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To cite this article: J. Plata 2009 *EPL* **87** 50001

View the article online for updates and enhancements.

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September 2009

EPL, **87** (2009) 50001
doi: 10.1209/0295-5075/87/50001

www.epljournal.org

Delayed response of a fermion pair condensate to a modulation of the interaction strength

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received 22 January 2009; accepted in final form 19 August 2009
published online 18 September 2009

PACS 05.30.Fk – Fermion systems and electron gas
PACS 03.75.Ss – Degenerate Fermi gases

Abstract – The effect of a sinusoidal modulation of the interaction strength on a fermion pair condensate is analytically studied. The system is described by a generalization of the coupled fermion-boson model that incorporates a time-dependent intermode coupling induced via a magnetic Feshbach resonance. Nontrivial effects are shown to emerge depending on the relative magnitude of the modulation period and the relaxation time of the condensate. Specifically, a nonadiabatic modulation drives the system out of thermal equilibrium: the external field induces a variation of the quasiparticle energies, and, in turn, a disequilibrium of the associated populations. The subsequent relaxation process is studied and an analytical description of the gap dynamics is obtained. Recent experimental findings are explained: the delay observed in the response to the applied field is understood as a temperature effect linked to the condensate relaxation time.

The study of ultracold atomic gases has led to a remarkable series of experimental realizations of fundamental effects [1]. Essential to many of these achievements has been the control of the interaction strength via a Feshbach resonance (FR), which has allowed the emergence of these systems as a practical testing ground for quantum-statistical and many-body physics. Specially relevant has been the realization, with a two-component Fermi gas of atoms, of the crossover from a molecular Bose-Einstein condensate (BEC) to a Bardeen-Cooper-Schrieffer (BCS) superfluid of loosely bound atom pairs [2–11]. By applying a magnetic FR, the interaction is led to change from repulsive in the BEC phase to attractive in the BCS side. Whereas two-body physics supports a bound molecular state in the BEC side, the formation of pairs in the BCS regime occurs only due to many-body effects. Significant advances have been made in the understanding of different aspects of this transition. In particular, the role of thermal fluctuations has been extensively analyzed [12]. Despite those advances, further work on the characterization of nonequilibrium aspects of the crossover is required. Indeed, the extension of the experiments on scattering length variations to unexplored time-dependent regimes and the setup of an expanded theoretical framework where the emergent effects can be understood are current challenges. Here, we aim at explaining recent experimental results on the nontrivial dynamics resulting from a sinusoidal modulation of the interaction strength [13]. This will require a precise characterization of the different time scales, in particular, of the relaxation time of the condensate. The interest of the study is not restricted to the field of ultracold atomic gases. In fact, the expansion of the BCS model proposed in our approach can have broad applicability: the effect of a disequilibrium population on the gap dynamics, which is the central issue in the analysis, is relevant to topics ranging from nonequilibrium superconductivity [14,15] to quench dynamics in superfluid ^{3}He [16].

As starting point we take the experiments reported in ref. [13]. In them, the interaction strength of a gas of ultracold ^{6}Li atoms in the BCS regime was sinusoidally modulated through a magnetic FR. (The (broad) FR at 834G between the two lowest hyperfine states was used.) The system response, which consisted in a damped oscillation of the condensate fraction with the modulation frequency, was found to be delayed with respect to the applied field. The delay showed no appreciable changes at different cycles of the external field; moreover, it presented the same scale for widely different frequencies. Additionally, the damping time was observed to be much longer than the driving period. In a preliminary analysis, the deferred response was conjectured to be rooted in the finite relaxation time of the condensate; furthermore, the

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decay of the oscillation amplitude was linked to heating resulting from the nonadiabaticity (on the gap time scale) of the process. (In the present paper, "adiabatic" will also be applied to a modulation much slower than the condensate relaxation.) To evaluate those conjectures, the measured characteristic times were compared with related theoretical predictions. However, that analysis was not conclusive about the origin of the recorded behavior because of the limitations of the available models, specially, of the lack of an appropriate description of finite-temperature effects. Our objective is to provide a theoretical framework where the observed features can be understood, and, in particular, the previous conjectures can be assessed. To this end, we concentrate on conditions which allow an analytical description of the dynamics, and, consequently, a clear identification of the dominant mechanisms.

We consider a gas of ultracold Fermi atoms with two hyperfine states coupled to a molecular two-particle state via a magnetic FR. Our methodology to deal with magnetic-field modulations combines three main elements. First, the standard framework, namely, the coupled fermion-boson model [17–20], is expanded by incorporating a time-dependent intermode coupling. Second, a perturbative scheme, valid for a sufficiently small modulation amplitude, is introduced in the Hartree-Fock-Bogoliubov (HFB) description. In this approach, finite-temperature effects are tackled and the out-of-equilibrium situation induced by the modulation is characterized. Finally, a method for describing the evolution of the quasiparticle populations, and, subsequently, the gap dynamics is presented. Accordingly, we start from the unmodulated system: the grand-canonical Hamiltonian reads

$$\begin{aligned} H-\mu N =& \sum_{\mathbf{k},\sigma}\varepsilon_{\mathbf{k}} a^\dagger_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma} + V_{int} \\ &\times \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} a^\dagger_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow} a^\dagger_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow} a_{\frac{\mathbf{q}}{2}-\mathbf{k}',\downarrow} a_{\frac{\mathbf{q}}{2}+\mathbf{k}',\uparrow} \\ &+\sum_{\mathbf{q}} \left(\varepsilon^m_{\mathbf{q}} + \hbar\nu_0\right) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} \\ &+ g\sum_{\mathbf{q},\mathbf{k}} \left(b_{\mathbf{q}} a^\dagger_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow} a^\dagger_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow} + \text{h.c.}\right), \end{aligned} \quad (1)$$

where μ is the chemical potential, N is the total number of bare Fermi atoms, $a^\dagger_{\mathbf{k},\sigma}$ ($a_{\mathbf{k},\sigma}$) denotes a fermionic creation (annihilation) operator of an atom with momentum $\mathbf{k}$ and spin σ, ($\sigma \in \{\uparrow,\downarrow\}$), and $b^\dagger_{\mathbf{q}}$ ($b_{\mathbf{q}}$) is a bosonic operator that creates (destroys) a molecule with momentum $\mathbf{q}$. The populations corresponding to the two hyperfine states are assumed to be equal. The free dispersion relations for fermions and bosons are $\varepsilon_{\mathbf{k}} = \hbar^2\mathbf{k}^2/2m - \mu$ and $\varepsilon^m_{\mathbf{q}} = \hbar^2 q^2/4m - 2\mu$, respectively. The binary attractive interaction potential between fermions is characterized by $V_{int}(<0)$. Additionally, g represents the FR coupling between the closed- and the open-channel states, ν_0 being the detuning of the boson resonance state from the collision continuum.

Initially, the system is at equilibrium at a finite temperature T. In that situation, a sinusoidal modulation of the detuning from the FR is applied. Correspondingly, ν_0 is replaced by $\nu(t) = \nu_0 + A\sin\omega_p t$. It is assumed that V_{int}, which characterizes the attractive pairing interaction resulting from nonresonant processes, is not affected by the applied detuning of the FR. Through the unitary transformation $U(t) = e^{i\frac{A}{\omega_p}\cos\omega_p t \sum_{\mathbf{q}} b^\dagger_{\mathbf{q}} b_{\mathbf{q}}}$, the Hamiltonian is transformed into $H' = U^\dagger H U - i\hbar U^\dagger \dot{U}$; consequently, eq. (1), (with ν_0 replaced by $\nu(t)$), is rewritten as

$$\begin{aligned} H'-\mu =& \sum_{\mathbf{k},\sigma}\varepsilon_{\mathbf{k}} a^\dagger_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma} + V_{int} \\ &\times \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} a^\dagger_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow} a^\dagger_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow} a_{\frac{\mathbf{q}}{2}-\mathbf{k}',\downarrow} a_{\frac{\mathbf{q}}{2}+\mathbf{k}',\uparrow} \\ &+\sum_{\mathbf{q}} (\varepsilon^m_{\mathbf{q}} + \hbar\nu_0) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} \\ &+ \left(g\sum_{\mathbf{q},\mathbf{k}} e^{i\frac{A}{\omega_p}\cos\omega_p t} b_{\mathbf{q}} a^\dagger_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow} a^\dagger_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow} + \text{h.c.}\right). \end{aligned} \quad (2)$$

Our procedure to analyze the dynamics starts, like the standard HFB approach [18,20], with the introduction of three mean fields: $n \equiv \sum_{\mathbf{k}} \langle a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}\rangle$ for the spin density, $\Delta \equiv |V_{int}| \sum_{\mathbf{k}} \langle a_{-\mathbf{k},\downarrow} a_{\mathbf{k},\uparrow}\rangle$ for the pairing field, and $\phi_m \equiv \langle b_{\mathbf{q}=\mathbf{0}}\rangle$ for the boson field. (We take $\mathbf{q}=\mathbf{0}$ as we focus on the condensed molecular field.) Next, a perturbative scheme is set up as follows. From eq. (2), it is apparent that the effect of the magnetic modulation can be understood as a time variation of the intermode coupling strength: we can work with the effective strength $g_{eff} \equiv g e^{i\frac{A}{\omega_p}\cos\omega_p t} = g + \delta g(t)$, where $\delta g(t)$ stands for the modulation-induced increment. (Note that the time dependence of g_{eff} prevents the effective one-channel reduction applicable, for a broad FR, to the undriven coupled fermion-boson model.) Furthermore, from the expansion $e^{i\frac{A}{\omega_p}\cos\omega_p t} = \sum_{l=-\infty}^{\infty} i^l J_l(A/\omega_p) e^{il\omega_p t}$, and taking into account the properties of the Bessel functions, it follows that, for $A/\omega_p \ll 1$, we can make the approximation $\delta g \simeq i2gJ_1(A/\omega_p)\cos\omega_p t$, the magnitude of δg being much smaller than that of g. (Higher-order terms will be discussed later on.) In turn, the previously defined mean fields can be expressed as $n = n_0 + \delta n$, $\Delta = \Delta_0 + \delta\Delta$, and $\phi_m = \phi_{m,0} + \delta\phi_m$, where n_0, Δ_0, and $\phi_{m,0}$ are the respective values in the absence of the magnetic variation, and, δn, $\delta\Delta$, and $\delta\phi_m$ are the corresponding modulation-induced increments. To first order, the complete Hamiltonian can be split as $H' - \mu N \simeq H_0 + H_{per}$. The zero-order term, which describes the unmodulated system, is given by $H_0 = \sum_{\mathbf{k},\sigma} V_k a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} - \sum_{\mathbf{k}} (\tilde{\Delta}_0 a^\dagger_{\mathbf{k},\uparrow} a^\dagger_{-\mathbf{k},\downarrow} + \text{h.c.})$ and corresponds to an effective BCS model with mode energy $V_k \equiv \varepsilon_{\mathbf{k}} - \mu + V_{int} n_0$ and gap $\tilde{\Delta}_0 \equiv \Delta_0 - g\phi_{m,0}$. The first-order correction reads $H_{per} = -\delta\tilde{\Delta}(t)\sum_{\mathbf{k}} a^\dagger_{\mathbf{k},\uparrow} a^\dagger_{-\mathbf{k},\downarrow} + \text{h.c.}$, where $\delta\tilde{\Delta}(t) = \delta\Delta - g\delta\phi_m - \delta g\phi_{m,0}$ is the increment of the generalized order parameter $\tilde{\Delta} \equiv \Delta - g\phi_m$. (We have neglected the variation in the atomic density, *i.e.*, we have taken $\delta n \simeq 0$, which is justified for a broad FR [21]. The generalization required to deal with a narrow FR is straightforward.) To complete the mean-field description, we must add the equation for the evolution of the boson mode, namely, $i\hbar\frac{\mathrm{d}(\phi_{m,0}+\delta\phi_m)}{\mathrm{d}t} = (\nu_0 - 2\mu)(\phi_{m,0} + \delta\phi_m) + \frac{g_{eff}}{|V_{int}|}(\Delta_0 + \delta\Delta)$, which, through the application of our perturbative approach and taking into account that the equilibrium molecular field is given by $\phi_{m,0} = \frac{g\Delta_0}{|V_{int}|(2\mu-\nu_0)}$, is converted into

$$i\hbar\frac{\mathrm{d}\delta\phi_m}{\mathrm{d}t} = (\nu_0 - 2\mu)\delta\phi_m + \frac{g}{|V_{int}|}\delta\Delta + \frac{\delta g}{|V_{int}|}\Delta_0. \quad (3)$$

Here, the presence of the driving term $\frac{\delta g}{|V_{int}|}\Delta_0$ points to the oscillation of $\delta\phi_m$ with frequency ω_p.

H_0 is standardly diagonalized through a Bogoliubov transformation (BT) characterized by the fermionic operators $c_{\mathbf{k},\uparrow} = \cos\theta_k a_{\mathbf{k},\uparrow} - \sin\theta_k a^\dagger_{-\mathbf{k},\downarrow}$ and $c^\dagger_{-\mathbf{k},\downarrow} = \sin\theta_k a_{\mathbf{k},\uparrow} + \cos\theta_k a^\dagger_{-\mathbf{k},\downarrow}$, where θ_k is defined by $\tan(2\theta_k) = |\tilde{\Delta}_0|/V_k$ [18,20]. With this BT, H_0 is cast into $H_0 = \sum_{\mathbf{k}} E_{k,0} \times (c^\dagger_{\mathbf{k},\uparrow}c_{\mathbf{k},\uparrow} + c^\dagger_{\mathbf{k},\downarrow}c_{\mathbf{k},\downarrow}) + \text{constant}$. Importantly, $c^\dagger_{\mathbf{k},\uparrow}$ ($c_{\mathbf{k},\uparrow}$) corresponds to the creation (annihilation) operator of a quasi-particle excitation with momentum $\mathbf{k}$ and spin $\uparrow$ from the BCS state, which acts as an effective vacuum state. The associated excitation energies are $E_{k,0} = \sqrt{V_k^2 + \tilde{\Delta}_0^2}$. The excitation gap $\tilde{\Delta}_0$ combines the mean field Δ_0, obtained from the BCS equation $\Delta_0 = \frac{|V_{int}|}{2}\sum_{\mathbf{k}}(2f_k - 1)\sin 2\theta_k$, with the equilibrium molecular field $\phi_{m,0}$, which results from the equation for the boson mode in the absence of driving. As thermal equilibrium is assumed for the system before the application of the magnetic modulation, the populations of the quasiparticle states $\{f_k\}$ are initially given by the Fermi distribution function $f^{eq}_{k,0} = 1/(1 + e^{E_{k,0}/k_BT})$.

Through the previously defined BT, the perturbation Hamiltonian is converted into

$$\begin{aligned} H_{per} = & \sum_{\mathbf{k}} \delta E_k(t)\left(c^\dagger_{\mathbf{k},\uparrow}c_{\mathbf{k},\uparrow} + c^\dagger_{\mathbf{k},\downarrow}c_{\mathbf{k},\downarrow}\right) \\ & + \left(G_k(t)c^\dagger_{\mathbf{k},\uparrow}c^\dagger_{-\mathbf{k},\downarrow} + \text{h.c.}\right), \end{aligned} \quad (4)$$

where $\delta E_k(t) \equiv \frac{1}{2}\delta\tilde{\Delta}(t)\sin 2\theta_k + \text{c.c.}$, and $G_k(t) \equiv -\delta\tilde{\Delta}(t) \times \cos^2\theta_k + \delta\tilde{\Delta}^*(t)\sin^2\theta_k$. From the form of H_{per}, a first picture of the dynamical implications of the field modulation can be drawn. The (time-dependent) diagonal terms lead to a time variation of the quasiparticle energies, which become $E_k(t) = E_{k,0} + \delta E_k(t)$. The non-diagonal terms represent modulation-induced interactions between the vacuum state and a doubly-excited state. Importantly, these coupling terms, which oscillate with the external frequency ω_p, are relevant only when they can induce an effective resonance between the BCS state and the two-excitation configuration, *i.e.*, only when $\omega_p \geqslant 2\tilde{\Delta}_0$, ($\hbar = 1$). Here, in order to isolate the mechanisms responsible for the delayed response, we concentrate first on the regime defined by $\omega_p < 2\tilde{\Delta}_0$. In this frequency range, the interaction terms can be discarded, and the perturbation Hamiltonian can be approximated as $H_{per} = \sum_{\mathbf{k}} \delta E_k(c^\dagger_{\mathbf{k},\uparrow}c_{\mathbf{k},\uparrow} + c^\dagger_{\mathbf{k},\downarrow}c_{\mathbf{k},\downarrow})$. Hence, the quasiparticle states of the unmodulated system still provide a diagonal representation of the driven Hamiltonian. The appearance of heating effects outside this regime will be discussed later on.

Now we turn to incorporate finite-temperature effects in the above framework. The modulation drives the system out of equilibrium as the initial populations, *i.e.*, the thermal values associated with the unmodulated energies, do not correspond to the Fermi distribution $f^{eq}_k(t) = 1/(1 + e^{E_k(t)/k_BT})$ for the actual (time-varying) energies. The description of the effect of this quasi-particle disequilibrium on the gap dynamics requires a self-consistent approach since the energies and the gap are interdependent. Indeed, as shown by the expression $E_k(t) = E^{(0)}_k + \left(\frac{\tilde{\Delta}_0}{2E_{k,0}}\delta\tilde{\Delta}(t) + \text{c.c.}\right)$, the quasi-particle energies are affected by the gap evolution and by the molecular-field variation; in turn, the $\{E_k(t)\}$ enter the general gap equation [18,20],

$$\Delta(t) = \frac{|V_{int}|}{2}\sum_{\mathbf{k}}[2f_k(t) - 1]\sin 2\theta_k, \quad (5)$$

via the (changing) associated populations $\{f_k(t)\}$. We will see that it is precisely the evolution of the populations, more specifically, their relaxation towards equilibrium, that gives the keys to understanding the experimental results. An important aspect of this problem can be understood by now: a finite relaxation time τ_f of the $\{f_k\}$ is necessary for the appearance of the gap delay. In fact, for a sudden relaxation, the populations follow adiabatically (on the relaxation time scale) the equilibrium values $\{f^{eq}_k(t)\}$ associated with the time-dependent energies. The evolution corresponds then to a sequence of equilibrium states where time enters as a parameter, the associated gap dynamics being "trivial": no delay between the gap evolution and the external field emerges. Therefore, to reproduce the delayed response, we must go beyond that adiabatic regime. Accordingly, we present a self-contained derivation of the dynamics with no constraints on time scales. The evolution of the populations is assumed to be governed by the equation [14]

$$\frac{\mathrm{d}f_k}{\mathrm{d}t} = -\frac{1}{\tau_f}\left[f_k(t) - f^{eq}_k(t)\right], \quad (6)$$

where $1/\tau_f$ represents the effective thermalization rate. The relaxation mechanism can be conjectured to be rooted in collisions between excited particles. Here, we do not go into details of the dependence of τ_f on the system characteristics; instead, as we focus on general aspects of the role of the population thermalization in the condensate relaxation, we consider a generic τ_f. Here, a comment on the stability of the temperature is in order. One must take into account that the standard trapping conditions allow assuming the stability of the temperature for the considered small variations of the scattering length. In this sense, we recall that a grand-canonical description, which is routinely applied in this context as it can incorporate the possible exchange of particles between the condensate fraction and the thermal cloud, implies that a fixed temperature can be reasonably assumed. Also, it is worth stressing that a parallel treatment of the fluctuations of the condensate field would be necessary to formally complete our description. However, it is shown that, in our perturbative regime, the non-condensate fraction has a second-order effect on the gap dynamics. (For a systematic treatment of different aspects of the role of fluctuations, see ref. [12].)

Equation (6) is exactly solved to give $f_k(t) = f_k^{eq}(t) - \int_{-\infty}^{t} e^{-(t-t')/\tau_f} \frac{\mathrm{d}f_k^{eq}}{\mathrm{d}t}(t')\mathrm{d}t'$. Next, this expression for the populations is introduced into eq. (5) to give the following integral-differential equation for the order parameter:

$$\Delta_0 + \delta\Delta(t) = \frac{|V_{int}|}{2} \sum_{\mathbf{k}} \left[2\left(f_k^{eq}(t) - \int_{-\infty}^{t} e^{-(t-t')/\tau_f} \frac{\mathrm{d}f_k^{eq}}{\mathrm{d}t}(t')\mathrm{d}t' \right) - 1 \right] \sin 2\theta_k. \tag{7}$$

Here, one must take into account that $f_k^{eq}(t)$ contains $\delta\Delta(t)$ and $\delta\phi_m$, and $\frac{\mathrm{d}f_k^{eq}}{\mathrm{d}t}$ contains $\delta\dot{\Delta}$ and $\delta\dot{\phi}_m$. Hence, we have obtained a description of the gap evolution, albeit in implicit form. Given the complexity of this picture, the problem of identifying the origin of the deferred response, reduced at this point of the study to that of uncovering the connection between the delay time and τ_f, is still nontrivial. However, it simplifies considerably in the following regime, where an explicit characterization of the gap evolution is feasible. Specifically, for $E_k \sim \Delta \ll T \approx T_c$, $(k_B = 1)$, where T_c is the temperature for the BCS transition, we can make the approximations $f_k^{eq}(t) \simeq f_{k,0}^{eq} + \frac{\mathrm{d}f_k^{eq}}{\mathrm{d}E_k}\delta E_k(t)$ and $\frac{\mathrm{d}f_k^{eq}}{\mathrm{d}E_k} \simeq -\frac{1}{4T_c}$. Then, with the expression for the unperturbed gap $\Delta_0 = \frac{|V_{int}|}{2}\sum_{\mathbf{k}}[2f_{k,0}^{eq} - 1]\sin 2\theta_k$ and the approximation $\frac{\mathrm{d}f_k^{eq}}{\mathrm{d}t} \simeq -\frac{1}{4T_c}\frac{\mathrm{d}E_k}{\mathrm{d}t} = -\frac{1}{4T_c}\sin 2\theta_k(\delta\dot{\Delta} - g\delta\dot{\phi}_m)$, eq. (7) is cast into

$$\delta\Delta = -\frac{|V_{int}|}{8T_c} \sum_{\mathbf{k}} \sin^2 2\theta_k \times \left[(\delta\Delta - g\delta\phi_m) - \int_{-\infty}^{t} e^{-(t-t')/\tau_f} (\delta\dot{\Delta} - g\delta\dot{\phi}_m)\mathrm{d}t' \right]. \tag{8}$$

Now, following a standard procedure, this integral-differential equation is converted into the differential equation

$$\frac{\mathrm{d}\delta\Delta}{\mathrm{d}t} = -\frac{1-\chi}{\tau_f}\delta\Delta - \frac{\chi}{\tau_f} g\delta\phi_m, \tag{9}$$

where $\chi \equiv \frac{|V_{int}|}{2\pi^2}\frac{1}{4T_c}\int_0^K \sin^2 2\theta_k k^2 \mathrm{d}k$ encapsulates the overall effect of the quasi-particle states on the gap response. (K is the upper limit of the momentum summation required by the standard renormalization procedure.) In the considered regime, $\chi \simeq \frac{k_F|a|}{2}\frac{\Delta_0}{T_c} \ll 1$ [22], where k_F is the Fermi wave number and a is the background scattering length. Equation (9) along with eq. (3) for the molecular field constitute a closed set of equations for the system evolution. From them, it is apparent that $\delta\Delta$ is determined by the combination of effective driving, coming from the term $\frac{\chi}{\tau_f}g\delta\phi_m$, and damping with rate $\frac{1-\chi}{\tau_f}$. As the driving is continuously taking the system out of equilibrium, the relaxation mechanism is permanently activated. The combined effect of both mechanisms can be expected to produce a nondirect following to the external field. Approximate analytical solutions confirm these predictions: the gap evolution is given by

$$\frac{\Delta(t)}{\Delta_0} = 1 - C[e^{-t/\tau_R}\sin\varphi + \sin(\omega_p t - \varphi)], \tag{10}$$

where $C = 2\chi\frac{g^2}{|V_{int}|}\frac{J_1(A/\omega_p)}{\omega_p\sqrt{1+(\omega_p\tau_f)^2}} (>0)$ determines the amplitude of the induced oscillations, $\varphi = \arctan(\omega_p\tau_f)$ is a phase shift with respect to the applied magnetic field, and $\tau_R = \frac{\tau_f}{1-\chi}$ appears as the condensate relaxation time. (The meaning of τ_R becomes evident in a simplified scenario: for the system with no external driving, a sudden perturbation of the gap is shown to relax to equilibrium with characteristic time τ_R.) In the considered regime, namely, near the critical temperature and for a perturbative gap variation, it is found that $\tau_R \simeq \tau_f$. The correspondence of these results with the experimental findings is summarized in the following points.

i) The system response contains a transitory decay with characteristic time τ_R, and, as observed in the experiments, a secular oscillatory behavior with ω_p. The amplitude, which combines in a nontrivial way parameters of the external field and characteristics of the unperturbed system, reflects the complex character of the driving mechanism. The following to the external field is not instantaneous: there is a delay time associated with the phase shift φ and given by $\tau_D = \frac{\varphi}{\omega_p} = \tau_R\left[1 + \mathcal{O}\left((\omega_p\tau_R)^2\right)\right]$. Hence, as conjectured in ref. [13], τ_D approximately corresponds to the condensate relaxation time. The delay presents no changes at different cycles of the external field. Furthermore, the small magnitude of the correction $\mathcal{O}\left((\omega_p\tau_R)^2\right)$ for the conditions of the experiments explains the detected invariance of the delay scale with the modulation frequency. As reflected by the minus sign before C, there is an extra phase shift π between the gap oscillation and the driving field. This corresponds exactly to the results presented in figs. 2 and 4 in ref. [13], where an inversion of the magnetic-field axis was introduced to facilitate the observation of the delay.

ii) The connection between the different time scales is uncovered. Since τ_R gives the time for the condensate to reach the thermal equilibrium, it is directly related to the thermalization rate of the populations. Notice that τ_R can significantly differ from τ_f outside the considered regime with $\chi \ll 1$. This can be understood taking into account the intricate interdependence of the gap and the populations, which implies, in general, a complex nonlinear contribution of the populations to the gap relaxation [14].

iii) The mechanism responsible for the deferred response is rooted in the finite reaction time of the gas to a variation in the quasiparticle energies. (Note that the adiabatic limit corresponding to a sudden relaxation, *i.e.*, to $\tau_f \to 0$, is consistently recovered in eq. (10).) Moreover, as conjectured in a preliminary analysis, the observed delay is a temperature effect. At zero temperature, there is no initial population of the excited states; furthermore, as the modulation in the assumed regime does not induce a transfer from the fundamental state, the excited states are never populated. Hence, at $T = 0$, there is no population relaxation, and, consequently, no delay in the gap evolution [23].

iv) It is worth discussing the effects that can be expected outside the considered regime of nonexciting frequencies ($\omega_p < 2\tilde{\Delta}_0$) and perturbative amplitudes. First, for $\omega_p \geqslant 2\tilde{\Delta}_0$, the magnetic field can induce an effective resonance between the vacuum state and a doubly-excited state. As a consequence, the interaction between those states, represented by the non-diagonal terms in eq. (4), becomes important, and significant heating can result. (See refs. [24] and [25] for related work.) Second, as the amplitude is increased, the contribution of the terms of order higher than one in the expansion of the exponential in eq. (2) grows. Given that the frequency of each term is a multiple of ω_p, a resonance between the ground state and the excitations can eventually be reached for increasing order, which, again, can lead to an irreversible loss of population.

At this point some aspects of our approach must be recalled. Importantly, a perturbative regime has been considered: the system, which is initially in the BCS side, is assumed to be inside that regime during the whole modulation process. Hence, the unitary limit of large scattering length is never reached and neither is attained the BEC side. Our approach, directly set up from the fundamental theory, has some similarities with former studies in superconductivity where the effect of a population disequilibrium on the gap dynamics was tackled by introducing an operative changing temperature in the static Ginzburg-Landau (GL) equation [14,15]. The effective time-dependent GL equation thus obtained was shown to satisfactorily explain the relaxation process. That description, as ours, is basically built from the incorporation of formal solutions for the evolving populations into the gap equation. In the uniform case and in the perturbative regime for the gap variation, the analogy with our self-contained approach is complete. Incidentally, we stress that the success of our uniform description in reproducing the experimental results, which, in fact, were obtained for a harmonic confinement, reflects the robustness of the identified physical mechanisms against spatial nonuniformities. The generalization through a local-density approximation is straightforward.

In summary, we have presented an analytical explanation for the delayed response of a Fermi condensate to a modulation of the interaction strength. Although a more quantitative comparison with the experiments of ref. [13] requires additional information on the temperature and the amplitude of the applied field, the study uncovers fundamental aspects of the out-of-equilibrium dynamics of the condensate, in particular, the nontrivial role of the relaxation in a time regime of experimental and theoretical interest. Our analysis can have direct practical implications. One of the main motivations for the experiments was the validation of the detection schemes based on a projection of the fermion pair condensate into a molecular condensate. Those schemes are applicable only if the response time of the condensate to variations in the interaction strength is much larger than the sweep time. The relevance of the nonzero-temperature character of the delay to the projection techniques is clear: an out-of-equilibrium situation and the subsequent relaxation process of the condensate can be induced not only by changing the temperature but also by manipulating the system with external fields. Our picture provides the theoretical basis for the design of methods for measuring the different time scales, and, consequently, for defining the appropriate ranges for the projection. Furthermore, the identification of the dominant mechanisms opens the way to develop variations of the basic arrangement as probing tools for different aspects of the dynamics. Given the generality of the applied model, the applicability of the study in parallel contexts can be expected.

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