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Non-Markovian spin relaxation in two-dimensional electron gas

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Abstract. – We analyze by Monte Carlo simulations and analytically the spin dynamics of a two-dimensional electron gas (2DEG) interacting with short-range scatterers in nonquantizing magnetic fields. It is shown that the spin dynamics is non-Markovian with the exponential spin relaxation followed by an oscillating tail due to the electrons residing on closed trajectories. The tail relaxes on a long time scale due to an additional smooth random potential and inelastic processes. The developed analytical theory and Monte-Carlo simulations are in quantitative agreement with each other.

Spin-dependent phenomena in semiconductors and semiconductor nanostructures have been attracting an increasing interest during last decade (for review see, *e.g.*, ref. [1]). The understanding of the mechanisms of spin decoherence, usually considered as a Markovian process, as well as the possibilities to control it is an important problem in the field. In the two-dimensional (2D) zincblende structures the electron spin dynamics is governed by the spin-orbit (SO) splitting of the conduction band. It originates from the lack of the inversion center either in the bulk material as represented by the bulk inversion asymmetry (BIA or Dresselhaus term [2, 3]) or the structural asymmetry of the heteropotential revealed by the Rashba term [4]. In all the cases the SO interaction gives rise to the effective Zeeman magnetic field which is characterized by the electron wave vector \mathbf{k} -dependent spin precession vector $\mathbf{\Omega}_{\mathbf{k}}$ whose direction determines the axis and the absolute value is the spin precession rate. In the collision dominated regime of electron motion with $\Omega\tau \ll 1$ (where Ω is the typical value of $|\mathbf{\Omega}_{\mathbf{k}}|$ and τ is the scattering time) the small spin rotation angles $\Omega\tau$ between successive collisions are not correlated, thus the spin relaxation rate τ_s^{-1} is proportional to the $\Omega^2\tau$ [3, 5]. This D'yakonov-Perel' spin relaxation mechanism, which represents a Markovian process, is the most important one in the wide range of temperatures and carrier concentrations [1, 6–12]. At given sample parameters there are two possibilities to control the spin relaxation rate with this mechanism. First, one may apply an external electric field in order to tune the Rashba constant [9, 13]. Second, an external magnetic field can suppress the spin relaxation due to the cyclotron rotation, Larmor spin precession or their combination [14, 15]. Although the main mechanisms of the magnetic field effects on the Markovian spin relaxation are understood [16], the peculiarities in the spin dynamics caused by the fascinating non-Markovian transport phenomena in magnetic field, lavished attention only very recently [17–19], and are scarcely known. Non-Markovian spin dynamics can arise for electrons interacting with nuclear spins [20], however, with a qualitatively different mechanism of the memory.

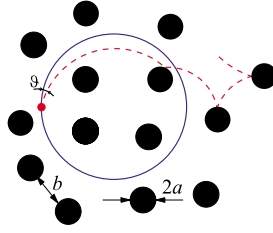


Fig. 1 – Electron’s motion along the circling trajectory. The black circles are the scattering centers. The solid line is the initial “circling” trajectory. The dashed line is the wandering trajectory after the inelastic scattering event with scattering angle ϑ . Here a is the scatterer radius and $b \sim N^{-1/2}$ is the mean distance between scatterers. Figure shown not to scale.

In this letter we point out and study a new non-Markovian effect in electron spin relaxation in the magnetic field, namely, the appearance of a long-living spin polarization tail. We consider a degenerated 2DEG with SO coupling scattered by short-range impurity centers (or antidots) with radius a and concentration N in a presence of a magnetic field \mathbf{B} directed perpendicular to the structure plane. Assuming specular reflection by the scatterers, the kinetics of electrons is characterized by the total scattering length $l = 1/(2Na)$ and the transport mean free path $l_{tr} = 3l/4$. As shown in the pioneering work [21] (see also refs. [22–24] and references therein) a fraction of electrons $P_c(B)$ moving with the velocity v does not experience scattering travelling along closed circular orbits. In the $Na^2 \ll 1$ limit it is given by

$$P_c(B) = \exp[-2\pi R_c/l], \quad (1)$$

where $R_c = v/\omega_c$ is the cyclotron radius, $\omega_c = eB/mc$ is the cyclotron frequency, e is the elementary charge and m is the electron effective mass. Physically, eq. (1) gives the probability to find no scatterers in the area between the circles of radii $R_c \pm a$, see fig. 1. As the electron finished the cyclotron revolution without a collision it will stay on this orbit infinitely long, thus demonstrating a non-Markovian behavior with an infinitely long memory. The fraction of these electrons increases with the magnetic field resulting in the classical magnetoresistance [23]. Here we discuss the role of the closed orbits in the spin relaxation. We show that the spin dynamics of the electrons on such trajectories is strictly periodical and the total spin falls exponentially down to the value determined by the fraction of circular orbits. Therefore, a tail in the spin polarization can be observed. Further, we develop a theory describing the decay of this tail caused by the scattering processes which transfer electrons between closed and open “wandering” [23] orbits. Finally, we discuss the role of the quantum effects.

In order to obtain quantitative results, we consider a $\hat{z} \parallel [001]$ -grown zincblende quantum well (QW) and assume that the SO interaction is dominated by the Dresselhaus term [3]:

$$\mathcal{H}_{SO}(\mathbf{k}) = \frac{\hbar}{2}(\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_{\mathbf{k}}), \quad \boldsymbol{\Omega}_{\mathbf{k}} = \Omega_0[\cos \varphi_{\mathbf{k}}, -\sin \varphi_{\mathbf{k}}, 0]. \quad (2)$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli matrix vector, Ω_0 is the spin precession frequency at the Fermi level and $\varphi_{\mathbf{k}}$ is the angle between the electron wave vector \mathbf{k} and the $\hat{x} \parallel [100]$ axis.

We assume classical dynamics of the electrons and their spins and subdivide the electrons into two groups. The first one corresponds to wandering electrons, while the second one corresponds to circling electrons, their relative fractions are $1 - P_c(B)$ and $P_c(B)$, respectively. An accurate description of the wandering electrons in the moderate magnetic fields can be done by using the kinetic equation and their spin decays exponentially with the rate [14]

$$\frac{1}{\tau_s(B)} = \frac{\Omega_0^2 \tau}{1 + (\omega_c \tau)^2}, \quad (3)$$

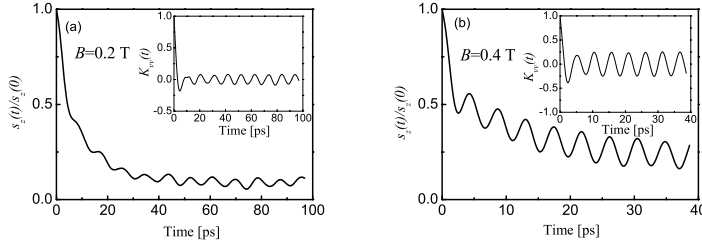


Fig. 2 – The results of the Monte Carlo simulation for 4096 electrons of the spin dynamics $s_z(t)/s_z(0)$ and the velocity autocorrelation function $K_{vv}(t)$. The scatterer radius is $a = 2 \times 10^{-6}$ cm, $v = 2 \times 10^7$ cm/s, $N = 3 \times 10^9$ cm $^{-2}$, and $B = 0.2$ T, $\Omega_0 = 4 \times 10^{11}$ s $^{-1}$ (panel (a)) and $B = 0.4$ T, $\Omega_0 = 8 \times 10^{11}$ s $^{-1}$ (panel (b)); $\omega_c = 3.0 \times 10^{12}$ s $^{-1}$ /T.

where $\tau = l_{tr}/v_F$. In the derivation of eq. (3) we assumed $\Omega_0\tau \ll 1$ ⁽¹⁾ and neglected the Larmor effect on spin relaxation as it is usually small in GaAs-based QWs. The Larmor effect can be taken into account in a standard way [16].

The spin of the single electron with $\mathbf{k}(t)$ -dependent wave vector is described by the equation

$$\frac{\partial \mathbf{s}}{\partial t} + \mathbf{s} \times \boldsymbol{\Omega}_{\mathbf{k}(t)} = 0. \quad (4)$$

Equation (4) can be readily solved assuming that \mathbf{k} rotates in the QW plane with frequency ω_c . For the initial condition $2\mathbf{s}(0) = (0, 0, 1)$, we obtain for the z -component

$$2s_z(t) = \frac{\omega_c^2}{\omega_c^2 + \Omega_0^2} + \frac{\Omega_0^2}{\omega_c^2 + \Omega_0^2} \cos\left(\sqrt{\Omega_0^2 + \omega_c^2}t\right). \quad (5)$$

The solutions for $s_{x,y}$ are rather cumbersome and not presented here. It is seen in eq. (5) that on the circling trajectory $2s_z$ oscillates around the mean value $\omega_c^2/(\omega_c^2 + \Omega_0^2) \rightarrow 1$ (in high fields, $\omega_c \gg \Omega_0$) with frequency $\sqrt{\Omega_0^2 + \omega_c^2}$, and the amplitude of the oscillations is $\Omega_0^2/(\omega_c^2 + \Omega_0^2) \rightarrow 0$ at $\omega_c \gg \Omega_0$. Note that these oscillations are insensitive to the initial phase of the \mathbf{k} rotation and thus all the spins of the circling electrons oscillate synchronously.

Therefore, if in the 2DEG the transitions between circling and wandering trajectories are strictly forbidden, at $t \gg \tau_s(B)$ (defined by eq. (3)) the total spin does not decay to zero as was assumed previously, but oscillates around the non-zero value

$$s_z^t = s_0 P_c(B) \frac{\omega_c^2}{\omega_c^2 + \Omega_0^2}, \quad (6)$$

where s_0 is the initially excited spin. Further, we consider the case of the strong enough magnetic fields, $\omega_c \gg \Omega_0$, where the oscillations of the circling electron spin are small and spin relaxation is followed by a non-vanishing tail with magnitude $s_z^t = P_c(B)s_0$. In order to have an insight into the tail formation, we performed a Monte Carlo simulation of the electron transport and spin dynamics with the results shown in fig. 2. It is seen that the exponential decay of the velocity autocorrelation function $K_{vv}(t) = \langle \mathbf{v}(t)\mathbf{v}(0) \rangle / v^2$ (shown in the inset) is followed by the oscillations due to the circling electrons. The magnitude of the oscillations corresponds to the value of $P_c(B)$. The polarization exhibits an oscillatory tail in agreement with eqs. (5), (6). We mention that in the nonquantizing field $B = 0.5$ T the tail has a considerable magnitude, $P_c(B) > 0.1$, already at low mobilities $\mu = 5 \times 10^4$ cm 2 /Vs.

Now we turn to the decay of the spin polarization tail. Any additional scattering process such as inelastic scattering or scattering by a random potential destroys the tail. With

⁽¹⁾This condition can be relaxed provided $\Omega_0 \ll \omega_c$ (M. M. Glazov, to be published).

the relaxation time approximation the z spin component is described by the coupled kinetic equations

$$\begin{aligned}\frac{\partial s_z^c}{\partial t} &= -\frac{s_z^c[1 - P_c(B)] - s_z^w P_c(B)}{\tau_j(B)}, \\ \frac{\partial s_z^w}{\partial t} &= \frac{s_z^c[1 - P_c(B)] - s_z^w P_c(B)}{\tau_j(B)} - \frac{s_z^w}{\tau_s(B)}.\end{aligned}\quad (7)$$

Here superscript c (w) corresponds to circling (wandering) electrons, respectively, and $\tau_s(B)$ and $\tau_j(B)$ are the spin relaxation time for wandering electrons and the intermixing time between the closed and wandering trajectories, respectively. The first term in the numerator of the right-hand side of eqs. (7) denotes the transfer of the electron from the circling to the wandering trajectory and is proportional to the scattering intermixing rate and the fraction of the wandering trajectories $1 - P_c(B)$ and the second term refers to the inverse process. In deriving eqs. (7) we have neglected small oscillations of s_z^c and took into account D'yakonov-Perel' spin relaxation of the wandering electrons (second term of the second line in (7)). The eigenrates of the system $\tau_{1,2}^{-1}(B)$ are

$$\frac{1}{\tau_{1,2}(B)} = \frac{1}{2} \left(\Gamma_{js} \pm \sqrt{\Gamma_{js}^2 + 4 \frac{P_c(B) - 1}{\tau_j(B)\tau_s(B)}} \right), \quad (8)$$

where $\Gamma_{js} = \tau_j^{-1}(B) + \tau_s^{-1}(B)$. Thus, the total electron spin $s_z(t)$ relaxes according to the two exponential law: $s_z(t) = \alpha \exp[-t/\tau_1(B)] + \beta \exp[-t/\tau_2(B)]$, where coefficients α and β depend on the initial condition and on B . Since $\tau_2(B) \geq \tau_1(B)$, the long-time $s_z(t)$ asymptotics of $s_z(t)$ is determined by $\tau_2(B)$ which can be identified with the tail relaxation time.

For example, in the limit of weak magnetic fields, $P_c(B) \ll 1$ and if $\tau_s(B) \ll \tau_j(B)$, the relaxation times are $\tau_1(B) = \tau_s(B)$ and $\tau_2(B) = \tau_j(B)$ and the total electron spin rapidly decays to the tail value $P_c(B)s_0$ and then slowly relaxes during the time $\tau_j(B)$ (as almost every “additional” scattering transfers an electron from the circling to the wandering trajectory where the spin is quickly lost).

When the magnetic field becomes stronger with $\omega_c \tau \gg 1$, $\tau_s(B) = \tau_s(0)(\omega_c \tau)^2$ increases quadratically with the field and the fraction $P_c(B) = 1 - 3\pi/(2\omega_c \tau)$ tends to unity. Two limiting regimes as a function of B are possible. In weak fields, where the intermixing τ_j is still much longer than $\tau_s(B)$, the spin tail relaxation time is field independent. With the increase of B , the intermixing time becomes smaller than $\tau_s(B)$, and, therefore, the tail disappears. In such a case only a small fraction ($\sim \omega_c \tau$) of electrons is losing at a given moment their spin, however circling and wandering electrons are intermixed on a short timescale. Therefore the tail is not observed but the spin relaxation time is

$$\tau_2(B) \approx \frac{l}{2\pi R_c} \tau_s(B) \approx \frac{2(\omega_c \tau)^2}{3\pi \Omega_0^2 \tau}, \quad (9)$$

being proportional to B^3 (contrary to the standard result B^2 [14]). Here $\tau_2(B)$ does not depend on the inelastic scattering time, despite the presence of strong inelastic scattering which is necessary here only to intermix the trajectories sufficiently quickly. These results are shown in fig. 3a illustrating the evolution of the spin relaxation process $s_z(t)$ and the rate with the change in the number of wandering trajectories governed by the B -dependent ratio l/R_c .

In sufficiently strong fields, the number of trajectories colliding several times with the same impurity, increases, and at a critical field such as $4\pi N(a + R_c)^2 \approx 4.4$ and $(R_c/l)^2 = 4.4Na^2/\pi \ll 1$ [22] all the paths become confined. For the parameters considered in fig. 2, this corresponds to $B > 0.8$ T. In this case the spin relaxation is strongly suppressed [25]. The detailed analysis of this regime is beyond the scope of this letter.

Below, we consider two kinds of the microscopic processes which contribute to the intermixing time $\tau_j(B)$ between closed and wandering trajectories. Namely, we discuss the destruction of the tail due to: i) the presence of the smooth potential, ii) inelastic processes.

i) In real QWs the typical disorder can be considered as a superposition of the short-range scattering centers⁽²⁾, the long-range smooth potential of the remote dopants and the interface roughness⁽³⁾. The correlation radius of this potential d is the distance between the dopant layer and the QW [26,28]. We will be interested in the weak smooth disorder where the transport scattering time by the long-range potential τ_l satisfies the condition $\tau_l\omega_c \gg 1$, and, therefore, the circling trajectories remain at least approximately intact. The displacement of the orbit center in the smooth potential, provided $R_c \gg d$, is $\delta r \sim l_l/(\omega_c\tau_l)^{3/2} \sim \sqrt{R_c^3/d}(W/E_F)$, where $l_l = v_F \tau_l$ is the transport mean free path there and W is the potential amplitude [29]. The transfer between trajectories is governed by the ratio $\delta r/a$. The rich transport [26] and spin dynamics of the electrons in the system with two types of disorder are beyond the scope of this letter. Here we consider the simplest situation where $\delta r \ll a$ and thus $\delta r \ll d$. Therefore, the electron drifts in the potential gradient averaged over its path and sweeps, during n revolutions, an area $S \sim 2\pi R_c n \delta r$. It will be transferred from the circling trajectory to the wandering as soon as it meets a short-range center in this area, *i.e.* $SN \sim 1$. So, the intermixing time can be estimated as

$$\tau_j^{sp}(B) \sim \frac{T_c}{NR_c\delta r} \propto B^{3/2}, \quad \delta r/a \ll 1, \quad (10)$$

where $T_c = 2\pi/\omega_c$ is the cyclotron period and it is assumed that $al_{tr}/(dR_c) \ll 1$. For the purpose of the present paper, we note that the electron drift in the smooth potential leads to spin relaxation even for circular trajectories, therefore the applicability of eq. (10) together with the system (7) is guaranteed, provided the intermixing time is much longer than the spin relaxation time of the circling electrons.

ii) The inelastic processes (electron-electron or electron-phonon scattering) change randomly electron wave vector and energy and thus transfer electrons between circling and wandering trajectories. In the low-temperature regime where the Fermi surface is well defined, $E_F \gg k_B T$, where E_F is the Fermi energy and $k_B T$ is the temperature measured in the energy units, all the scattering processes are statistically quasi-elastic [30], *i.e.* the transferred energy $\sim k_B T$ is much smaller than electron kinetic energy E_F . The crucial question is to determine the probability that the scattering at an angle ϑ (fig. 1) is not accompanied by an electron transition from the circling to the wandering orbit. From fig. 1 it is clear that the displacement of the circular orbit by a distance of the order of the impurity separation b will lead to the electron collision with some scattering center, thus the probability should have a peak in the vicinity of $\vartheta = 0$ with an angular width of the order of $b/R_c \sim 1/(N^{1/2}R_c)$. At larger angles the probability saturates at a value close to $P_c(B)$, representing the fraction of the circling electrons. This prediction is corroborated by the Monte Carlo calculation, see fig. 3b.

In a degenerate 2D electron gas with carrier concentration ranging in the interval $10^{11}-10^{12} \text{ cm}^{-2}$ the Coulomb interaction is screened, the electron-electron scattering is almost isotropic and its temperature dependency is determined by the Pauli-exclusion principle only. At not too low temperatures, $k_B T \gtrsim \hbar c_s \sqrt{\langle k_z^2 \rangle}$, where c_s is the speed of sound and $\sqrt{\langle k_z^2 \rangle}$ is the root mean square of the electron wave vector in the growth direction, the electron-acoustic phonon scattering is dominated by the deformation-potential coupling and is isotropic as well. Thus we further assume that the inelastic scattering is isotropic and can be described in the

⁽²⁾These centers can be introduced artificially as antidots, see, *e.g.*, [17,26,27].

⁽³⁾The monolayer fluctuations of the QW interfaces induce the potential $\delta V \sim 1-10 \text{ meV}$ with lateral size $50-100 \text{ \AA}$. Thus, the scattering length on the interface roughnesses a is of the order of 10^4 \AA .

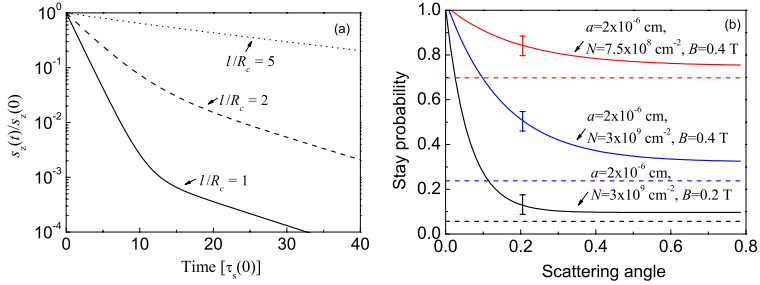


Fig. 3 – (a) Electron spin s_z time dependence calculated from eqs. (7), for different values of the magnetic field: $l/R_c = 1, 2, 5$ from bottom to top, $\tau_j = 10\tau_s(B = 0)$. (b) The scattering probability found by the Monte Carlo simulation (solid lines) and the values of $P_c(B)$ (dashed lines). The parameters used in the calculations are shown above each curve. Other parameters are same as in fig. 2. The accuracy of the Monte-Carlo computation is shown by the error bars.

framework of the relaxation time τ_j , which can be estimated as for the electron-electron and electron-phonon scattering, respectively [31, 32]:

$$\tau_j^{ee} = \zeta_e \frac{\hbar}{E_F} \left(\frac{E_F}{k_B T} \right)^2, \quad \tau_j^{ph} = \zeta_{ph} \frac{\rho c_s^2 \hbar^3}{D^2 m k_B T \sqrt{\langle k_z^2 \rangle}}. \quad (11)$$

Here $\zeta_e = 4/[\pi \ln(E_F/k_B T)]$ for Fermi level electrons, D is the deformation potential constant, ρ is a material density, and $\zeta_{ph} \sim 1$ is a numerical coefficient. At lower temperatures the electron-phonon scattering is partially suppressed due to Pauli-exclusion principle and by the phonon-freeze-out, and the coefficient ζ_{ph} becomes strongly temperature dependent. Its exact calculation represents an extremely difficult numerical tasks, but simple estimates show that in the temperature range $T \leq 20$ K and for electron concentration $N_e \sim 10^{11}$ cm $^{-2}$ the electron-electron scattering is dominant. We note that the fraction of the circling electrons $P_c(B)$, eq. (1), has a purely geometric character, thus the tail is insensitive to the thermal broadening of the electron distribution, provided inelastic processes are weak enough.

Finally, we would like to comment on the quantum effects neglected above. First, the quantization of electron orbits leads to an oscillatory B -dependence of the spin relaxation rate (3) [33]. This effect is small for high Landau levels ($E_F/\hbar\omega_c \gg 1$) and vanishes as temperature increases. Second, the quantization of the electron orbits modifies the relaxation rates of inelastic scattering in eq. (11). However, for semiclassical Landau levels this modification is negligible [30]. Third, the magnetic field leads to spin polarization either due to Zeeman effect or as a result of the SO coupling and Landau quantization [4]. The former contribution is proportional to $g\mu_B B/E_F$, where g is the electron g -factor and μ_B is Bohr magneton. For the typical GaAs structures g is small, besides it can be adjusted to zero with the structure parameters [6]. The contribution $|\mathcal{P}_L| = |\langle \psi_{L,\pm} | \sigma_z | \psi_{L,\pm} \rangle|$ of the latter effect can be evaluated for the L -th Landau level $E_{L,\pm} = \hbar\omega_c(L \pm \sqrt{\gamma^2 L + 1/4})$ with $\gamma = \hbar\Omega_0/(2\sqrt{E_F}\hbar\omega_c)$ as $|\mathcal{P}_L| = (2\sqrt{\gamma^2 L + 1/4})^{-1}$ and is negligible for high Landau levels with $\gamma^2 L \gg 1$.

In conclusion, we have theoretically analyzed the spin dynamics of the 2D electron gas scattered by short-range defects in classically strong magnetic fields. We have shown that a tail in the spin polarization appears as a result of the collisionless circular motion of the fraction of electrons. It was demonstrated that the tail decays due to the presence of an additional weak long-range potential or due to inelastic scattering. In strong magnetic fields the long-time spin relaxation is slowed down as a cube of the field. The predicted phenomena open new perspectives in the spin dynamics control in 2DEG.

* * *

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REFERENCES

- [1] ZUTIC I., FABIAN J. and SARMA S. D., *Rev. Mod. Phys.*, **76** (2004) 323.
- [2] DRESSELHAUS G., *Phys. Rev.*, **100** (1955) 580.
- [3] D’YAKONOV M. I. and KACHOROVSKII V. YU., *Sov. Phys. Semicond.*, **20** (1986) 110.
- [4] RASHBA E. I., *Sov. Phys. Solid State*, **2** (1964) 1874.
- [5] D’YAKONOV M. I. and PEREL’ V. I., *Sov. Phys. Solid State*, **13** (1972) 3023.
- [6] IVCHENKO E. L., *Optical Spectroscopy of Semiconductor Nanostructures* (Alpha Science) 2005.
- [7] AVERKIEV N., GOLUB L. and WILLANDER M., *J. Phys.: Condens. Matter*, **14** (2002) R271.
- [8] MALINOWSKI A., BRITTON R. S., GREVATT T., HARLEY R. T., RITCHIE D. A. and SIMMONS M. Y., *Phys. Rev. B*, (2000) 13034.
- [9] KARIMOV O., JOHN G., HARLEY R., LAU W., FLATTE M., HENINI M. and AIREY R., *Phys. Rev. Lett.*, **91** (2003) 246601.
- [10] BRAND M. A., MALINOWSKI A., KARIMOV O. Z., MARSDEN P. A., HARLEY R. T., SHIELDS A. J., SANVITTO D., RITCHIE D. A. and SIMMONS M. Y., *Phys. Rev. Lett.*, **89** (2002) 236601.
- [11] SONG P. H. and KIM K. W., *Phys. Rev. B*, **66** (2002) 035207.
- [12] OHNO Y., TERAUCHI R., ADACHI T., MATSUKURA F. and OHNO H., *Phys. Rev. Lett.*, **83** (1999) 4196.
- [13] MILLER J. B., ZUMBUHL D., MARCUS C., LYANDA-GELLER Y. B., GOLDBABER-GORDON D., CAMPMAN K. and GOSSARD A. C., *Phys. Rev. Lett.*, **90** (2003) 076807.
- [14] IVCHENKO E. L., *Sov. Phys. Solid State*, **15** (1973) 1048; IVCHENKO E. L., KOP’EV P. S., KOCHERESHKO V. P., URALSTEV I. N. and YAKOVLEV D. R., *JETP Lett.*, **47** (1988) 486.
- [15] WILAMOWSKI Z. and JANTSCH W., *Phys. Rev. B*, **69** (2004) 035328.
- [16] GLAZOV M. M., *Phys. Rev. B*, **70** (2004) 195314.
- [17] PERSHIN Y. V. and PRIVMAN V., *Phys. Rev. B*, **69** (2004) 073310.
- [18] LYUBINSKIY I. S. and KACHOROVSKII V. YU., *Phys. Rev. B*, **70** (2004) 205335; LYUBINSKIY I. S. and KACHOROVSKII V. Y., *Phys. Rev. B*, **73** (2006) 041301.
- [19] GLAZOV M. M. and SHERMAN E. Y., *Phys. Rev. B*, **71** (2005) 241312.
- [20] MERKULOV I. A., EFROS AL. L. and ROSEN M., *Phys. Rev. B*, **65** (2002) 205309; KHAETSKII A., LOSS D. and GLAZMAN L., *Phys. Rev. B*, **67** (2003) 195329.
- [21] BASKIN E. M., MAGARILL L. N. and ENTIN M. V., *Sov. Phys. JETP*, **48** (1978) 365.
- [22] BASKIN E. and ENTIN M., *Physica B*, **249** (1998) 805.
- [23] DMITRIEV A., DYAKONOV M. and JULLIEN R., *Phys. Rev. Lett.*, **89** (2002) 266804.
- [24] BOBYLEV A. V., MAAO F. A., HANSEN A. and HAUGE E. H., *J. Stat. Phys.*, **87** (1997) 1205.
- [25] LYUBINSKIY I. S., *JETP Lett.*, **83** (2006) 336.
- [26] POLYAKOV D. G., EVERS F., MIRLIN A. D. and WOLFLE P., *Phys. Rev. B*, **64** (2001) 205306.
- [27] SOTOMAYOR CHOQUE N. M., GUSEV G. M., LEITE J. R., BYKOV A. A., LITVIN L. V., MOSHEGOV N. T., TOROPOV A. I., MAUDE D. K. and PORTAL J. C., *Phys. Rev. B*, **66** (2002) 035324.
- [28] MIRLIN A. D., POLYAKOV D. G., EVERS F. and WOLFLE P., *Phys. Rev. Lett.*, **87** (2001) 126805.
- [29] MIRLIN A. D., WILKE J., EVERS F., POLYAKOV D. G. and WOLFLE P., *Phys. Rev. Lett.*, **83** (1999) 2801.
- [30] GANTMAKHER V. and LEVINSON Y., *Carrier Scattering in Metals and Semiconductors* (North-Holland, Amsterdam) 1987.
- [31] GLAZOV M. M. and IVCHENKO E. L., *JETP*, **99** (2004) 1279.
- [32] PRICE P. J., *Ann. Phys. (N.Y.)*, **133** (1981) 217.
- [33] BURKOV A. A. and BALENTS LEON, *Phys. Rev. B*, **69** (2004) 245312.