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Anderson localization in correlated fermionic mixtures

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Abstract – A mixture of two fermionic species with different masses is studied in an optical lattice. The heavy fermions are subject only to thermal fluctuations, the light fermions also to quantum fluctuations. We derive the Ising-like distribution for the heavy atoms and study the localization properties of the light fermions numerically by a transfer-matrix method. In a two-dimensional system one-parameter scaling of the localization length is found with a transition from delocalized states at low temperatures to localized states at high temperature. The critical exponent of the localization length is $\nu \approx 0.88$.

The question of Anderson localization in an ultracold gas has attracted considerable attention recently by a number of experimental groups [1–3]. Although the phenomenon itself has been studied in great detail over the last 50 years by many theoretical groups for various physical systems [4–6], its experimental observation has been difficult. One of the reasons is that Anderson localization is an interference effect of waves due to elastic scattering in a random environment (disorder). Real systems, however, experience also substantial inelastic scattering (*e.g.* absorption of electromagnetic waves by the scattering atoms, Coulomb interaction in electronic systems etc.) This may hamper the direct observation of Anderson localization significantly. Another reason is that random scattering is difficult to control in a real system. This is important in order to distinguish Anderson localization from simple trapping due to local potentials. It requires some kind of averaging over an ensemble of randomly distributed scatterers.

Ultracold gases offer conditions, where most physical parameters are controllable. Since the atoms are neutral, there is no Coulomb interaction, and at sufficiently high dilution the interatomic collisions are negligible. Moreover, a periodic potential (optical lattice) can be applied by counterpropagating laser fields. This enables us to control the kinetic properties of the gas atoms by creating a specifically designed dispersion. Disorder could be created by disturbing the periodicity of the optical lattice. In practice, however, this is not easy because real disorder would require infinitely many laser frequencies. A first attempt is to study the superposition of two laser fields with "incommensurate" frequencies (*i.e.* the ratio of the two frequencies is an irrational number) [2]. An alternative is to randomize the laser field by sending it through a diffusing plate [3].

Recent progress in atomic mixtures [7] has offered another possibility to create disorder in an atomic system. Mixing of two different atomic species, where one is heavier than the other, creates a situation where the light atoms are scattered by the randomly distributed heavy atoms [8–12]. An optical lattice is applied in order to keep the heavy atoms in quenched positions. Due to their higher mass, the heavy atoms behave classically in contrast to the light atoms, which can tunnel in the optical lattice. A crucial question is what determines the distribution of the heavy atoms. The most direct distribution is obtained by putting atoms randomly in the optical lattice "by hand", each of them with independent probability [8]. This case corresponds to uncorrelated disorder. Another possibility is to fill the optical lattice with both atomic species and consider a repulsive (local) interaction between them. Then, the two species have to arrange each other such that the total atomic system presents a grand-canonical ensemble at a given temperature and a given lattice filling. Máska *et al.* have discussed several scenarios in which a mixture of light and heavy atoms can behave according to the Falicov-Kimball model [10]. A very likely one consists of an atomic cloud, for which an optical lattice is slowly turned on. Once the optical lattice is fully applied, the cloud is not in a low-energy state yet. Therefore, energy

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will be transferred between the atoms to reduce the total energy of the cloud. The main processes are tunneling of light atoms and scattering of light atoms by heavy atoms. Then heavy atoms absorb energy in the scattering process which will be released by classical (thermal) jumps in the optical lattice. The latter, of course, will take place on a much larger time scale than the tunneling of the light atoms such that they can be included by a thermal average.

In the presence of interparticle interaction within each atomic species there is a complex interplay of interaction and localization effect. This makes it difficult to isolate the effect of Anderson localization. In order to avoid interaction within each species we choose spin-polarized fermions in an optical lattice. Then, only the Pauli principle controls the short-range interaction within each species, and the remaining interaction is only between the different fermionic species. It has been shown that then the light atoms are subject to a quenched average with respect to a thermal distribution of the heavy atoms, and that the distribution is related to an Ising-like model [9,10,13]. The latter implies (strong) correlations between the heavy atoms. For systems in more than one dimension there is a critical temperature T_c at which the correlation length diverges. This system provides several interesting features for studying Anderson localization. Although it is a many-body system, the light atoms behave effectively like independent (spinless fermionic) quantum particles in a random potential. The correlation of the randomness can be controlled by temperature, where the correlation length decreases with increasing temperature for temperatures $T > T_c$, or by the strength of the interspecies scattering.

In the following we shall study diffusion and Anderson localization in a grand-canonical ensemble of two spin-polarized fermionic species. Motivated by a recent experimental study on a dilute BEC in $d = 1$ [3], we consider the Fermi gas in equilibrium and calculate the scaling properties of the localization length in one and two dimensions.

Model. – $c^\dagger$ (c) are creation (annihilation) operators of the light fermionic atoms, $f^\dagger$ (f) are the corresponding operators of the heavy fermionic atoms. The physics of the mixture of atoms is defined by the asymmetric Hubbard Hamiltonian

$$H = -\bar{t}_c \sum_{\langle r,r'\rangle} c_r^\dagger c_{r'} - \bar{t}_f \sum_{\langle r,r'\rangle} f_r^\dagger f_{r'} - \sum_r \left[\mu_c c_r^\dagger c_r + \mu_f f_r^\dagger f_r - U f_r^\dagger f_r c_r^\dagger c_r\right]. \quad (1)$$

The effective interaction within each species is controlled by the (repulsive) Pauli principle, whereas the interaction strength of different atoms is U. If the f atoms are heavy, the related tunneling rate is very small. The limit $\bar{t}_f = 0$ is known as the Falicov-Kimball model, which has been studied in great detail using the coherent-potential approximation (CPA) and the dynamical mean-field theory (DMFT) [10,14–17]. CPA as well as DMFT are based on the infinite dimensional limit which gives a reliable description on spectral properties such as the gap opening at the metal-insulator transition. Since we are interested in properties of the wave functions in one- and two-dimensional realizations of the FK model, however, we cannot use these approximation schemes here but must employ a numerical scaling method.

A grand-canonical ensemble of fermions at the inverse temperature $\beta = 1/k_B T$ is defined by the partition function

$$Z = \mathrm{Tr} e^{-\beta H}.$$

In the FK limit $\bar{t}_f = 0$ the Hamiltonian of the light atoms depends only on the real numbers $\{n_r\}$ ($n_r = 0, 1$), representing the presence or absence of a heavy atom at lattice site r. Then the Hamiltonian is given by a quadratic form with respect to the c operators of the light atoms:

$$H_c(\{n_r\}) = \sum_{r,r'} h_{c;rr'} c_r^\dagger c_{r'} = -\bar{t}_c \sum_{\langle r,r'\rangle} c_r^\dagger c_{r'} + \sum_r (U n_r - \mu_c) c_r^\dagger c_r, \quad (2)$$

where the second equation is the definition of the matrix h_c. This means that the density fluctuations $n_r = f_r^\dagger f_r$ have been replaced by classical variables $n_r = 0, 1$. Thus $H_c(\{n_r\})$ describes non-interacting fermions which are scattered by heavy atoms, represented by n_r. The trace Tr_c in the partition function can be evaluated and gives a fermion determinant:

$$Z = \sum_{\{n_r\}} e^{\beta\mu_f \sum_r n_r} \mathrm{Tr}_c \left(e^{-\beta H_c(\{n_r\})}\right) = \sum_{\{n_r\}} e^{\beta\mu_f \sum_r n_r} \det\left(\mathbf{1} + e^{-\beta h_c}\right). \quad (3)$$

The right-hand side is a sum over (non-negative) statistical weights. After normalization we can define

$$P(\{n_r\}) = \frac{1}{Z} e^{\beta\mu_f \sum_r n_r} \det\left(\mathbf{1} + e^{-\beta h_c}\right), \quad (4)$$

which gives $\sum_{\{n_r\}} P(\{n_r\}) = 1$. Thus $P(\{n_r\})$ is a probability distribution for correlated disorder and describes the distribution of the heavy atoms. In the strong-coupling regime $\bar{t}_c^2/2U \gg 1$ the distribution becomes that of an Ising model with nearest-neighbor coupling. At half-filling (*i.e.* $\mu_f = \mu_c = U/2$) it reads [9]

$$P(\{S_r\}) \propto \exp\left(-\beta(\bar{t}_c^2/2U) \sum_{\langle r,r'\rangle} S_r S_{r'}\right), \quad (5)$$

where $S_r = 2n_r - 1$.

Anderson localization. – A trapped atomic cloud, concentrated around the center of the optical lattice, is the initial state $|i\rangle$ of our system. After switching off the trapping potential the dynamics of the light atomic cloud is described by the evolution equation $|\Psi_t\rangle = e^{-iHt}|i\rangle$. The optical lattice remains present during the evolution of the cloud. We assume that the thermal excitations are slow in comparison with the tunneling dynamics. This is the case when the tunneling energy $\bar{t}_c$ is large in comparison with the thermal energy $k_B T = 1/\beta$. Moreover, a slow adiabatic expansion is studied. Now we consider a light atom inside the expanding cloud and follow its movement: using the equilibrium state of the entire system $|0\rangle$, we add one particle to create the initial state $|i\rangle = c_0^\dagger|0\rangle$. Then, the local density of particles at site r with respect to the state $|\Psi_t\rangle$ reads

$$N_r = \langle\Psi_t|c_r^\dagger c_r|\Psi_t\rangle = \langle i|e^{iHt}c_r^\dagger c_r e^{-iHt}|i\rangle.$$

The equilibrium state $|0\rangle$ can be expanded in terms of energy eigenfunctions and Boltzmann weights at the inverse temperature β as

$$\langle N_r\rangle = \frac{\sum_k e^{-\beta E_k}\langle E_k|c_0 e^{iHt}c_r^\dagger c_r e^{-iHt}c_0^\dagger|E_k\rangle}{\sum_k e^{-\beta E_k}} = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}c_0 e^{iHt}c_r^\dagger c_r e^{-iHt}c_0^\dagger\right]. \quad (6)$$

For the FK model this expression can also be written as a quenched average with respect to the distribution of heavy particles [13]

$$\langle N_r\rangle = \langle\mathcal{G}_{0r}^\dagger(t)\mathcal{G}_{r0}(t)\rangle_f \quad (7)$$

with the single-particle Green's function

$$\mathcal{G}_{rr'}(t) = [e^{-ith_c}(\mathbf{1}+e^{-\beta h_c})^{-1}]_{rr'}.$$

$\langle\ldots\rangle_f$ is the average with respect to the statistical weight of eq. (4) or eq. (5). For a given configuration $\{n_r\}$ of heavy atoms the Green's function can also be expressed by eigenfunctions of the single-particle Hamiltonian h_c in eq. (2) ($h_c\phi_k = e_k\phi_k$). The spatial properties of these eigenfunctions determine the spreading of the average density $\langle N_r\rangle$ through the Green's function:

$$\mathcal{G}_{r0}(t) = \sum_k e^{-ie_k t}\frac{\phi_{k,r}^*\phi_{k,0}}{1+e^{-\beta e_k}}. \quad (8)$$

The denominator represents the Fermi function, reflecting the fact that our atoms are fermions. At low temperatures all states with $e_k > 0$ (*i.e.* states with energy above the chemical potential acccording to eq. (2)) contribute equally to the Green's function.

According to the localization theory, it can be assumed that $|\phi_{k,r}| \sim e^{-|r|/\xi_k}$, where ξ_k is the localization length. After a Fourier transformation of the time-dependent density in eq. (7), the $\omega = 0$ Fourier component of the time-dependent density $\langle N_r\rangle$ reads

$$\bar{N}_r(\omega=0) = \sum_k\left\langle\frac{|\phi_{k,r}^*\phi_{k,0}|^2}{(1+e^{-\beta e_k})^2}\right\rangle_f \sim \frac{e^{-2|r|/\xi}}{(1+e^{-\beta e_{k_0}})^2} \quad (r\sim\infty),$$

where ξ is the largest localization length and e_{k_0} the corresponding energy level. Thus the expansion of the wave packet on large scales is controlled by ξ. This result suggests that the spatial expansion of an atomic cloud is governed by the largest length scale of the system, after having removed the characteristic size given by the trapping potential.

The localization length can be studied under the change of length scales of a finite optical lattice of length L and width M [6], representing the adiabatically expanding atomic cloud. In particular, we analyze the change of the localization length with respect to the width M. For this purpose, we define the reduced (or normalized) localization length of light atoms as $\Lambda_M = \xi/M$ and calculate this quantity by means of a numerical transfer-matrix approach for a given realization $\{n_r\}$ of heavy atoms [18]. For the latter statistically relevant realizations are chosen by Monte Carlo sampling of the distribution in eq. (5). For this purpose, the $L\times M$ strip is divided into $M\times M$ squares and the sampling is performed starting on one end and proceeding recursively along the strip. Since for sufficiently large values of L (we use typically $L\sim 10^8$) the localization length is presumably self-averaging, there is no need for additional averaging over different realizations. We use open boundaries along the action of the transfer matrix and periodic boundary conditions in the perpendicular direction.

Λ_M either increases (delocalized states) or decreases (localized states) with the width M, depending on the system parameters (*e.g.* the inverse temperature β). There can also be a marginal behavior (*e.g.* for a special value β_c'), where Λ_M does not change with M. The latter indicates the existence of a phase transition from localized to delocalized states. A quantitative description of the behavior near β_c' can be based on the one-parameter scaling hypothesis [6,19]. This states that $\ln\Lambda_M$ can be expanded in a vicinity of the critical point β_c' as [18]

$$\ln\Lambda_M = \ln\Lambda_c \pm A|\beta-\beta_c'|M^{1/\nu}. \quad (9)$$

For $A>0$ the positive (negative) sign corresponds to delocalized (localized) behavior. The exponentiation of this equation and using $\zeta = |\beta-\beta_c'|^{-\nu}$ give

$$\Lambda_M = \Lambda_c\exp\left[\pm A\left(\frac{\zeta}{M}\right)^{-1/\nu}\right] \equiv g\left(\frac{\zeta}{M}\right), \quad (10)$$

where g is the scaling function. Our numerical transfer-matrix approach allows us to determine the critical point β_c' and the exponent ν, depending on the interspecies coupling parameter U.

Results. – First, we analyze a one-dimensional system of length M. In this case heavy atoms are always disordered due to thermal fluctuations. The reduced localization length Λ_M decreases with increasing length of the system (cf. fig. 1) at any temperature. This reflects that all states are localized. On the other hand, the localization length decreases monotonically with temperature,

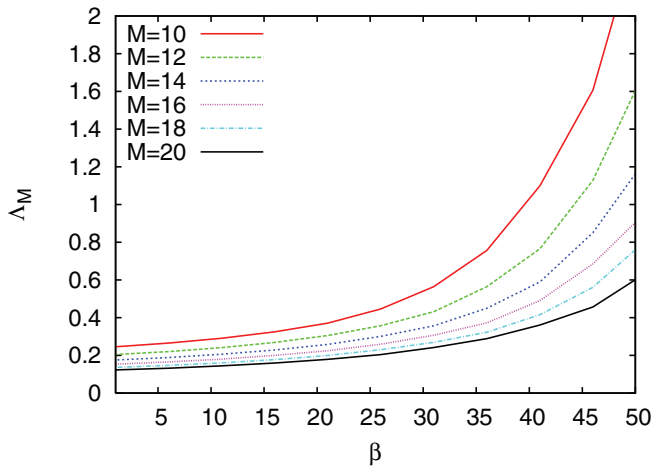

Fig. 1: The reduced localization length Λ_M of light atoms in $d=1$ as a function of the inverse temperature for interaction strength $U=9$. Λ_M decreases for increasing system size M, which indicates localized states.

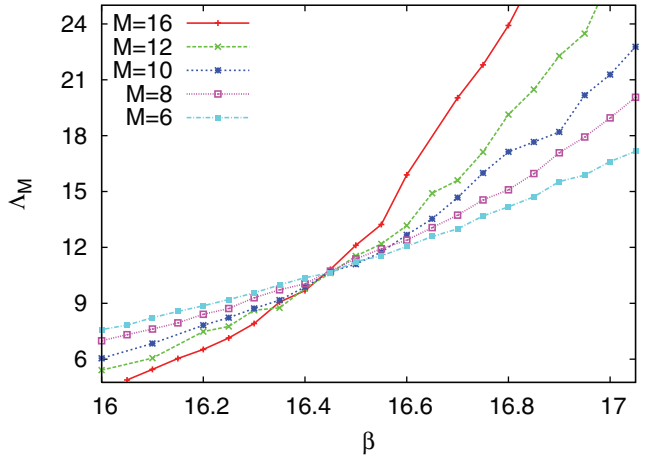

Fig. 2: Reduced localization length of light atoms for $d=2$, $U=9$, and system length $L=10^8$. There is a critical inverse temperature $\beta_c' \approx 16.5$, where an Anderson transition occurs. A finite-size effect appears for $M \leqslant 4$ (not shown here), where the curves do not cross in the same point. For $M \geqslant 6$, however, the crossing point is remarkably robust with respect to size changes.

as a consequence of the increasing disorder. Therefore, at sufficiently low temperature the localization length can be larger than the size of a finite system. This could be relevant in experiments, where we have a finite optical lattice.

In two dimensions the behavior is more complex. First of all, the heavy atoms can form an ordered state at low temperatures and a disordered state at high temperatures [9,10]. As long as $T>0$, thermal excitations in the ordered state lead to correlated fluctuations of heavy atoms. There is a second-order phase (Ising) transition with a divergent correlation length at the critical temperature T_c. The corresponding distribution of heavy atoms provides a complex random environment for the light atoms. Our numerical transfer-matrix approach finds a transition from localized states at high temperatures to delocalized states at low temperatures, indicated by a qualitative change of the scaling behavior (cf. fig. 2). There is a critical temperature T_c', where this transition takes place. For instance, at low temperatures and half-filling (*i.e.* for $\mu_f=\mu_c=U/2$), the heavy atoms are arranged in a staggered configuration with weak thermal fluctuations. Using the approximated distribution of eq. (5), the effective spin-spin coupling $\bar{t}_c^2/2U$ leads to the critical temperature $T_c \propto \bar{t}_c^2/2U$. The result for the reduced localization length at $U=9$ (measured in units of $\bar{t}_c$) is shown in fig. 2. All curves cross at $\beta_c' \approx 16.5$, indicating a localization transition. With these parameters the Ising transition is at $\beta_c \approx 15.9$. Therefore, the localization transition occurs in the ordered phase of the heavy atoms. The one-parameter scaling function of eq. (10) with

$$\Lambda_c \approx 10.9, \qquad A \approx 0.09, \qquad \nu \approx 0.88 \tag{11}$$

fits the data of the transfer-matrix calculation (cf. fig. 3).

In conclusion, we have discussed a mixture of two fermionic species with different masses in an optical lattice,

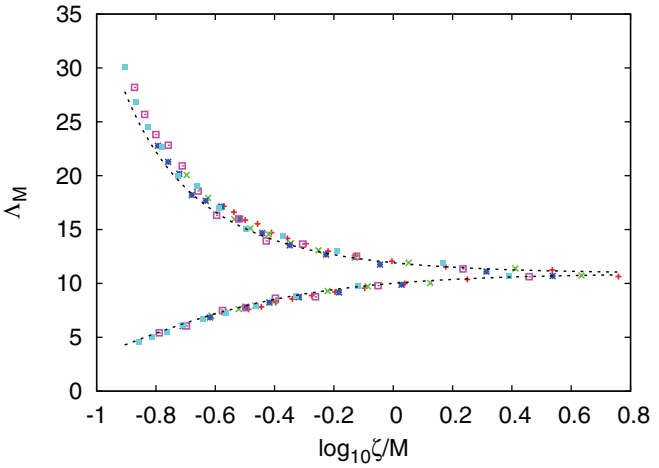

Fig. 3: One-parameter scaling for $d=2$ and $U=9$. The lower (upper) branch represents (de-) localized states. The data from the transfer-matrix calculation approach the scaling function of eq. (10) with $\Lambda_c \approx 10.9$, $A \approx 0.09$ and $\nu \approx 0.88$ (dashed curves).

using the Falicov-Kimball model. The heavy atoms are represented as Ising spins and the light atoms as quantum particles. The latter tunnel in a random environment which is provided by a correlated distribution of heavy atoms. The distribution of the heavy atoms is given by an Ising-type model, which undergoes a second-order phase transition in $d=2$ from staggered order to disorder. Depending on the dimensionality ($d=1,2$) of the atomic system and the physical parameters (*e.g.* temperature or interaction strength), the quantum states of the light atoms are either localized or delocalized. All states of light atoms in a one-dimensional fermionic mixture are localized. In a two-dimensional mixture these states are

localized at high temperatures and delocalized at low temperatures. Such a system can be realized experimentally as a mixture with two spin-polarized fermionic species.

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