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Quantum mechanics and geodesic deviation in the brane world

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Abstract – We investigate the induced geodesic deviation equations in the brane world models, in which all the matter forces except gravity are confined on the 3-brane. Also, the Newtonian limit of induced geodesic deviation equation is studied. We show that in the first Randall-Sundrum model the Bohr–Sommerfeld quantization rule is as a result of consistency between the geodesic and geodesic deviation equations. This indicates that the path of test particle is made up of integral multiples of a fundamental Compton-type unit of length h/mc.

The mission to formulate a consistent quantum theory of gravity has maintained physicists busy since the first attempt by Rosenfeld in 1930. In spite of much work, no definitive progress has been made. Nowadays, there are many interesting attempts to quantize gravity. In this paper we take an opposite direction: we will show that quantum objects can be constructed from gravitational-geometrical effects. Actually, the idea of geometrization of quantum mechanics has been considered in different approaches. For example, one can increase the number of dimensions of spacetime in Kaluza-Klein (KK) models of gravity [1], Weylian spacetime [2], scalar-tensor theories of gravity [3] or other possible extensions of Einstein general relativity. Recently, it has been shown that the existence of non-compact extra dimensions leads to quantum effects in the classically induced 4-dimensional (4D) physical entities [4]. In [5], to construct semi-classical quantum gravity from geometric properties of brane, the authors have used the Induced Matter Theory (IMT) which is an extension of the KK theory. In this approach, not only the gauge fields are unified with gravity (geometry) but also the matter fields are unified with gravity and have geometrical origin, constructed from extrinsic curvature [6]. The origin of quantum effects in fact is the fluctuation of matter fields around 4D spacetime.

In this paper we discuss the existence of quantum effects in the most famous model of brane gravity. In this model and its extensions, the presence of non-compact extra dimension is not in fact for the unification shame, but for the explanation of hierarchy problem without using supersymmetry [7].

The idea that our familiar 4D spacetime is a hypersurface (brane) embedded in a 5D bulk has been experiencing a phenomenal interest during the last decade. The behavior of geodesics and the Newtonian limit of linearized gravity for the Randall-Sundrum (RS) and an alternative brane background have been investigated extensively [8]. Also, ref. [9] has looked into the geodesic motions of a test particle in the bulk spacetime in RS scenario. The induced 4D geodesic equation on the brane, to which we assume that the matter fields except gravity is confined, is given by [10]

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}=0, \tag{1}$$

where τ is the proper time defined on the brane and $\Gamma^\mu_{\alpha\beta}$ are 4D Christoffel symbols derived from the induced metric. (Here and throughout we shall use $A,B=0,1,2,3,5$ to denote 5D coordinates, $\mu,\nu=0,1,2,3$ to denote the standard 4D ones and $\bar{A}=1,2,\ldots,5$, $\bar{\mu}=1,2,3$ denotes spacelike counterparts).

Note that in eq. (1) the effect of the existence of bulk space is hidden in the induced metric which one can obtain via induced Einstein field equations [11]. To obtain the induced geodesic (1), we usually start from the geodesic equation of a test particle in the bulk space and then reduce it to the 4D hypersurface. One can use the same procedure to acquire induced geodesic deviation (GD) equation. For example in the Kaluza-Klein theory authors

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of [12] used the same method to obtain GD on this kind of compact models. Hence, we start with the GD equation associated to the bulk space, namely

$$\frac{^{(5)}\mathrm{D}^2\xi^A}{\mathrm{D}S^2}=\mathcal{R}^A{}_{BCD}\frac{\mathrm{d}x^B}{\mathrm{d}S}\frac{\mathrm{d}x^C}{\mathrm{d}S}\xi^D, \tag{2}$$

where $\mathcal{R}^A{}_{BCD}$ is the Reimann tensor for the bulk space, ξ^A is an infinitesimal GD vector, $\mathrm{D}/\mathrm{D}S$ denotes the pull-back of covariant derivatives and S is an affine parameter for the bulk space. To induce eq. (2) on the brane we need induced components of the Reimann tensor of the bulk space on the brane, *i.e.* Gauss-Codazzi equations. In the Gaussian normal frame, explicit calculation directly gives

$$\mathcal{R}^\mu{}_{\alpha\beta\gamma}=R^\mu{}_{\alpha\beta\gamma}+K_{\alpha\beta}K^\mu{}_\gamma-K_{\alpha\gamma}K^\mu{}_\beta, \tag{3}$$

and

$$\mathcal{R}^\mu{}_{4\alpha4}=K^\mu{}_{\alpha,4}-K^\sigma{}_\alpha K_\sigma{}^\mu, \tag{4}$$

where $R^\mu{}_{\alpha\beta\gamma}$ is the 4D Reimann tensor and $K_{\mu\nu}$ denotes the extrinsic curvature. Inserting eqs. (3) and (4) into the eq. (2) gives

$$\begin{aligned}\frac{\mathrm{D}^2\xi^\mu}{\mathrm{D}S^2} =& \left(R^\mu{}_{\alpha\beta\gamma}+K_{\alpha\beta}K^\mu{}_\gamma-K_{\alpha\gamma}K^\mu{}_\beta\right)\frac{\mathrm{d}x^\alpha}{\mathrm{d}S}\frac{\mathrm{d}x^\beta}{\mathrm{d}S}\xi^\gamma\\ &+\epsilon\left(K^\mu{}_{\alpha,4}-K_{\alpha\sigma}K^{\sigma\mu}\right)\left[\frac{\mathrm{d}x^4}{\mathrm{d}S}\frac{\mathrm{d}x^\alpha}{\mathrm{d}S}\xi^4-\left(\frac{\mathrm{d}x^4}{\mathrm{d}S}\right)^2\xi^\alpha\right].\end{aligned} \tag{5}$$

Now the derivatives with respect to the 5D line element $\mathrm{d}S$, should be replaced by derivatives with respect to the 4D Affine parameter. To this end, we rewrite eq. (5) with a general parameter λ, which parameterizes 4D motion as

$$\frac{^{(5)}\mathrm{D}^2\xi^\mu}{\mathrm{D}S^2}=\left(\frac{\mathrm{d}\lambda}{\mathrm{d}S}\right)^2\frac{^{(5)}\mathrm{D}^2\xi^\mu}{\mathrm{D}\lambda^2}+\frac{\mathrm{d}\lambda}{\mathrm{d}S}\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}S}\right)\frac{^{(5)}\mathrm{D}\xi^\mu}{\mathrm{D}\lambda}, \tag{6}$$

where the relation between 5- and 4–dimensional covariant differentiations is given by

$$\begin{aligned}\frac{^{(5)}\mathrm{D}\xi^\mu}{\mathrm{D}\lambda}=&\frac{\mathrm{d}\xi^\mu}{\mathrm{d}\lambda}+{}^{(5)}\Gamma^\mu_{AB}\frac{\mathrm{d}x^A}{\mathrm{d}\lambda}\xi^B=\frac{\mathrm{D}\xi^\mu}{\mathrm{D}\lambda}\\ &-\epsilon K^\mu{}_\alpha\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\xi^4-\epsilon K^\mu{}_\alpha\frac{\mathrm{d}x^4}{\mathrm{d}\lambda}\xi^\alpha,\end{aligned} \tag{7}$$

so that in the second equality, 5D Christoffel symbols have been replaced by their 4D counterparts using the relations obtained in ref. [10]. Now, from eqs. (5), (6) and (7) we obtain

$$\begin{aligned}\frac{\mathrm{D}^2\xi^\mu}{\mathrm{D}\lambda^2}=&R^\mu{}_{\alpha\beta\gamma}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda}\xi^\gamma\\ &+\left(K_{\alpha\beta}K^\mu{}_\beta-K_{\alpha\gamma}K^\mu{}_\beta\right)\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda}\xi^\gamma\\ &+\epsilon\left(K^\mu{}_{\alpha,4}-K^\rho{}_\alpha K^\mu{}_\rho\right)\left[\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^4}{\mathrm{d}\lambda}\xi^4-\left(\frac{\mathrm{d}x^4}{\mathrm{d}\lambda}\right)^2\xi^\alpha\right]\\ &-\left[\frac{\mathrm{D}\xi^\mu}{\mathrm{D}\lambda}-\epsilon K^\mu{}_\alpha\left(\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\xi^4+\frac{\mathrm{d}x^4}{\mathrm{d}\lambda}\xi^\alpha\right)\right]\left(\frac{\mathrm{d}\lambda}{\mathrm{d}S}\right)^{-1}\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}S}\right).\end{aligned} \tag{8}$$

The above-induced GD equation can be used in various brane models. For example, in the IMT [4,13], test particles are not in general, confined to the specific fixed brane [14]. In this case, since the extra component of velocity of test particle, $u^4=\mathrm{d}x^4/\mathrm{d}\lambda$, does not vanish, all the extra terms on the right-hand side of eq. (8) will be present. Another important point in the IMT is the choice of λ, the parameterization of the path. Usually, it is assumed that the line element of the brane, which is defined here as the proper time "$\mathrm{d}\tau$", is logical and convenient. However, the non-integrability property of the induced physical quantities on the brane dictates that the parameterization of the path is, in general, diferent from the 4D proper time [14]. On the other hand, in the brane phenomenological models where matter fields are confined to the fixed brane, the 4D proper time defined on the brane is required as a suitable parameterization of the motion. In this paper, we would like to study GD in brane models based on the Horava and Witten theory [15], hence, we will assume that all the matter fields, except gravity, are confined on the fixed brane. Therefore, in eq. (8), $\mathrm{d}\lambda$ will be substituted by $\mathrm{d}\tau$, the proper time defined on the brane. Furthermore, we assume that the velocity of test particles along the extra dimension vanishes. Imposing the above assumptions on eq. (8), we obtain

$$\begin{aligned}\frac{\mathrm{D}^2\xi^\mu}{\mathrm{D}\tau^2} =& R^\mu{}_{\alpha\beta\gamma}u^\alpha u^\beta\xi^\gamma\\ &+\left(K_{\alpha\beta}K^\mu{}_\gamma-K_{\alpha\gamma}K^\mu{}_\beta\right)u^\alpha u^\beta\xi^\gamma,\end{aligned} \tag{9}$$

where $u^\alpha=\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}$ denotes the 4-velocity of test particles defined on the brane.

In general relativity, the Newtonian limit of GD equation leads us to the form of the field equations [8]. Hence we derive and analyze the Newtonian limit of eq. (8). We elaborate on tensor equation (2) in the local rest frame for one of the two test particles A_1 and A_2 with coordinates $x^A(s,\eta)$ and $x^A(s,\eta+\delta\eta)$, respectively. In this frame $\mathcal{G}_{AB}=\eta_{AB}$ and $\mathrm{d}S=\mathrm{d}t$. This means that A_1 promotes its clock to the master clock indicating coordinates time. Also, $^{(5)}\mathrm{D}/\mathrm{D}S=\mathrm{d}/\mathrm{d}t$, $x^A=(t,0,0,0,0)$ and $u^A=(1,0,\dots,0)$. We are left with

$$\frac{\mathrm{d}^2\xi^{\bar{A}}}{\mathrm{d}t^2}=\mathcal{R}^{\bar{A}}{}_{00\bar{B}}\xi^{\bar{B}}\quad(\bar{A}=1,2,3,4). \tag{10}$$

At this point A_1 recalls that according to classical mechanics both he and A_2 move in a stationary gravitational field: $\ddot{\mathbf{r}}_{A_1}=\mathbf{F}(\mathbf{r}_{A_1})$ and $\ddot{\mathbf{r}}_{A_2}=\mathbf{F}(\mathbf{r}_{A_2})$. Setting $\xi^B=\mathbf{r}^B_{A_2}-\mathbf{r}^B_{A_1}$ gives

$$\begin{aligned}\frac{\mathrm{d}^2\xi^{\bar{A}}}{\mathrm{d}t^2} =& F^{\bar{A}}(\mathbf{r}_A+\xi)-F^{\bar{A}}(\mathbf{r}_A)\simeq\\ &F^{\bar{A}}{}_{,\bar{B}}\xi^{\bar{B}}=-\Phi^{,\bar{A}}{}_{,\bar{B}}\xi^{\bar{B}},\end{aligned} \tag{11}$$

where Φ is the gravitational potential in the bulk space. Comparing eqs. (10) and (11) gives

$$\mathcal{R}^{\bar{A}}{}_{00\bar{B}} = -\Phi^{,\bar{A}}{}_{,\bar{B}}. \tag{12}$$

Now, using this equation and recalling eqs. (3) and (4) we find

$$\begin{aligned} &R^{\bar{\mu}}{}_{00\bar{\mu}} + K_{00}K - K_{0\bar{\mu}}K^{\bar{\mu}}{}_0 - K_{,5} \\ &-K^{\bar{\mu}}{}_{\bar{\nu}}K_{\bar{\mu}}{}^{\bar{\nu}} = -\Phi^{,\bar{A}}{}_{,\bar{A}} \quad (\bar{\mu}=1,2,3). \end{aligned} \tag{13}$$

The classical field equations in the bulk space is

$$\Phi_{,\bar{A}\bar{A}} = -\Lambda + (-\sigma + k_5^2\rho)\delta(x^5), \tag{14}$$

where according to the spirit of brane models, we have assumed existence of the bulk cosmological constant Λ, tension of the brane σ and the matter density ρ. Consequently, we obtain

$$\begin{aligned} &R^{\bar{\mu}}{}_{00\bar{\mu}} + K_{00}K - K_{0\bar{\mu}}K^{\bar{\mu}}{}_0 - K_{,5} - K^{\bar{\mu}}{}_{\bar{\nu}}K_{\bar{\mu}}{}^{\bar{\nu}} = \\ &-\Lambda + (-\sigma + k_5^2\rho)\delta(x^4). \end{aligned} \tag{15}$$

Integration along normal direction gives the Newtonian limit of the Israel junction condition as

$$[K] = -k_5^2\rho + \sigma, \tag{16}$$

where $[X] := \lim_{x^4\to 0^+} X - \lim_{x^4\to 0^-} X$. Also, if we impose the Z_2 symmetry then we obtain

$$K^+ = \frac{1}{2}(k_5^2\rho - \sigma), \tag{17}$$

which is the Newtonian version of Israel junction condition obtained in [15]. Now, we obtain the GD equation in the RS brane world scenario. In the RS scenario, the 5D bulk space is described by the metric [7,16]

$$\mathrm{d}S^2 = e^{-2k|y|}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}y^2, \tag{18}$$

where $y = r\phi$ signifies the extra spacelike dimension with compactification radius r, $k = \sqrt{-\Lambda/12M^3}$ and Λ is the bulk cosmological constant and M is fundamental 5D Planck scale. The factor $e^{-2k|y|}$ is called warp factor and the geometry of the extra dimension is orbifolded by S^1/Z_2. In the RSI scenario it can be shown that even if Higgs or any other mass parameter in the 5D Lagrangian is of the order of Planck scale, $m_0 \simeq 10^{16}$ TeV, on the visible brane, it gets warped by a factor of the form

$$m = m_0 e^{-kr\pi}. \tag{19}$$

Thus by assuming $kr = 11.84$, one gets $m \simeq 1$ TeV. Using RSI metric (18) we obtain

$$K_{\mu\nu} = k\frac{|y|}{y}e^{-2k|y|}\eta_{\mu\nu}. \tag{20}$$

The constant slices at $y=0$ and $y=r\pi$ are known as the hidden and visible branes respectively, where the observable universe is identified with the latter. Therefore, the GD equation (2) on the visible brane becomes

$$\frac{D^2\xi^\mu}{D\tau^2} = \ddot{\xi}^\mu = k^2e^{-2\pi kr}(\eta_{\alpha\beta}\eta^\mu{}_\gamma - \eta_{\alpha\gamma}\eta^\mu{}_\beta)u^\alpha u^\beta\xi^\gamma, \tag{21}$$

where a dot denotes derivative with respect to the brane proper time. On the other hand, solving the geodesic equation (1) on this brane model gives the constant 4-velocity of test particles as $u^\mu = \text{const}$, which shows that the initially parallel geodesics will always remain parallel as a property of 4D Minkowski spacetime. The solution of equation (21) for massive test particles is

$$\xi^\mu = f^\mu e^{ike^{-\pi kr}\tau}, \tag{22}$$

where f^μ is the integration constant. Equation (22) implies that the distance between two geodesics oscillate contrary to the geodesic equation. The consistency of this solution with geodesic equation then impose the following restriction

$$cke^{-\pi kr}\tau = n\pi, \qquad n = 0,1,2,\ldots, \tag{23}$$

where c is the speed of light which is not considered, here, to be unity. Also it is well known that

$$\int p_\mu \mathrm{d}x^\mu = \int m u_\mu \mathrm{d}x^\mu = \int m\left(\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)^2\mathrm{d}\tau = mc^2\tau, \tag{24}$$

where p_μ is the induced 4-momentum of test particle and m is the rest mass. Comparing eqs. (23) and (24) gives

$$\int p_\mu \mathrm{d}x^\mu = n\pi\frac{mce^{\pi kr}}{k}. \tag{25}$$

Replacing m from eq. (19) with the above equation, and setting $k \sim 1/l_{Pl}$, eq. (25) reduces to

$$\int p_\mu \mathrm{d}x^\mu = nh, \tag{26}$$

which is similar to the old quantum theory quantization condition but is less stringent, for the old quantum conditions involve the integration being taken for a closed curve. We also note that condition (26) involves integration overtime component as well. On the other hand, eq. (23) leads to

$$\tau = n\frac{h}{mc^2}, \tag{27}$$

indicating that the proper time of test particle is made up of integral multiples of a fundamental Compton-type unit of length h/mc. This result suggests that the world-line of test particle is to be considered as have made up of such units of length, nothing smaller being observable directly or indirectly in experiments. Note that according to [17] it could be concluded from (27) that the smallest interval of time and distance are given by

$$\begin{aligned}\delta t &= \frac{h}{mc^2}\frac{1}{\sqrt{1-\beta^2}},\\ \delta l &= \frac{h}{mc}\frac{\beta}{\sqrt{1-\beta^2}},\end{aligned} \tag{28}$$

where $\beta = \frac{v}{c}$ and the following uncertainty relations:

$$\Delta p_\mu \Delta x_\mu \sim \frac{2h}{n-1}. \tag{29}$$

Note that the last relations in (28) are dependent upon the velocity of test particle. For velocities approaching the velocity of light they become very large which means that it is impossible to measure intervals of time and length in association with such rapidly moving particles. Hence it seems that the deduction from the existence of a fundamental proper time is that any accurate measurements on a particle moving with such velocity would be impossible. Also in eq. (29) the worst case is for $n=1$, but this has no practical significance for it corresponds to an observation of one fundamental unit of length which is recorded as corresponding to zero proper time. In this uncertainty relation for a large value of n, the right-hand side of (29) vanishes, *i.e.* this equation naturally contains classical limit. Since the minimum length and time intervals that can be measured are given by (28) then the maximum uncertainly on 3-momentum and energy becomes

$$\begin{aligned}\delta p &\sim \frac{2mc}{n-1}\frac{\sqrt{1-\beta^2}}{\beta},\\ \delta E &\sim \frac{2mc^2}{n-1}\sqrt{1-\beta^2}.\end{aligned} \tag{30}$$

The conclusion is that the above uncertainties vanish when the velocity of test particles reach the velocity of light, while the corresponding uncertainty on time and length tends to infinity, but their product remains finite. we have obtained the above uncertainty relations for massive test particles. Note that the existence of minimum spatial and causal structures will also appear where seeking for theories of quantum gravity such as the loop quantum gravity [18] or string theory [19]. The modification of special relativity in which a minimum length, which may be the Planck length, joins the speed of light as an invariant is done in ref. [20]. We now discuss the light quanta or massless particles. In this case we have $u_\mu u^\mu = 0$ and therefore eq. (21) becomes

$$\frac{D^2\xi}{D\tau^2} = -k^2 e^{-2\pi k r} u_\gamma u^\mu \xi^\gamma. \tag{31}$$

If we assume a solution like $\xi^\mu = f^\mu(\tau)$, then by inserting into the above equation and by considering null condition for 4-velocity we obtain $\mathrm{d}^2 f^\mu/\mathrm{d}\tau^2 = 0$ and consequently

$$\xi^\mu = A^\mu \tau + B^\mu, \tag{32}$$

where A^μ and B^μ are constants of integration. This result shows that the extension of the massive test particle case to the photons is not correct. The above solution shows classically propagating massless particles along or perpendicular propagating photons. Note that the case of massless particles can be driven in this approach and the Wesson suggestions [4] cannot lead us to this result. In fact the different behavior of photons proceeds from the confinement of gauge fields on the brane. Also as we know, the concepts of time in general relativity and quantum theory differ intensely from each other. Time in quantum theory is an external parameter, whereas in general relativity time is a dynamical one. Consequently, a consistent theory of quantum gravity should exhibit a new concept of time. In general relativity spacetime is dynamical and therefore there is no absolute time. Spacetime influences material clocks in order to allow them to show proper time. The clocks, in turn, react to the metric and change the geometry [21]. In this sense, the metric itself is a clock. A quantization of the metric can thus be interpreted as a quantization of the concept of time. In this paper we showed that the consistency of geodesic and geodesic deviation equations on the RS brane dictates the quantization of proper time or clock rate. Note that this quantity cannot be dealt with as operators in ordinary quantum theories. The advantage of this model is that it makes general relativity compatible with de Broglie ideas, allows a geometric interpretation of de Broglie waves without any generalization of Riemannian spacetime. Along this direction, the problem needs to be more accurately surveyed.

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