\Gamma = 5.0$ as a thin solid line. This distribution is heavily weighted to short separations. The peaks in $p(r)$ are the result of packing effects, the separation between the peaks corresponding closely to the diameter of the glass spheres. Beyond $r = 15$ mm the distribution decreases steadily with increasing r. These results strongly suggest the existence of an attractive interaction between the intruders.

However, since the intruders are placed close to one another at the start of every run, a small bias has been introduced into our data. In order to quantify the effect of this bias on $p(r)$, we carried out simulations to predict $p(r)$ for two *non-interacting* intruders. Initially, the

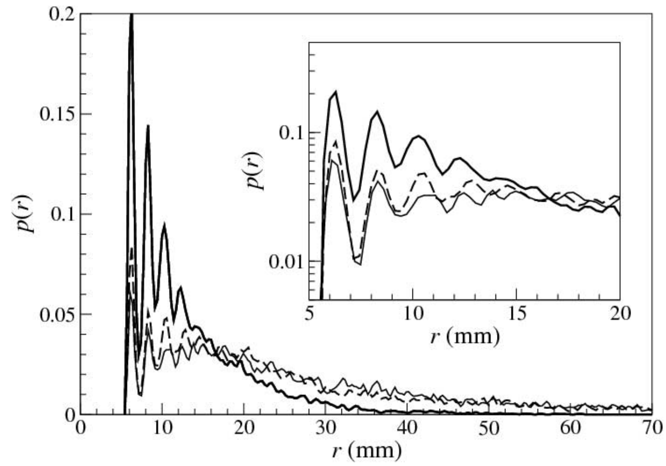

Fig. 3 – Experimentally determined intruder separation probability density, $p(r)$, *vs.* separation r in mm for three values of Γ: $\Gamma = 4.0$ (thick solid line), $\Gamma = 5.0$ (dashed line), and $\Gamma = 6.0$ (thin solid line). The insert shows the small-r behaviour on a log-linear plot.

simulated intruder particles were placed near the centre of the cell with a separation of roughly two intruder diameters. We used the approximate values of D, $\overline{u}_x$, and $P(z)$ obtained from the single-intruder experiments to model the intruders' motion within the cell, using over-damped Langevin dynamics. In a mirror of the experimental procedure, the simulation was stopped if either of the intruder particles reached the edge of the cell or if 15 minutes simulated time elapsed. For each run the trajectory of the intruders can be calculated assuming that there was no interaction between them, apart from a hard-core repulsion for separations less than an intruder diameter. By averaging over many such simulations we were able to calculate the probability of finding the two intruders at a separation between r and $r + \mathrm{d}r$.

The predicted probability for two *non-interacting* intruders is shown as the heavy solid line in fig. 2. This line indicates that our experimental procedure does indeed produce a small bias as there is an enhanced probability of finding the intruders at shorter separations. The experimental results clearly show a much higher probability of finding the intruders close together than is predicted for non-interacting particles. This confirms the existence of an attractive interaction between intruders in our experiments.

In fig. 2, both the experimental (thin solid line) and simulated (thick solid line) probability distributions have been normalised so that the area under the curves is equal to one. However, in order to determine the range of the intruder attraction, one must compare the experimentally measured $p(r)$ with the simulated $p(r)$ for non-interacting intruders, scaled so that their values coincide at large separations. This is because one would expect the intruders in the experiments to be non-interacting only at large separations. The dashed curve in fig. 2 shows the simulated $p(r)$ scaled down so that it matches the experimental data for $r \approx 70\,\mathrm{mm}$. The experimental and scaled simulation data are in good agreement for separations from 70 mm down to 40 mm, whereas for $r \leq 36\,\mathrm{mm}$, the experimental and scaled simulated data begin to differ significantly. This indicates that the attraction extends up to $\sim 36\,\mathrm{mm}$, or about 6 intruder diameters.

In fig. 3 we show $p(r)$ from experiment for three different values of Γ. The data for $\Gamma = 4.0$ shows a much stronger attraction with much larger peaks than for $\Gamma = 5.0$. Under these conditions, the intruders appear visibly bound. In fact the data for $\Gamma = 4.0$ is in

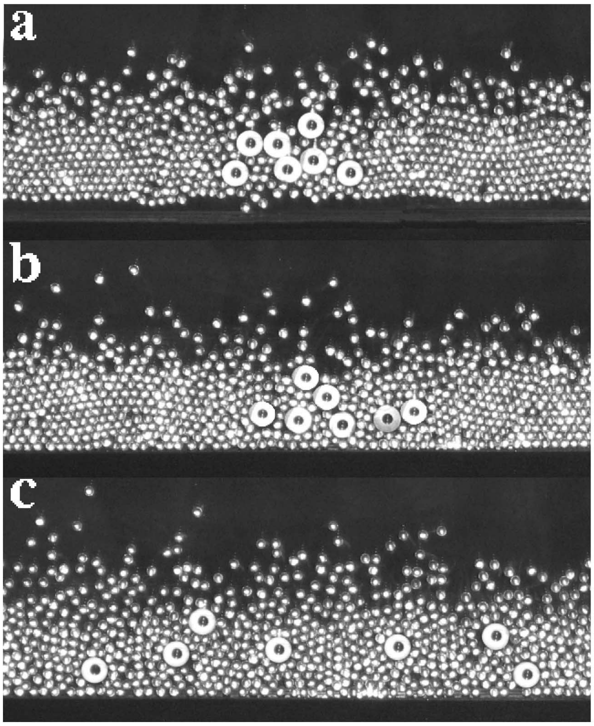

Fig. 4 – Snapshots showing typical configurations of the granular bed with seven intruders present: (a) $\Gamma = 4.0$, (b) $\Gamma = 5.0$ and (c) $\Gamma = 6.0$. The images have been enhanced to improve the contrast between intruder particle and the granular bed.

excellent qualitative agreement with the predictions of 2D computer event-driven simulations presented in [14]. However, a more direct quantitative comparison is not possible because in our experimental setup, the particles are influenced by friction with the front and back faces of the cell [17]. The experimental data for $\Gamma = 6.0$ shows a slightly weaker attraction than for $\Gamma = 5.0$ and the peaks are somewhat smaller, indicating that the greater agitation reduces the intruder interaction. Indeed for $\Gamma = 7.0$ the intruders appear to move completely independently of one another.

Finally, the experiment was run with seven intruder particles placed in the central third of the cell. When the cell was vibrated at $\Gamma = 4.0$, the intruders quickly formed a tightly bound cluster in the centre of the cell. This is shown in fig. 4a. If the experiment was run for a long time the cluster sometimes drifted to one of the cell walls. Very rarely would any of the intruders break away from the main cluster and those that did quickly rejoined it. The same experiment was conducted at $\Gamma = 5.0$ (fig. 4b) and $\Gamma = 6.0$ (fig. 4c). For $\Gamma = 5.0$ one central cluster regularly formed. However, this cluster was not as tightly bound as in the case of $\Gamma = 4.0$. For $\Gamma = 6.0$, the tendency to cluster is much reduced.

Our experimental results have confirmed the prediction that neutrally buoyant intruders are attractive. We have measured the influence of this attraction on a pair of such intruder particles and have demonstrated that this interaction is significant up to separations of ~ 6 intruder diameters. We have also shown that the attractive force between intruders diminishes with increasing Γ. Furthermore, we have observed the clustering of multiple intruders and investigated the range of Γ over which this phenomenon occurs. The system we have considered is possibly the simplest example of a granular mixture. By tuning the intruders' size and density appropriately, we have attempted to eliminate the established mechanisms for granular segregation. However, instead of achieving a completely mixed system, we observe an attractive interaction which can lead to horizontal segregation. Our results highlight the difficulties faced when trying to mix different types of granular material.

REFERENCES

[1] Jaeger H. M., Nagel S. R. and Behringer R. P., *Rev. Mod. Phys.*, **68** (1996) 1259.

[2] Willis B. A., *Mineral Process Technology*, 6th edition (Butterworth and Heinmann, Boston) 1997.

[3] For a review see Rosato A. D. *et al.*, *Chem. Eng. Sci.*, **57** (2002) 265 and references therein.

[4] Rosato A. *et al.*, *Phys. Rev. Lett.*, **58** (1987) 1038.

[5] Knight J. B., Jaeger H. M. and Nagel S. R., *Phys. Rev. Lett.*, **70** (1993) 3728; Duran J., Rajchenbach J. and Clément E., *Phys. Rev. Lett.*, **70** (1993) 2431; Vanel L., Rosato A. D. and Dave R. N., *Phys. Rev. Lett.*, **78** (1997) 1255; Liffman K. *et al.*, *Granular Matter*, **3** (2001) 205.

[6] Möbius M. E. *et al.*, *Nature*, **414** (2001) 270; Naylor M. A., Swift M. R. and King P. J., *Phys. Rev. E*, **68** (2003) 012301; Möbius M. E. *et al.*, *Phys. Rev. Lett.*, **93** (2004) 198001.

[7] Shinbrot T. and Muzzio F. J., *Phys. Rev. Lett.*, **81** (1998) 4365.

[8] For a review see Kudrolli A., *Rep. Prog. Phys.*, **67** (2004) 209 and references therein.

[9] Hong D. C., Quinn P. V. and Luding S., *Phys. Rev. Lett.*, **86** (2001) 3423; Shishodia N. and Wassgren C. R., *Phys. Rev. Lett.*, **87** (2001) 084302; Krouskop P. E. and Talbot J., *Phys. Rev. E*, **68** (2003) 021304; Jullien R., Meakin P. and Pavlovitch A., *Phys. Rev. Lett.*, **69** (1992) 640; Biswas P. *et al.*, *Phys. Rev. E*, **68** (2003) 050301(R).

[10] Jenkins J. T. and Yoon D. K., *Phys. Rev. Lett.*, **88** (2002) 194301; Trujillo L. and Herrmann H. J., *Physica A*, **330** (2003) 519; Trujillo L., Alam M. and Herrmann H. J., *Europhys. Lett.*, **64** (2003) 190; Willits J. T. and Arnarson B. Ö., *Phys. Fluids*, **11** (1999) 3116; Both J. A. and Hong D. C., *Phys. Rev. Lett.*, **88** (2002) 124301.

[11] Breu A. P. J. *et al.*, *Phys. Rev. Lett.*, **90** (2003) 014302.

[12] Canul-chay G. A. *et al.*, *Phys. Rev. Lett.*, **89** (2002) 189601; Quinn P. V., Hong D. C. and Luding S., *Phys. Rev. Lett.*, **89** (2002) 189602; Walliser H., *Phys. Rev. Lett.*, **89** (2002) 189603; Quinn P. V., Hong D. C. and Luding S., *Phys. Rev. Lett.*, **89** (2002) 189604.

[13] Huerta D. A. and Ruiz-suárez J. C., *Phys. Rev. Lett.*, **92** (2004) 114301.

[14] Sanders D. A. *et al.*, *Phys. Rev. Lett.*, **93** (2004) 208002.

[15] Duran J. and Jullien R., *Phys. Rev. Lett.*, **80** (1998) 3547.

[16] Knight J. B., *Phys. Rev. E*, **55** (1997) 6016.

[17] Warr S., Huntley J. M. and Jacques G. T. H., *Phys. Rev. E*, **52** (1995) 5583.