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Least-action principle in gait

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Abstract – We apply the laws of human gait vertical ground reaction force and discover the existence of the phenomenon of least-action principle in gait. Using a capacitive mat transducer system, we obtain the variations of human gait vertical ground reaction force and establish a structure equation for the resultant of such a force. Defining the deviation of vertical force as an action function, we observe from our gait optimization analysis the least-action principle at half of the stride time. We develop an evaluation index of mechanical-energy consumption based upon the least-action principle in gait. We conclude that these observations can be employed to enhance the accountability of gait evaluation.

Gait analysis explores laws of body movement by biomechanical methods so that it can serve the clinical diagnosis and rehabilitation. Gait parameters refer to physical quantities while walking, for example, the space-time characteristics, kinesiological quantities and kinetic quantities [1,2]. The relative symmetry of gait parameters is a notable feature of normal human natural gait [3,4]. Phase symmetry index was proposed as gait quantization index and mathematical model and it has brought satisfactory result in the study of walking function of hemiplegic patients [5–7]. With the development of gait measurement equipment, more gait evaluation indices of gait parameters have been defined [8–10]. Gait evaluation index of vertical ground reaction force (VGRF) distribution and parameters of peak value based upon data collected from equipment such as force plate or planar pressure measurement system has been employed constantly to analyze gait [9,11–14], which provides foundation for medical rehabilitation. The essence of gait index evaluation is to compare, that is, to take the normal human gait as the standard of rehabilitation evaluation. However, the non-complete symmetry of human shape, coordinate and strength has formed the uniqueness of human gait [15], which has brought difficulty to the establishment of gait parameter index standard. Consequently, exploring a law that is irrelevant to one's age, gender or physical shape in gait has become very important when applying gait to scientifically evaluate the rehabilitation.

Bipedal walking has enabled the continuous evolution of human gait [16], which eventually brought about the optimized gait [17] and formed a human behavioral trait. Nature always minimizes certain important quantities when a physical process takes place [18]. In the continuous evolution of this human behavioral trait, explanations to issues such as how it observes the least-action principle (hereinafter referred to as LAP) and what the action function in gait is remain controversial. In this letter we address the issue of the principles of least action in gait and propose a more reliable and standard gait evaluation index system.

Gait is under the control of the nervous system. Its musculoskeletal system generates resultant force, which acts on the ground by foot, and in turn, the consequent VGRF enables the human body to move [19]. The vertical change of mass center can be analyzed either by the space-time relation of human segments or by VGRF [20].

Following the human gait characteristics, the equation of certain moment's VGRF resultant force[1] is

$$F_{GRF}(t) = F_z^r(t) + F_z^l(t), \qquad (1)$$

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[1] We have assumed that the VGRF of both the left and right foot is the same.

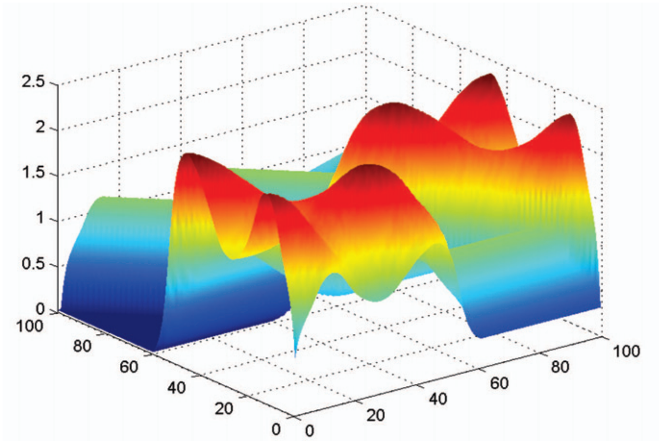

Fig. 1: (Colour on-line) Relationship between the landing sequences of the foot and its variations of VGRF. The variation range of both t and t_0 is defined over the range $[0, T]$.

where $F_z^r(t) = F_z(t)$ and $F_z^l(t) = F_z(t + t_0)$, for $0 \leqslant t \leqslant T$, are the VGRFs of the right and left foot, respectively and T is one foot stride time. Using eq. (1), one can obtain the landing sequences of the foot and its variations of VGRF. Rating the cycle time as a percentage and normalizing the reaction force by $F_z(t)/mg$ and $F_z(t + t_0)/mg$, we display the relationship between the variation of VGRF and the landing sequences of the foot in fig. 1.

Analyzing the spectrum in fig. 1, we notice, after determining each foot's variation of vertical ground reaction force, that $\frac{1}{Tmg}\int_0^T F_{GRF}(t)\,\mathrm{d}t - 1 = 0$ in each stride cycle. In this way, the change of t_0 leads to the change of the distribution of VGRF in a stride cycle. The distribution of F_{GRF} actually determines the walking movement. For example, when $t_0 = 0$, the walking will become jumping. In order to analyze the influence of foot land sequence to the distribution of VGRF resultant, we turn to the concept of deviation distribution. Using the deviation of VGRF resultant force, $\sqrt{\frac{\sum(F_z(t)/mg+F_z(t+t_0)/mg)^2-Tf}{Tf-1}}$ (f being the collection frequency of the measurement system), we develop the relationship between the landing sequence of one foot and its deviation of VGRF resultant force. Using the optimization method, we found that the action function $\Psi(t_0)$ $(= \sum_{t=1/f}^{T}(F_Z(t) + F_Z(t + t_0) - mg)^2)$, of the resultant of VGRF in a gait cycle reaches an optimum value around $\frac{1}{2}T$. As can be seen in fig. 2, this change is symmetric and has a minimal value about $\frac{1}{2}T$. Therefore we conclude that in gait, when the starting time of one foot stride cycle time falls right at the half of the other foot stride time, the deviation of VGRF is the minimal.

The movement of human segment is done by a combination of prime movers, antagonist, and synergists. The extension and bending of a segment's movement consume mechanical energy. Since in a stride cycle, $E_g = 0$ and $E_k = \frac{1}{Tf}\sum\frac{1}{2}mv_t^2 > 0$ thus we use E_k to describe the vertical mechanical energy (VME) consumption in gait. Figure 3 illustrates the effect the landing sequences of the foot impose upon the variation of vertical mechanical energy. It follows that while walking, one rearfoot touches the ground exactly at the half of the other foot's stride time and consecutive gaits consume the least VME. This is the so-called least-action principle in gait (LAPG).

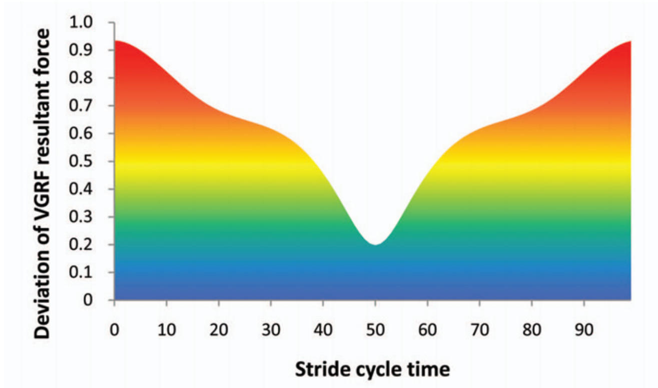

Fig. 2: (Colour on-line) Plot showing the deviation of the resultant vertical ground reaction force as a function of the stride cycle time. The stride time is rated as percentage by $(t/T) \times 100$.

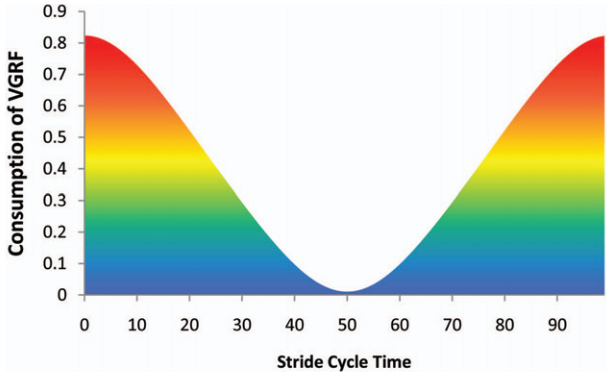

Fig. 3: (Colour on-line) Consumption of the vertical mechanical energy as a function of the stride cycle time.

Using the variation of VGRF to evaluate the consumption of VME in gait, we propose an energy consumption index of the following form:

$$ID_E = \frac{\sum_{i=1}^{n} T_i \int_0^T |F_z(t) + F_z(t + t_0) - mg|\mathrm{d}t}{T\int_0^{T_1+\cdots+T_n} |F_{GRF}(t) - mg|\mathrm{d}t}, \qquad (2)$$

where n is the number of stride cycles. For $n = 3$, the above definition reproduces Zebris FDM Gait Analysis System[2]. The advantage with the above definition is that one does not need to calculate vertical accelerated velocity, velocity and displacement of mass center.

Using the Zebris FDM-System Measurement System for Gait Analysis, we collect the data that best represent the usual normal gaits and meet the essential requirements of the test. We test the gait of one male adult with slight injury in his left ankle, a second male and one elderly

[2]Sensor area: 608 cm $\times$ 56 cm ($L \times W$); number of sensors: 34048 (1 sensor/cm^2, individually calibrated capacitive force sensors); force range 1–120 N/cm^2; sampling rate: 100 Hz; accuracy: ± 0.07.

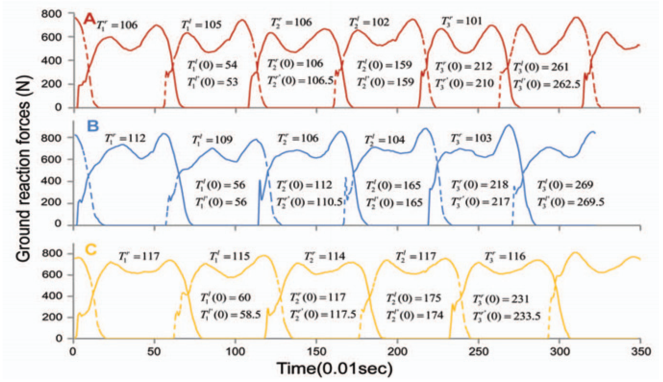

Fig. 4: (Colour on-line) Shown are the plots of (A) feet pressure of a slightly injured adult, (B) feet pressure of an adult and (C) feet pressure of an elderly with rheumatoid arthritis.

Table 1: Mean $\pm$ S.D age, height and body mass of the subjects.

Gender	Sample size	Age (years)	Height (m)	Body mass (Kgs)
Male	95	21.1 ± 1.31	1.72 ± 0.64	61.8 ± 8.3
Female	78	21.8 ± 1.3	1.61 ± 0.58	51.3 ± 7.6

male with rheumatic arthritis. The analysed results are displayed in fig. 4. We notice that when a fit adult walks, the starting time of one foot in a stride cycle will spontaneously fall right at the half of the other foot's stride cycle time. Applying the VME to evaluate the energy consumption in gait for these subjects, we obtain the estimates of 0.931, 0.998 and 0.743, respectively. A comparison with the earlier results [9] shows that the established evaluation index can provide better accountability to different gaits. In order to examine its universality, we enlarged our sample size to 173 subjects (95 male and 78 female students)[3]. The collected data are tabulated in table 1. The statistical analysis to the results for gait, using WinFDM, yield the stride times of 1.01(6) s and 1.00(6) s for the left and right foot, respectively. The ratio between the starting time calculated by LAPG and that of the tested result for the left and right foot is estimates as 0.998(21) and 0.993(21), respectively. The excellent agreement between the estimates confirms the signature of the universality of LAPG.

We normalized the subjects weight and rated the stride time by percentage. Figure 5 is the variations of male and female subjects VGRF, VME, center of mass velocity and position in a stride cycle, respectively. Figure 5 shows that when the sample size is big enough (like the one in our study), the variations of male and female VGRF, VME, center of mass velocity and position in a stride cycle are quite similar. Thus it confirms that the evaluation index in this study is valid to both male and female subjects.

[3]Each subject's medical history was inquired and subjects were screened for orthopaedic and neuromuscular disorders so as to make sure that their physical condition meets the requirements of the test.

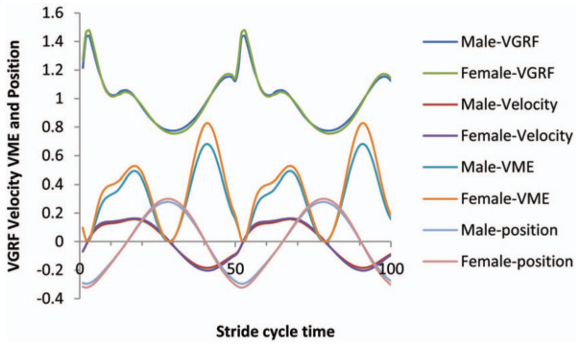

Fig. 5: (Colour on-line) Variations of VGRF, VME, center-of-mass velocity and position in a stride cycle. The unit of VGRF is a multiple of weight, that of velocity m/s. To make this figure more attractive, we multiplied VME values by 40, and center-of-mass position values by 20.

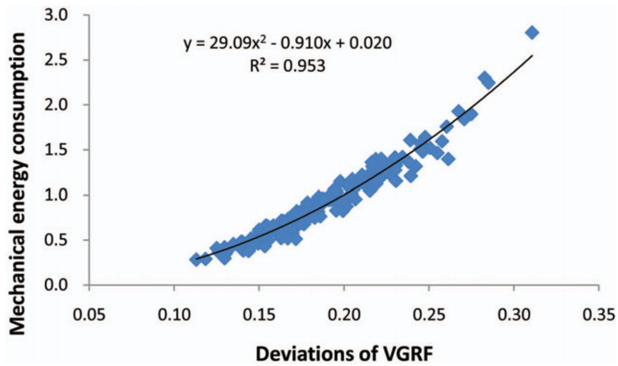

Fig. 6: (Colour on-line) Consumption of VME as a function of subject's deviation of VGRF. The solid curve is a quadratic fit to the data.

To examine the effect that the landing sequences of the foot exert upon both the deviation of VGRF resultant and upon the consumption of VME, we display our proposed relationship between the subject's deviation of VGRF and the consumption of VME in fig. 6. An analysis of correlation reveals that the two quantities are highly correlated ($R = 0.976$, $p < 0.001$), a least deviation of VGRF results in least consumption of VME.

In this study, we have developed a relationship between the deviation of VGRF and its consumption of VME and established a description of VGRF resultant force. From our analysis we conclude that when the deviation of VGRF in gait is least, its consumption of VME is also the least. We have also developed an evaluation index of mechanical-energy consumption based up the LAPG. Our results indicate the universality of least-action principle in gaits.

A normal adult's gait has nothing to do with their physiology (*e.g.* gender, age), body shape (*e.g.* height, weight) or their gait (*e.g.* cadence, velocity). One foot's stride starting time always begins at the half of the next foot's stride cycle time, which consumes the least VME. A regression equation of foot VGRF in a stride cycle has been set up. Representing the deviation of VGRF by the action function, we have discovered the LAPG by the optimization analysis method. This signature was confirmed by analyzing the consecutive gaits of 173 subjects. Our results suggest that the evolution of gait, in addition to its adaptation to the natural environment [16,21], is a consequence of following LAP. Human present physical condition uses the most energy-saving gait, even after a slight injury to the ankle. This could be considered as a human instinct. In sport rehabilitation, the therapy should be focused on the recovery of physical function. The uniqueness of each individual's gait is shaped when deviations of human-body inertial parameters, muscle strength, and motion coordination do exist, but they all follow LAPG. A research into the variations of natural gait (bare-footed) shear stress would enrich the study of LAPG. This study has convinced us that LAP has profoundly influenced the natural evolution, even the evolution of life. Lamarck's mechanism for the evolution of life use and disuse is an expression of LAP. A further research into LAPG will be significant to the studies such as sport rehabilitation, biometric identification techniques and the control of biped robots gaits [22,23].

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