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Shearing a fermionic gas and quantized friction

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Abstract – The frictional forces in sheared fermionic gas are investigated. The gas is sheared by two sliding surfaces. Except for a small imperfection (bulge) on one of them, the surfaces are totally smooth. We show that when the bulge is extremely small, and the gas is also confined in the lateral dimension, the frictional force (F) is *quantized*. That is, $F = \mu_F v$, where v is the sliding velocity with respect to the lubricant gas, $\mu_F \equiv 2mn_c^2 h$ is the friction coefficient, h is the Planck constant and m is an integer. It is also shown that the coefficient n_c depends only on the bulge's properties (it does not depend on either the gas properties or the sliding velocity).

Introduction. – Recent developments in micro-electromechanical systems have raised practical interest in research involving quantum friction [1–13]. However, the investigation of quantum friction has revealed some surprising fundamental effects. For example, two smooth dielectric surfaces moving laterally (parallel to one another) experience frictional forces as if the vacuum between them were a viscous fluid [14–16]. Therefore, friction measurements can contain a great deal of information on the nature of vacuum fluctuations. Sliding surfaces, however, are seldom smooth; in fact, they are usually rough and corrugated. The most common remedy for such roughness is, of course, lubrication, and in modern micro-machinery, the lubricants are usually gases [17,18]. Also, the possibility of confining a fermionic gas in space with a laser beam has been shown experimentally (see, for example, refs. [19,20]).

In this paper, we investigate the onset of frictional forces between two adjacent surfaces. The two surfaces are sliding against one another at zero temperature, whilst the shearing is lubricated by a fermion (say, atoms or electrons) gas. On one of the surfaces there is a small bulge. The collisions between the particles and the bulge create the frictional force. The main argument of (and motivation for) this paper is the following: a small fraction from each of the transversal modes is reflected from the bulge. The modes, which are propagating against the bulge, are reflected from it with a slightly higher energy. When the sliding velocity is low (in comparison to the Fermi velocity), the energy increase is proportional to the particles' momentum, and therefore, the power activated upon the bulge is proportional to p^2R (p is the momentum and R is the modes' reflectivity coefficient– see a detailed discussion further on). Now, since for a very weak protrusion $R \sim p^{-2}$ (probably the simplest one is the delta-function potential. See also refs. [22], and eq. (9)), one can conjecture that each mode makes *exactly* the same contribution to the frictional force, which therefore should be quantized. In this paper, we show that for a very weak protrusion (the bulge), this is indeed the case.

The system's description. – Imagine a lubricating fermionic gas confined between two infinitely long surfaces at zero temperature (they are infinitely long in the x-direction and have a finite width in the z-direction, see fig. 1). The fermionic gas is thus confined to a rectangular cross section, whose width corresponds to the surfaces' width (w) and whose height is equal to the distance between them (d in the y-direction). In the following, we discuss only very close surfaces, *i.e.*, $d \ll w$. On one of the surfaces, say the upper one, there is a small protrusion (bulge or bump, for example, see fig. 1). For simplicity, it is assumed that the protrusion has no features in the transverse direction, *i.e.*, it is independent of z, and thus can be fully characterized by its cross section in the x-y plane. Since in most cases the friction depends very strongly on the size of the bulge (when the bulge is a small impurity the force is proportional to the fourth power of the size of the bulge [21]), even if there are many bulges, the largest one will have the dominant effect on

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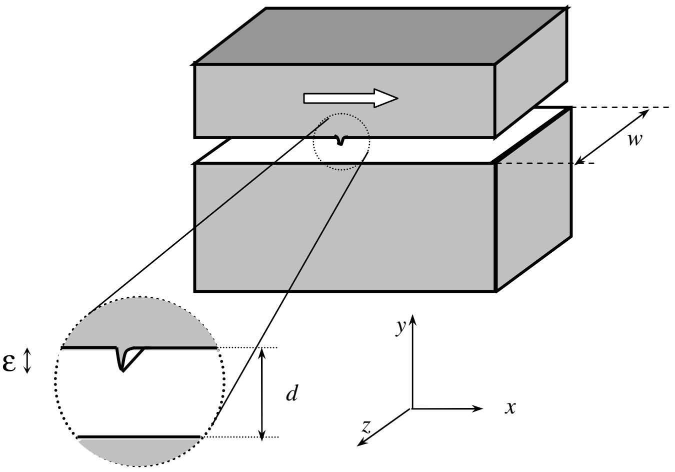

Fig. 1: An illustration of the two. On the upper (sliding) surface there is a small bulge, which is magnified in the inset. The fermionic gas is confined between the two surfaces.

the friction, and the rest can be ignored (therefore, it does not really matter whether the bulge is microscopic or macroscopic). It is also assumed, for simplicity, that the lower surface has an infinite mass so that it is always at rest with respect to the gas. Therefore, when the two surfaces slide against each other, with a relative velocity v (*i.e.*, the bulge slides with a velocity v with respect to the lubricant gas), the protrusion's resistance to the sliding is manifested by a frictional force. If not for the bulge, the shearing process would not influence the lubricant. Classically, the particles of the lubricant gas, which are bouncing back and forth in their confinement zone prevent the bulge from sliding smoothly through the lubricant.

In many respects this system is related to the the surface conductivity of refs. [12], however, in this paper we do not use the semiclassical treatment, but instead solve the problem in the quantum regime. The Schrödinger equation, which describes the dynamics of the confined lubricant, reads

$$-\frac{\hbar^2}{2m_0}\nabla^2\psi+[V_{su}(y)+V_{co}(z)+V_{pr}(x,y)]\,\psi=i\hbar\frac{\partial\psi}{\partial t},\quad (1)$$

where $V_{su}(y)$ is the potential of the surfaces,

$$V_{co}(z)\equiv\begin{cases}0, & 0\leqslant z\leqslant w,\\ \infty, & \text{otherwise},\end{cases}$$

is the confinement potential in the z-direction and $V_{pr}(x,y)$ is the potential of the protrusion, which satisfies $V_{pr}\to 0$ for $|x|\to\infty$; note that it does not depend on z.

In the following discussion, we assume zero temperature, and a very sparse lubricant (very low density), *i.e.*, the density per unit length (n) satisfies $n\ll d^{-1}$. Therefore, only the first transversal mode (which is related to the coordinate y) is occupied. In the z-direction, however, many modes can survive, which will be identified by their quantum number j. For any given energy $E=\hbar\omega$ and a given channel j, the quantum wave function of the lubricant particles can be written:

$$\psi^{\pm}_{j,\omega}=\sin(\pi y/d)\sin(\pi zj/w)\exp(\pm ik^jx-i\omega t),\qquad (2)$$

where the longitudinal wave number k^j satisfies: $0\leqslant k^j\leqslant k_F^j$, when $\left(k_F^j\right)^2\equiv 2m_0E_F/\hbar^2-(\pi/d)^2-(\pi j/w)^2$, m_0 is the lubricant gas particles' mass; j is an integer that characterizes the transversal mode, E_F is the Fermi energy and the upper (lower) sign in eq. (2) refers to particles which propagate from the left (right) side of the bulge to its right (left) side.

In order to solve this problem, it is very convenient to choose a frame of reference in which the bulge is at rest. Consequently, the bulge "sees" (in the new frame of reference) on its right, particles emerging with the wave numbers $0\leqslant k_r\leqslant k_F^j+m_0v/\hbar$, for every channel j, while on its left the incoming particles have, for the same channel, the wave numbers: $0\leqslant k_l\leqslant k_F^j-m_0v/\hbar$ (see fig. 2).

It should be stressed that since the gas is not confined in the x-direction, it can be regarded as if it is connected to two infinite reservoirs. Therefore, the bulge is *always* in motion relative to the gas. In the moving frame of reference we will always see the picture of fig. 2. This reasoning is very similar to the one taken in conductance calculations (on the connection between friction and conductance, see, for example, [12] and [13] and references therein).

In the following we discuss only very low sliding velocities, *i.e.*, $m_0v/\hbar\ll k_F^j$, which means that the particles' maximum energy on the right is higher by $\Delta\varepsilon_j\cong 2k_F^jv\hbar$ (in the moving frame) than the particles' maximum energy on the left.

Clearly, in the first approximation, only the particles that propagate from the right (to the left) with momentum $k_F^j-m_0v/\hbar\leqslant k_r\leqslant k_F^j+m_0v/\hbar$ contribute to the energy wasted by friction.

Let us denote by $T_j\equiv T(k=k_F^j)$ the quantum transmission (for transport in the x-direction) through the bulge via the j-th channel. It should be pointed out here that the geometry prevents mode mixing. The reflected current of the j-th channel (note that here we are regarding a particle's current and not a charge current) from the bulge, which includes only the particles with $k_r\geqslant k_F^j-mv/\hbar$, is equal to

$$I_j=\Delta\varepsilon_j(1-T_j)/\hbar.\qquad (3)$$

Each of the reflected particles gains an energy quantum $2\Delta\varepsilon_j$, and therefore the power imposed upon the bulge is

$$P=2\sum_j\Delta\varepsilon_j^2(1-T_j)/\hbar.\qquad (4)$$

Therefore, the power produced during the shearing process maintains the relation $P=\frac{4}{\pi}v^2\hbar\sum_j(k_F^j)^2(1-T_j)$. Finally, the friction force reads

$$F=P/v=\mu_Fv,\qquad (5)$$

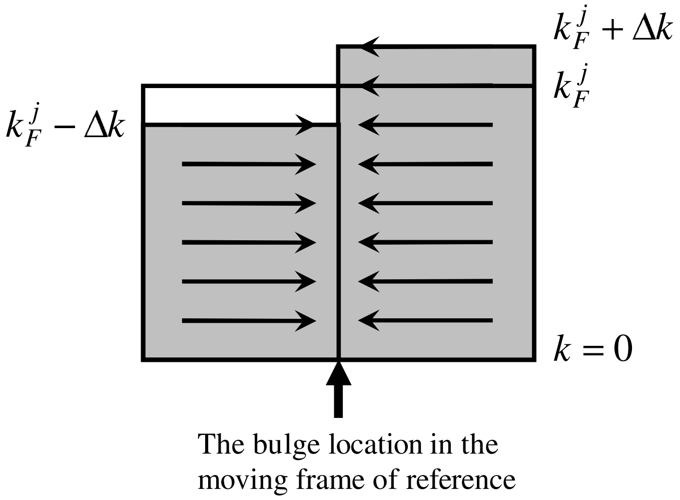

Fig. 2: At the moving frame (where the bulge is at rest), the particles on both sides of the bulge can be regarded as hitting it with slightly different momenta (for the j-th mode in the figure), $\Delta k \equiv m_0 v/\hbar$.

where the friction coefficient is

$$\mu_F \equiv 2h \sum_j n_j^2 (1 - T_j) \tag{6}$$

and $n_j \equiv (2/\pi) k_F^j$ is the particles' density per unit length of the jth channel (the prefix 2 comes from the spin degeneracy).

Equation (5) suggests a linear relationship between the frictional force and the shearing velocity. Notice that T_j is also a function of the lubricant density, since it is a function of the Fermi wave number. On the other hand, the transmission coefficient T has no dependence on the lubricant's characteristics other than its density, but it is highly sensitive to the bulge's properties, and therefore no general relation can be obtained. However, a general conclusion that does arise from eq. (6) is that

$$\mu_F \leqslant \mu_F^{max} \equiv 2h \sum_j n_j^2 \tag{7}$$

regardless of the bulge's characteristics or dimensions. Moreover, this result even holds true for an arbitrary number of protrusions, and thus can be applied to any rough surface. For any number of channels, eq. (7) can be evaluated (by calculating the sum in 7) as $\mu_F^{max} = 2h \left[\left(q^2 - \frac{4}{d^2}\right) m - \frac{2}{w^2} \frac{m(m+1)(2m+1)}{3} \right]$, where $q^2 \equiv (2m_0 E_F/\hbar^2)(4/\pi^2)$ and m, which is the integral part of $w(q^2/4 - 1/d^2)^{1/2}$, is the number of channels (note the difference between the number of channels m and the lubricant particles' mass m_0). When $m \to \infty$, *i.e.*, $w \to \infty$, it can be approximated by (simply by taking the limit)

$$\frac{\mu_F^{max}}{w} = \frac{2}{3} h \left(q^2 - 4/d^2\right)^{3/2}. \tag{8}$$

When the onset of friction is being considered the exact nature of the protrusion is very important. Of course, the transmission T cannot have a general expression; however, in a wide range of thin and small bulges (or bumps), the transmission can be expressed by [21]

$$T_j \cong \left(1 + (n_c/n_j)^2\right)^{-1}, \tag{9}$$

where n_c is a transition density, which characterizes the bulge. If the problem was a one-dimensional one, and the protrusion was a delta-function, then the approximation sign "$\cong$" would be replaced by the equality sign "$=$". Nevertheless, if the protrusion is a very small bulge, this expression can be applied with great accuracy, even to problems with more than a single dimension (see refs. [21,22]).

If, for example, the protrusion is a very shallow but long bulge, then $n_c \sim L^{-1}$, where L is the bulge's length. If the bulge is a point impurity (in the x-y plane and independent of z), then [22]

$$n_c = 4\pi^2 (\varepsilon^2/d^3) \ln^{-1}(\rho_0/\varepsilon C), \tag{10}$$

where ε is the distance from the surface and ρ_0 is a length parameter, which is proportional to the resonance wavelength of the impurity and is related to the eigen-energy (E_0) of the impurity by $E_0 = 8(\hbar/\rho_0)^2 m_0^{-1} \exp(-\gamma)$, $\gamma \cong 0.577$ is the Euler constant and $C \cong 5$ is a numerical constant (see refs. [21,22]). In any case, eq. (6) should read (by substituting eq. (9) in eq. (6))

$$\mu_F = 2hn_c^2 \sum_j \frac{1}{1 + (n_c/n_j)^2}. \tag{11}$$

For very sparse lubricant ($n_j \ll n_c$ for every channel j), the friction coefficient expectedly vanishes and the expression in eq. (7) is regained: $\mu_F = 2h \sum_j n_j^2$.

More important is the case of a point protrusion, that is $\varepsilon \to 0$, or $n_c \ll n$. This regime is important not only because it investigates *the onset of friction* due to a minuscule defect (bulge), but also because it even applies to relatively high densities (so long as the distance between the surfaces is small enough, *i.e.*, $n_j \ll d^{-1}$ for every j), in which case the interaction between the particles (if there is any) can be neglected.

The friction coefficient (eq. 11) as a function of the Fermi wave number $k_F^0 \equiv \sqrt{2m_0 E_F/\hbar^2 - \pi^2/d^2}$ (see definition in eq. (2) is presented in fig. 3 for a specific n_c. μ_F is measured in units of $2hn_c^2$ and k_F^0 is measured in units of π/w. The plot exhibits a staircase pattern and reveals the *quantization* of the friction coefficient μ_F, where the friction coefficient quantum is $2hn_c^2$. This quantity does not depend on either the lubricant's properties or its density. It depends only on properties of the bulge and the distance between the surfaces.

When $n_c \to 0$, the staircase pattern is more sharply delineated, and eq. (11) can be written

$$\mu_F = 2mn_c^2 h, \tag{12}$$

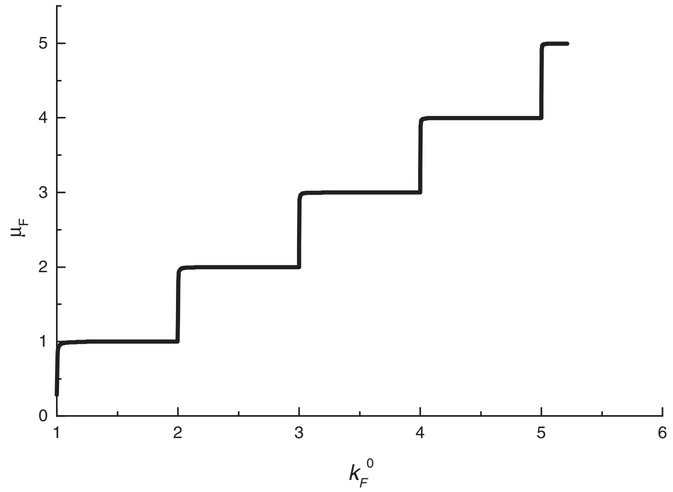

Fig. 3: A plot of the friction coefficient (μ_F) for the case $n_c = 0.1/w$ as a function of the normalized Fermi wave number ($k_F^0 \equiv \sqrt{2m_0E/\hbar^2 - \pi^2/d^2}$). μ_F is measured in units of $2hn_c^2$ and k_F^0 is measured in units of π/w. This plot manifests the quantization of the friction coefficient.

where m is the total number of propagating channels. This is the main result of this paper, which manifests the quantization of quantum friction for extremely weak protrusions. Each mode has exactly the same contribution to the friction force: $\delta F = 2n_c^2 hv$.

It is clear from the coefficient that this is a quantum effect, which disappears for classical particles. Moreover, if the particles were bosons, they would all accumulate at the lowest energy state (at $T = 0$) and the quantization would disappear.

Numerical evaluation and possible implementation. – If the bulge is a point protrusion (in the x-y plane), whose eigenenergy [21,22] is approximately equal to $E_0 = (h/\varepsilon)^2/m_0$ and, for example, $d \sim 1\text{nm}$, $\varepsilon/d \sim 0.3$ and $v = 1\text{m/s}$, then eq. (10) can be used and the friction quanta δF (*i.e.*, the force is $F = m\delta F$) can be larger than $\delta F > 10^{-13}\text{N}$– a small but measurable quantity [4,5].

Note that the friction quantum is independent (in the high-density regime) of the gas properties. Therefore, when the density increases, the force increases with it but not its quanta.

To enlarge δF, one can use more bulges[1] or more parallel channels (see fig. 4). Since the gas is confined only to the laser beam, it is possible to use many parallel beams, each of which makes the same contribution to the force (note that δF is independent of w). That is, the beam confines the gas only in the z-direction (clearly, in the y-direction the surfaces are responsible for the confinement).

[1]The number of bulges should not be too large, since if the distance from the first bulge to the last one is larger than the reciprocal of k_F, eq. (9) is invalid.

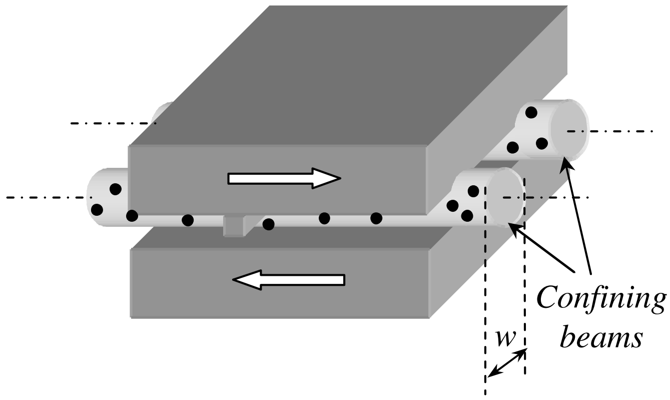

Fig. 4: The gas is confined by laser beams. Each beam makes the same contribution to the friction force. The beams in this case are polarized in the y-direction. The black dots represent the confined atoms (fermions).

Summary. – The frictional force which emerges in a lubricant fermionic gas due to shearing was investigated. The discussion focused on the onset of the frictional force. It was shown that when the smoothness of one of the sliding surfaces is damaged by a single small imperfection, the frictional force is proportional to the sliding velocity, *i.e.*, $F = \mu_F v$, where the friction coefficient is quantized $\mu_F = 2mn_c^2 h$ (m is an integer). The friction coefficient quantum $\delta\mu_F = 2n_c^2 h$ depends only on the geometry, *i.e.*, on the bulge and on the distance between the surfaces; it does not depend on either the lubricant gas properties or the sliding velocity.

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